Review Gr.9-10 = Unit #0 Journal

1.ⓐ FACTORING METHODS

COMMON - divide out the greatest common factor (G.C.F) out of all terms and place the GCF outside of the bracket.

ex. $\frac{16x^3y}{4x^2y} - \frac{4x^2y^2}{4x^2y} + \frac{20x^4y^3}{4x^2y}$ G.C.F = $4x^2y$

$= 4x^2y(4x - y + 5x^2y^2)$

* if negative is 1st pull it out.

DIFFERENCE of SQUARES

- you must have ① two terms ② one negative ③ even powers on variables only
- Create brackets (+)(−) and square root each term

ex. $144m^2 - 49n^2$
$= (12m + 7n)(12m - 7n)$

ex. $x^8 - 6$
$= (x^4 + \sqrt{6})(x^4 - \sqrt{6})$

TRINOMIAL FACTORING (Mrs.K's favorite method)

ex. $9x^2 - 24x + 16$

$1x$ $3x$
$9x$ $3x$

2 8 −4 16 1
8 2 −4 1 16
or two neg.

$= (3x-4)(3x-4)$

- Must have variable pattern as follows

first	middle	last
x^2	x	
x^2	xy	y^2
x^4	x^2y^3	y^6

etc

ex. $2x^2 + 3x - 20$

can drop variables in rough work

2
1

4 5 10 2 20 1
5 4 2 10 1 20
one neg.

$= (2x-5)(x+4)$

* even powers on 1st and last and ½ of each power in middle

- List factor combinations for 1st and last terms

Positive #	Negative #
+ +	− +
+ −	+ −

ex. $4x^6 - 27x^3y + 35y^2$

4 2
1 2

7 5 1 35
−5 7 35 1
or two neg

$= (4x^3 - 7)(x^3 - 5)$

- Criss-cross multiply
Is there a combination that adds to middle?

$a \quad b$
$c \quad d$

- Record $(a+b)(c+d)$
(top with top)(bottom with bottom)

* check work by expanding
* check if can factor more (remember common factor 1st to make #'s smaller)

1ⓑ EXPONENT RULES

$a^n \cdot a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^{n-m}$ * record answer in numerator!

$(a^nb^m)^k = a^{nk}b^{mk}$ * allowed to distribute for MONOMIALS only

$\frac{ab^{-n}c^m}{d^{-k}} = \frac{ac^md^k}{b^n}$ * move only the bases with negative exponents over the division line

$a^0 = 1$

ex. $\frac{(2x^0y^2)^{-3} \cdot 2yx^3}{(2x^{-3})^2}$

power of power

$= \frac{2^{-3}x^0y^{-6} \cdot 2^1yx^3}{2^2x^{-6}}$

mult/divide same bases can add/subtract exponents

$= 2^{-3+1-2}\, x^{0+3--6}\, y^{-6+1}$

$= 2^{-4}x^9y^{-5}$

negative exponent bases drop "downstairs"

$= \frac{x^9}{2^4y^5}$

$= \frac{x^9}{16y^5}$

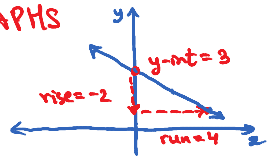
2 LINEAR RELATIONS

EQUATION with a slope $y = mx + b$ (m: slope, b: y-int $(0,b)$)

horizontal $y = \#$

vertical $x = \#$

GRAPHS



$y = mx + b$

$y = -\frac{2}{4}x + 3$

$y = -\frac{1}{2}x + 3$

$$\boxed{m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}}$$

WORD PROBLEMS

Look for
- initial value or flat rate number $= y\text{-int} = b$
- rate of change number $=$ slope $= m$

ex. Pizza costs \$6.00 plus \$0.50 per topping.

