 11U Date:____________ Name: ________________________

Functions Unit 1

Tentative TEST date_______________

Big idea/Learning Goals

This unit introduces you to several new concepts like: what is a function, how to use function notation, domain and range, as well as how to find an inverse of a function. All of these concepts are building the foundation for the grade 12 advanced functions course. The key characteristics you will study here will enable you to learn about more complex type functions in the next units of grade 11: quadratic, exponential, trigonometric, and discrete functions.

Corrections for the textbook answers:

Sec 1.4 #17 d) Range $20\sqrt{2} \le y \le 40$

Sec 1.8 #16 $y = 3\sqrt{-(x-5)} - 2$

Sec 1.8 #22 $-\frac{1}{4}(x+6)^2 + 2$

Success Criteria

Specific questions will not be assigned, since it will depend on your knowledge and skill (everyone is at a different level). The goal is to do all types of questions quickly and without reference to notes or back of textbook or another individual. BUT you may not have time to do every single question available… so… If you are a strong student you may just concentrate on harder TIPS or APP questions, while if you are a weak student you may want to use all your time practicing the basic KU or COMM questions. The number of questions done should also be proportional to your mark so far. If you have very low scores, more practice is required.

Date	pg	Topics	**# of quest. done?** You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-3	Relations and Functions Section 1.1		
	4-6	Function Notation Section 1.2 & Handout		
	7-8	Domain and Range Section 1.4 & two Handouts		
	9-10	Transformations Review with Quadratics Handout		
	11-13	Parent/Basic Functions Section 1.3		
	14-15	Transformations of New Functions Section 1.6 – 1.8 & three Handouts		
	16-18	Inverses Section 1.5 & Handout		
		REVIEW		

Reflect – DIAGNOSTIC TEST mark ___________.

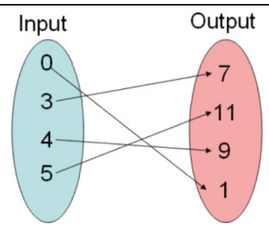
Relations and Functions

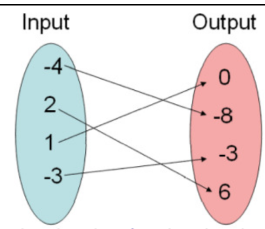
1. Mathematics is a study of relationships. These relationships when written with variables are called **relations**. Sometimes a problem may relate variables that have interdependence. We usually call ____ variable as independent and the ____ variable the dependent. The independent variable can also be called the ______________ and dependent variable the _______________. This terminology is often used when the relations are **functions.**

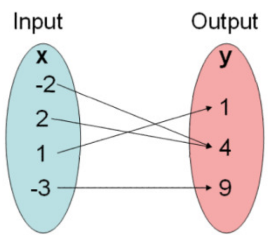
2. What is a function? Look at the 'yes' and 'no' examples to determine what makes something a function or not.

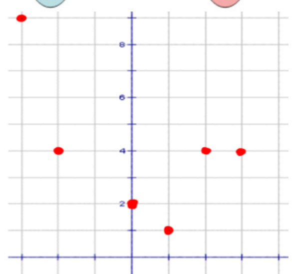
YES – these are functions	NO – these are not functions

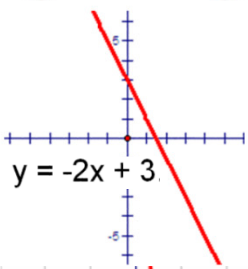
YES – these are functions

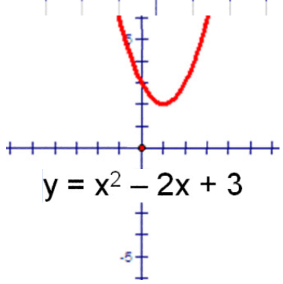


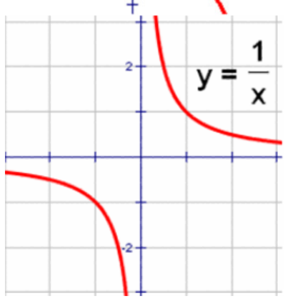


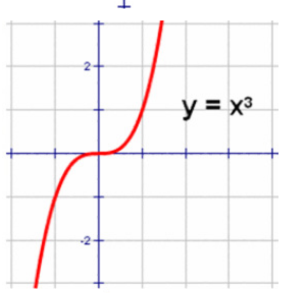


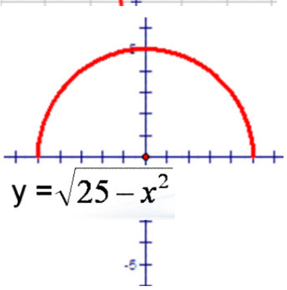


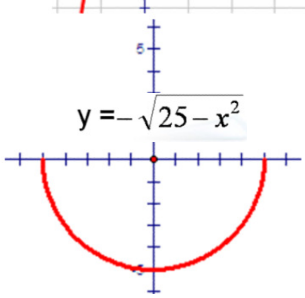




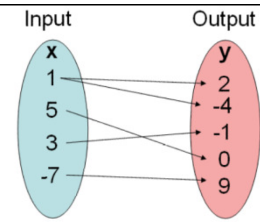


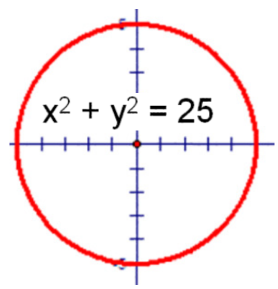


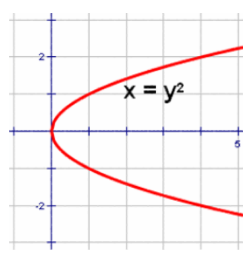


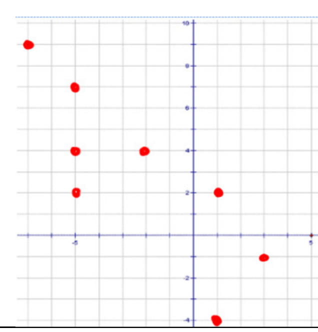


NO – these are not functions







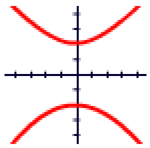


3. What is a function?

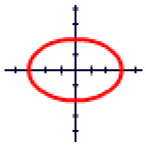
4. How does the **vertical line test** help you determine whether something is a function or not?

5. Determine if each of the following are functions or not.

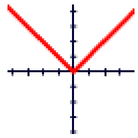
a.



b.



c



d. $\{(2,5),(3,5),(4,10),(5,0)\}$

e. $\{(-3,9),(-2,8),(-1,2),(-2,-4)\}$

f. $y = 3-6x$

g. $y = x^3 - 2x^2 + 1$

h. $x^2 + 2xy + y^2 = 8$

i. $y = x^{-1}$

j. $x - 6 = y^4 + x^2$

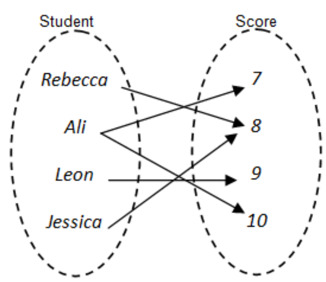
k. Sara asked each of her extended family members to measure his/her foot length. Then she graphed the relationship between foot length and age, using the age as an independent variable.

l. The input is the street address; the output is the postal code.

m. The input is the postal code, the output is the street address.

n. A vending machine produces pop, gum, chocolate bars, etc. depending on the button pressed.

o. A relation is shown in the arrow diagram. The input is a student's name and the output is their score out of 10 on a math quiz.



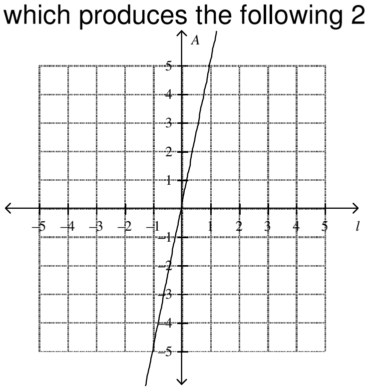
6. True or False? "All functions are relations." Explain if true and give a counterexample if false.