flat $= b$ rate per topping $= m$

$y = mx + b$

let $C =$ cost
$t = \#$ of toppings

$\therefore C(t) = 0.50t + 6.00$

SYSTEMS: SUBSTITUTION METHOD

- isolate one variable in one equation
- sub in other equation
- expand + solve for one variable
- sub in again + solve for other variable

ex. $x + 7y = 0$ ①
$2x - 8y = 22$ ②

①: $x = -7y$ sub in ②

②: $2(-7y) - 8y = 22$
$-14y - 8y = 22$
$-22y = 22$
$y = -1$

sub in ① $x = -7(-1)$
$x = 7$

$\therefore$ POI $(7, -1)$

ELIMINATION METHOD

- align like variables over top of each other (equals sign too)
- multiply the whole equation(s) to create the same coefficient on one variable
- add / subtract to eliminate that variable
- solve for the remaining variable + proceed as before

ex. ① $-17 - 5y - 11x = 0$
② $-15 = 9x + 4y$
① $-17 = 11x + 5y$

②×5: $-75 = 45x + 20y$
①×4: $-68 = 44x + 20y$

subtract $-7 = x$

sub in ② $-15 = 9(-7) + 4y$
$-15 = -63 + 4y$
$48 = 4y$
$12 = y$

$\therefore$ POI is $(-7, 12)$

3. QUADRATIC RELATIONS

a) EQUATIONS

Standard $y = ax^2 + bx + c$

direction of opening (a) — y-int $(0,c)$

Factored $y = a(x-r)(x-t)$

dir. (a) — x-int or zeros $(r,0)$ and $(t,0)$

* think, what # for x makes whole bracket zero

ex. $y = (2x-3)(4x+8)$

zeros $(\frac{3}{2},0)$ and $(-2,0)$

Vertex $y = a(x-h)^2 + k$

dir (a) — vertex (h,k)

* this form also tells you transformations

$|a|>1$ vertical stretch ex. $a=-2$ or $a=2$

$|a|<1$ vertical compression ex. $a=0.3$ or $a=-0.3$

$h>0$ right, $h<0$ left (switch sign!!)

$k>0$ up, $k<0$ down

b) QUADRATIC FORMULA

used to find zeros of standard form of quadratic must have equals to zero $0 = ax^2+bx+c$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Discriminant

if $b^2-4ac > 0$ two roots

< 0 no roots

$= 0$ one root

* resort to this formula only if you can't factor to see zeros.

What makes this problem dificult?

- You need to remember the steps and be able to work with fractions
- There are many steps and a small mistake can lead to a wrong answer
- You must understand the steps + how to factor instead of memorizing the steps

c) COMPLETING the SQUARE

- used to convert standard form to vertex form (when asked for tranformations, inverse or MAX/MIN)

ex. $y = -3x^2 + 4x + 1$

$y = -3(x^2 - \frac{4}{3}x \quad) + 1$

• divide out "a" from first two terms (not common factor x stays!)

• find special # $\left(\frac{b}{2}\right)^2$

$\left(-\frac{4}{3} \div 2\right)^2 = \left(-\frac{4}{3} \times \frac{1}{2}\right)^2 = \left(-\frac{4}{6}\right)^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

• add and subtract this # in the bracket

$y = -3\left(x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}\right) + 1$

• move last term outside of bracket by multiplying by "a"

$y = -3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - \frac{4}{9}(-3) + 1$

will always be # before you square

• factor trinomial left + simplify

$y = -3\left(x - \frac{2}{3}\right)^2 + \frac{7}{3}$

opens ↓

$\therefore$ vertex $\left(\frac{2}{3}, \frac{7}{3}\right)$

$\therefore$ MAX val $= \frac{7}{3}$ at $x = \frac{2}{3}$ axis of symmetry

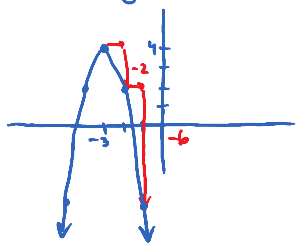
d) GRAPHS

- use of vertex and step pattern is the fastest way of graphing <u>vertex form</u>