7. True of False? "All relations are functions." Explain if true and give a counterexample if false.

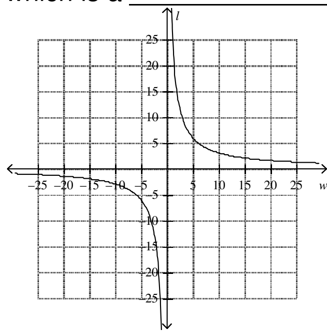
Function Notation

1. Formulas use several variables to describe a relationship. For example $A = lw$ has three variables. If this was graphed as a relationship between variables, it will produce a three dimensional graph – these are studied at universities in the 2nd year. For grade 11, we only look at the relationship between TWO variables.

Let's say width is given $w = 5$ then $A = 5l$ is a ______________ relationship, which produces the following 2D graph:



Let's say area was given $A = 30$ then $30 = lw$ or $l = \frac{30}{w}$ which is a ________________ relationship, with a graph:



2. Let's say you were not given the value of the constant at all. But you were told length is a constant, then $A = lw$ is a ___________________ relationship and the input is _____ and the output is _____

In the above question, the way the formula is written $A = lw$, it is not clear what is the input and what is the output unless you explicitly write it . Math is all about shorthand, so function notation was invented.

3. What is **function notation**?

4. You have seen function notation for specific functions, but you just didn't know it. Indicate what is the name of the function, the input and the output of each of the following
   a. $\cos(\theta)$.
   b. $\sin^{-1}(0.5)$
   c. $\sqrt[3]{(9)}$
   d. $(2x)^2$

5. Explain why it is incorrect to write the following, give corrected versions.
   a. $\cos = \frac{1}{2}$
   b. $\tan x = 0.5234$
      $x = 0.5234 \tan^{-1}$

6. When can function notation be used? Explain by using this example: "Pythagorean theorem with one side as output and hypotenuse as input."

7. What could be confusing when using function notation? Explain by using this example:

Evaluate $m(3)-n$

a) if m = 2 and n = 6

b) if $m(x) = x^2$ and n = 6

8. Find the following for the functions $f(x)=2x^4-3x^2$, $h(a)=\sqrt{a-5}$, $g(k)=\dfrac{6k}{k+1}$

a. $f(2)$

b. $h(69)$

c. $f(\sqrt{5})-h(30)$

d. $f(-3)+4g(3)$

e. $2g\left(\dfrac{1}{2}\right)f(-1)$

f. $g(3)^{h(6)}$

9. Isolate the output variable and record the answer in function notation.

a. $2x-6y=18$

b. $3x=\dfrac{y+1}{y-2}$

c. $4+\sqrt{2y-3x}=10$

10. Write down the formula, in function notation for the perimeter of a rectangle, considering perimeter as input and length as output.

11. Write down the formula, in function notation for the area of the triangle, considering area as input and height as output.

12. Write down the area of a circle as a function of its perimeter.

Bring colouring pencils to next class

Name: ________________________

Domain & Range

1. It is important to identify characteristics of functions in order to distinguish them from each other. Two of the characteristics you will study is

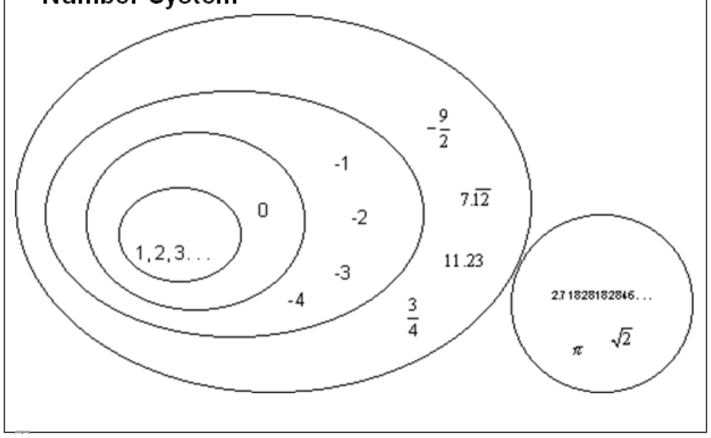
   Domain ____

   and **Range** ____

   In order to correctly write the domain and range you must also learn to record solutions in **set notation**.

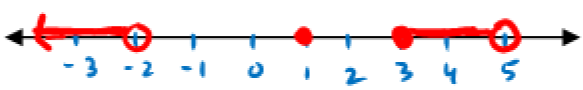
2. Quick recap of symbols:

Number System

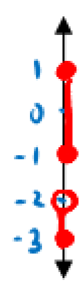


3. Use set notation to write the following solution sets:

   a.



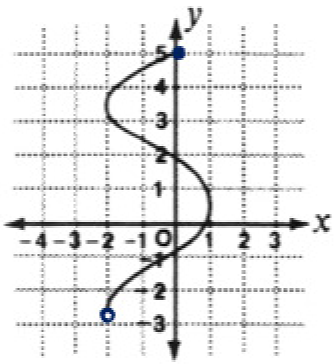
   b.



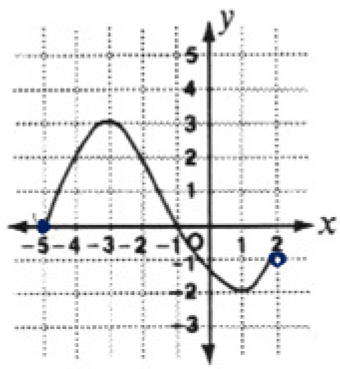
4. Explain why it is improper to record the following:
   $6 < x < -2$

5. Find the domain and range of the following relations, also identify if each is a function or not.

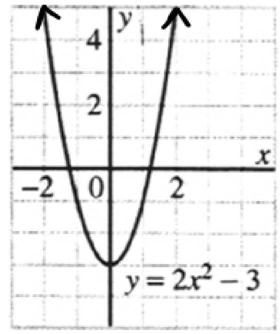
a.



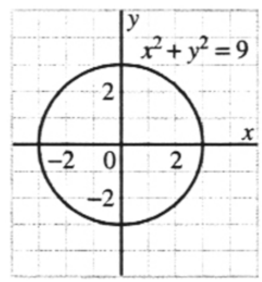
b.



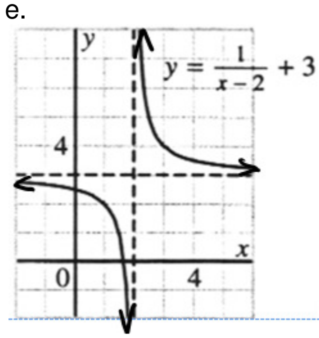
c.



d.



e.



6. Domain can be found without graphing. (Range is hard to find without graphing). What are two things that can make a relation undefined? How does that help you find domains of relationships without doing a graph?

7. Without graphing, find the domain of each of the following.

a. $y=\dfrac{x+5}{x^2+3x+2}$

b. $y=3x^2-8x+1$

c. $q=\sqrt{p-1}+5$

d. $c=\dfrac{2}{b^3-4b}$

e. $y=\sqrt{4-2x}-9$

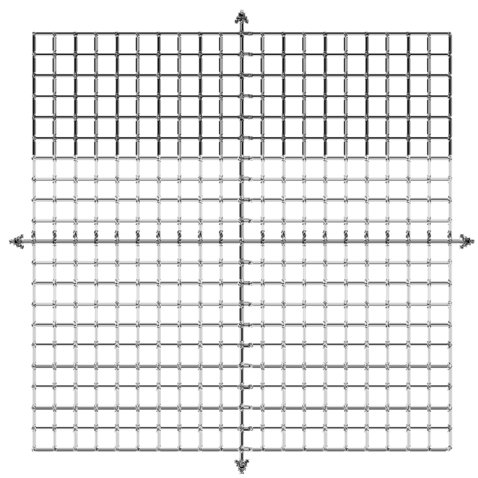
f. $y=\dfrac{1}{\sqrt{x-4}}$

Transformations REVIEW of Quadratics

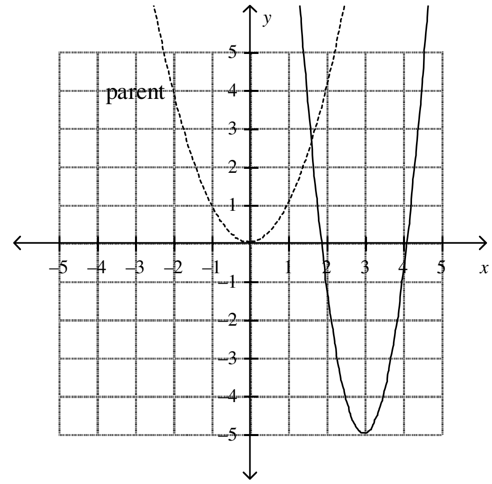
1. Vertex form of a quadratic tells you more than just the vertex. What do the constants represent in terms transfomations for $y = a(x-h)^2 + k$?