ex. $y = -2(x+3)^2 + 4$ vertex $(-3, 4)$

step $(1, 3, 5) \times a$
$= -2, -6, -10$

think of this as $\frac{\text{rise}}{\text{run}}$



- use zeros and vertex for <u>factored form</u>

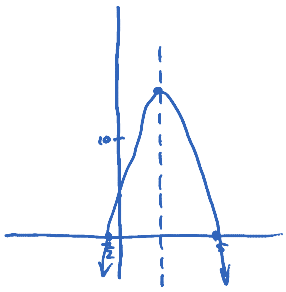
ex. $y = -(x-5)(2x+1)$

zeros $(5, 0)$ and $(-\frac{1}{2}, 0)$

$\boxed{a.o.f.s = \frac{\text{add zeros}}{2}}$ $\boxed{\text{opt. val} = \text{sub a.o.f.s into equation}}$

$a.o.f.s = \frac{5 + -0.5}{2} = 2.25$

$\text{opt. val} = -(2.25 - 5)(2(2.25) + 1)$
$= -(-2.75)(5.5)$
$= 15.125$ $\therefore$ vertex $(2.25, 15.125)$



4. <u>TRIGONOMETRY</u>

a) SOH CAH TOA *only used on Right Δ

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

ex. <u>solve for side</u>

$\cos 30° = \frac{7}{x}$ • CAH

$x \cos 30 = 7$ • cross multiply

$x = \frac{7}{\cos 30°}$ • solve

$x \doteq 8.1$ cm

make sure you're in DEGREE mode!!

ex. <u>solve for angle</u>

$\sin\theta = \frac{5}{8}$ • SOH

$\theta = \sin^{-1}\left(\frac{5}{8}\right)$ • inverse sine

$\theta \doteq 39°$

b) Pythagorean Th *only used on Right Δ

$\boxed{a^2 + b^2 = c^2}$ hypotenuse

ex.

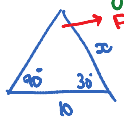
$7^2 + x^2 = 10^2$
$49 + x^2 = 100$
$x^2 = 100 - 49$
$x^2 = 51$
$x = \pm\sqrt{51}$
$x \doteq 7.1$
side length always positive

c) Sine Law * used on ANY Δ as long as you've given a pair of opposite side + angle

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

can flip the formula :)

ex.



Find 3rd angle so you can have a pair of opposites

$180° - 70° - 30° = 80°$

$$\frac{10}{\sin 80°} = \frac{x}{\sin 70°}$$

$$\frac{10 \sin 70°}{\sin 80°} = x$$

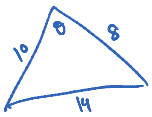
$$9.5 \doteq x$$

d) Cosine Law * used on ANY Δ as long as you've given SSS or SAS

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

$$\boxed{a^2 = b^2 + c^2 - 2bc \cos A}$$

ex.



$$\cos\theta = \frac{10^2 + 8^2 - 14^2}{2(10)(8)}$$

$$\cos\theta = \frac{-32}{160}$$

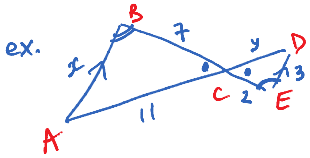
$$\theta = \cos^{-1}\left(\frac{-32}{160}\right)$$

$$\theta \doteq 102°$$

e) Similar Triangles

- same shape/angles
- different size

ex.



order matters

$\Delta ABC \sim \Delta DEC$

$$\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$$

set up proportion statement

$$\frac{x}{3} = \frac{7}{2} = \frac{11}{y}$$

cross multiply to solve.

$2x = 21$ $\quad$ $7y = 22$

$x = \frac{21}{2}$ $\quad$ $y = \frac{22}{7}$