2. State the transformations of $y = -0.5(x+6)^2 + 8$ in the order that you would apply them.

3. Sketch $y = -0.5(x+6)^2 + 8$ showing all steps using different colours.



If the quadratic is written in vertex form but x has a coefficient on it, like $y=(6-2x)^2-5$, it can be rewritten in two ways.



Method 1: Factor out the coefficient and keep it in the square function.
Method 2: Factor out the coefficient and pull it out of the square function completely:

$y=(6-2x)^2-5$	$y=(6-2x)^2-5$
$y=(-2x+6)^2-5$	$y=(-2x+6)^2-5$
$y=(-2(x-3))^2-5$	$y=(-2(x-3))^2-5$
	$y=(-2)^2(x-3)^2-5$
	$y=4(x-3)^2-5$

Both equations will yeild the same graph as shown

Method 2 gives a result that you know how to deal with since it is in the form $y=a(x-h)^2+k$. However when you study other types of functions, you might not be able to pull out the coefficient out of the function. So look at method 1 and think about what could the coefficient on x inside the square function could mean?

4. You will now study a new transformations form: $y=a(k(x-d))^2+c$. What do you think the constants represent here?

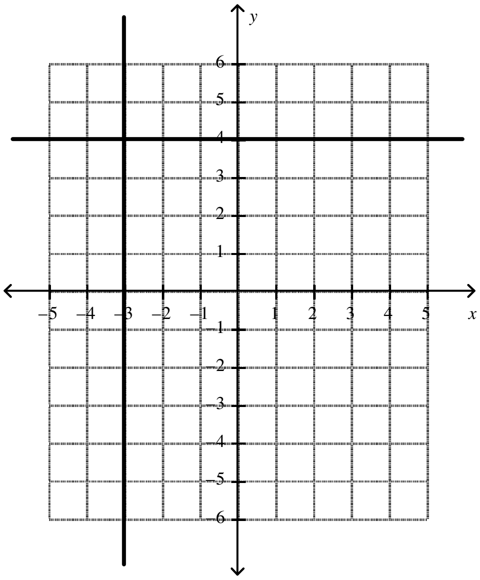
Bring colouring pencils to next class

Name: ________________________

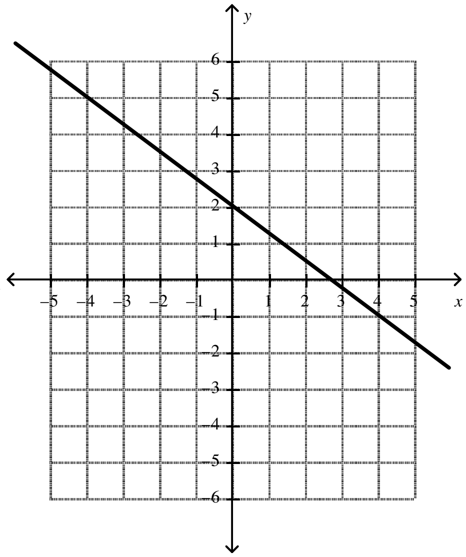
Parent/Basic Functions and their Properties

1. This course is a preparation course for the Advanced Functions course in grade 12. You will learn several new functions. Before we get into those, review the relations you already know. For the following, state the equations of the graphs drawn.

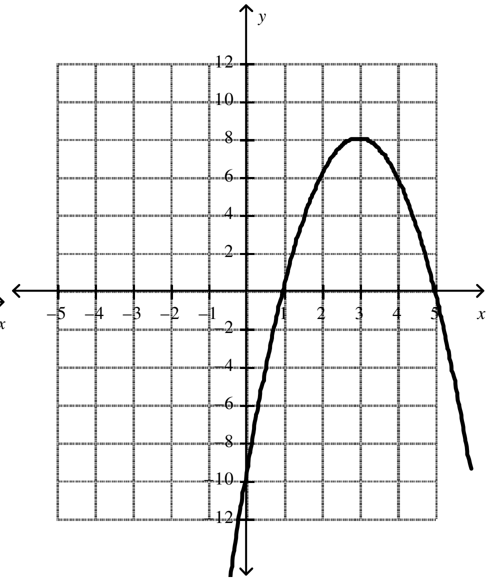
a. & b.



c.

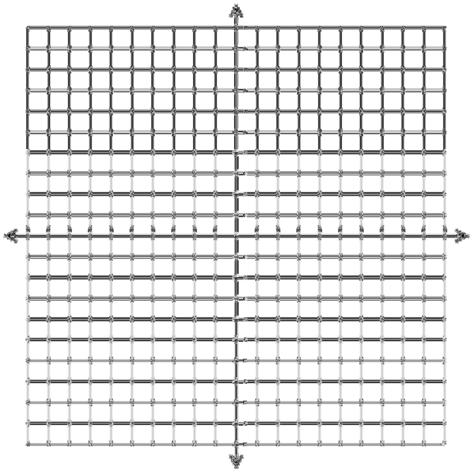


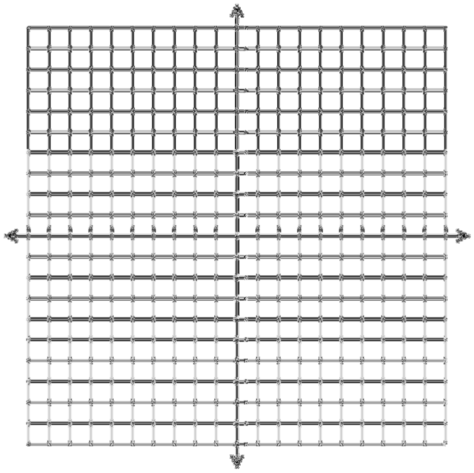
d.



There are many lines and parabolas possible. However each set belongs to a particular **family** or type. For each family of functions, there is a **parent** or a basic function.

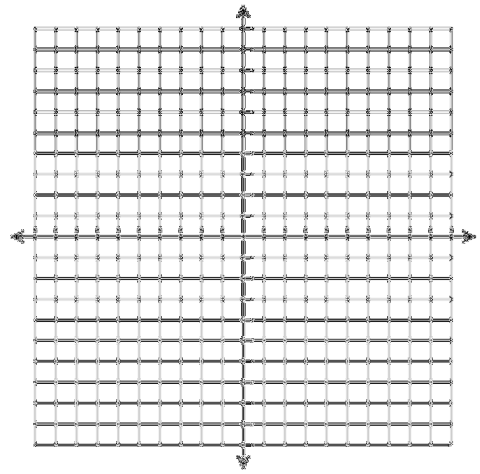
2. For the functions above, write the **parent** equations for linear and quadratic functions, create a table of values and a sketch. State the general equations for linear and quadratic functions and then state the domain and range for these general equations.



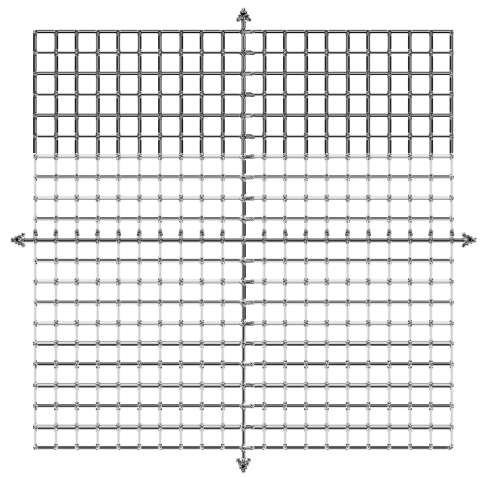


3. Develop the table of values and a sketch for each of the following new parent functions. State the transformed version of the equation and then state the domain and range for the transformed equation.

   a. Square Root Function $y = \sqrt{x}$



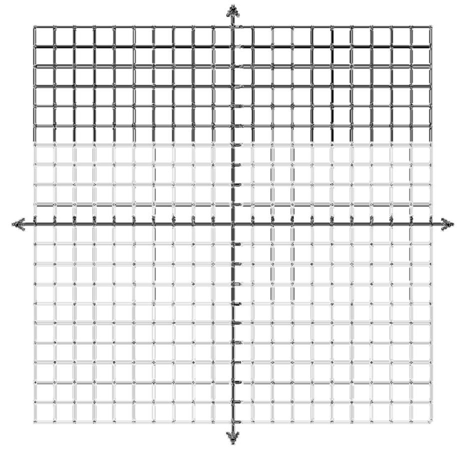
   b. Rational Function $y = \frac{1}{x}$



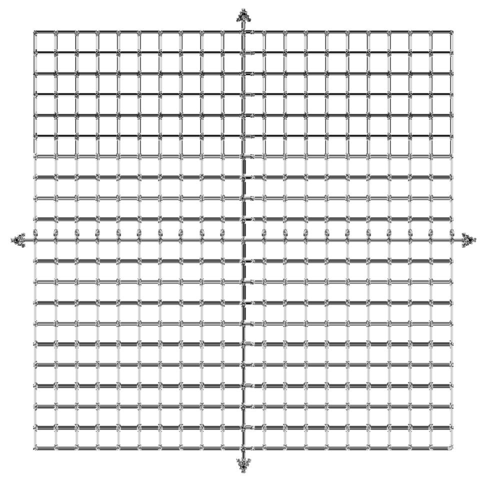
4. In the last question, lines $x = 0$ and $y = 0$ are called **asymptotes** for that function. Explain, what do you think are asymptotes?

5. Show a table of values, a sketch, a transformed version of the equation, and domain & range

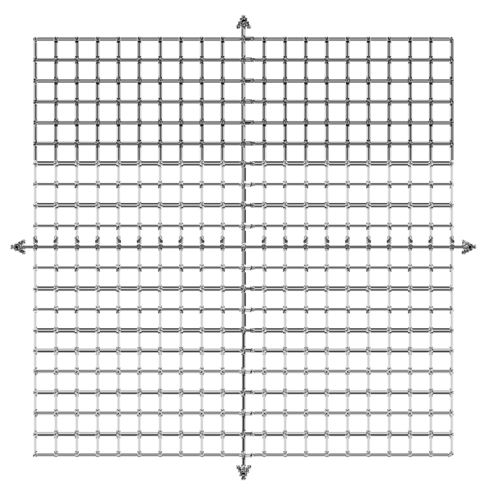
   a. Cubic Function $y = x^3$



   b. Cube Root Function $y = \sqrt[3]{x}$



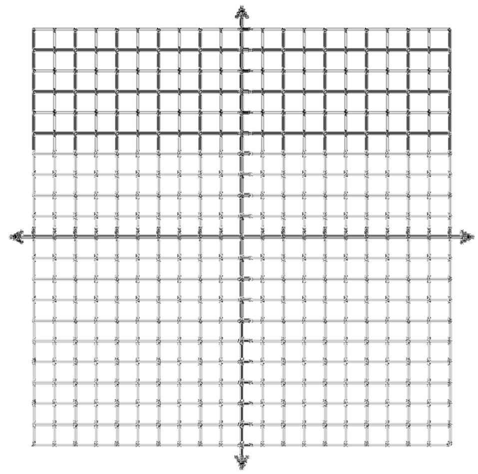
   c. Absolute Value Function $y = |x|$



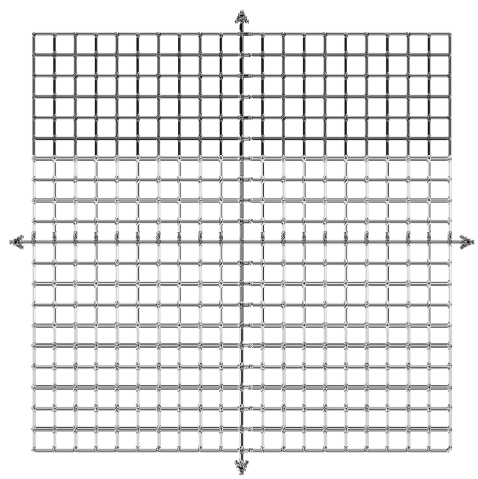
Transformations of New Functions

Identify the parent function, the transformations that were applied, domain&range, and sketch showing all steps using different colours.

1. $y = -3\sqrt{0.5x+2} - 5$



2. $y = \dfrac{5}{0.25x+1}$ Now try the short cut



3. Once you understand how to apply the transformations in the correct order, there are some shortcuts you can take. Summarize the steps to sketch the final function without sketching all the intermediate functions.

4. Write the equation given the parent function and the transformations. Sketch using the shortcut.

   a. $f(x)=|x|$, reflected in x-axis, horizontal compression by 2, shift up by 5

   b. $g(x)=x^3$, reflected in y-axis, horizontal stretch by 3, vertical compression by 2, shift left 4.

5. What are the transformations of $y=-3x^2+6x$

Inverses

1. You have already seen inveses in action. Identify the inverse operations that would be used to solve the following equations.

   a. $\cos\theta = \dfrac{4}{7}$

   b. $100 = \pi r^2$

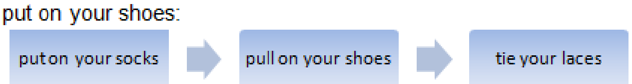
2. Before you go on, clarify the differences between the words that sometimes get confused.

OPPOSITE RECIPROCAL INVERSE

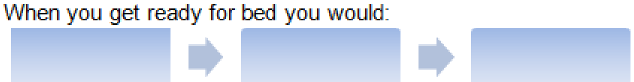
3. When finding inverses, it is crucial to remember to undo operations in BEDMAS backwards order. Look at this real life example to understand the importance of order. Then apply it in the mathematical example about temperature. Find the inverse for each of the following.

   a.

   When you get ready for school, you probably follow these steps to put on your shoes:



   When you get ready for bed you would:

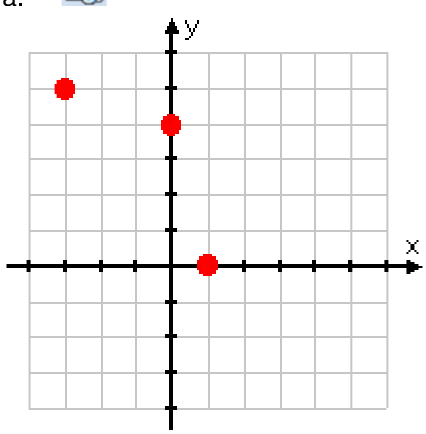


   b.

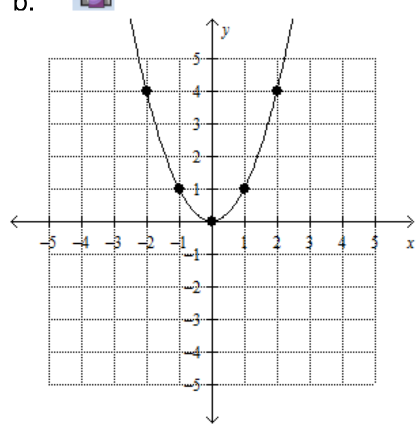
   American visitors to Canada would use this formula to convert Celsius to Fahrenheit: $F = \frac{5}{9}C + 32$. What would Canadians use to convert back?

4. Notice in both examples the input became the output and vise versa. Sketch the inverse of the following graphs

   a.



   b.



5. What type of graph will remain the same when input and output get switched? Why?

6. The inverse may or may not be a function. What kind of test can you do to ensure that both the graph and its inverse are functions?

7. What do you think will be the notation for the inverse function? Just be careful, what can it be confused with?

8. What do you think is the relationship between domain and range for the relation and its inverse?

9. What do you think will be the steps to find the inverse algebraically so that function notation can be used?

10. Demonstrate the steps you've outlined on the following $f(x) = 3(x+5)^2 + 2$

11. Find the inverse for each of the following

a. $g(x) = \dfrac{1-x}{2}$

b. $h(x) = -2(15-5x)^3 + 16$

Inverses can help you find range algebraically:

12. You have seen how to find domain of a function without graphing. Remind yourself how to find domain without graphing.

13. To find range without graphing there are TWO possibilities. What are they?

14. Find the domain and range of the function, (use inverses to help you, if neccesary).

 a. $f(x) = -2|3x-6|+7$

 b. $g(x) = \dfrac{-2x}{6-3x}+4$

15. Domain of $j(x)$ is $-4 \le x \le 6$ and the range is $0 \le y \le 10$. A new function $k(x)$ is formed when transformations are applied as follows: $k(x) = -2j(3x+6)-4$, what is the range of $k^{-1}(x)$?