S14FunctionsNOTESnew

June 21, 2014 12:14 PM

FunctionsNOTESnew

see below

Functions Unit 1

Tentative TEST date_______________

Big idea/Learning Goals

This unit introduces you to several new concepts like: what is a function, how to use function notation, domain and range, as well as how to find an inverse of a function. All of these concepts are building the foundation for the grade 12 advanced functions course. The key characteristics you will study here will enable you to learn about more complex type functions in the next units of grade 11: quadratic, exponential, trigonometric, and discrete functions.

Corrections for the textbook answers:

Sec 1.4 #17 d) Range $20\sqrt{2} \le y \le 40$

Sec 1.8 #16 $y = 3\sqrt{-(x-5)} - 2$

Sec 1.8 #22 $-\frac{1}{4}(x+6)^2 + 2$

Success Criteria

Specific questions will not be assigned, since it will depend on your knowledge and skill (everyone is at a different level). The goal is to do all types of questions quickly and without reference to notes or back of textbook or another individual. BUT you may not have time to do every single question available… so… If you are a strong student you may just concentrate on harder TIPS or APP questions, while if you are a weak student you may want to use all your time practicing the basic KU or COMM questions. The number of questions done should also be proportional to your mark so far. If you have very low scores, more practice is required.

Date	pg	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
Day 1	2-3	Relations and Functions Section 1.1 #1, 2, 7, 11		
	4-6	Function Notation Section 1.2 & Handout Handout 1, 6, 7, 9, 11, 15, 16	text 1.2 #2, 3a, 12a	
Day 2	7-8	Domain and Range Section 1.4 & two Handouts 2, 8, 9, 10, 13		
		Extra Day for above?		
	9-10	Transformations Review with Quadratics Handout all		
	11-13	Parent/Basic Functions Section 1.3		
	14-15	Transformations of New Functions Section 1.6 – 1.8 & three Handouts #6-10		
Day 3	16-18	Inverses Section 1.5 & Handout all		
		REVIEW		

Reflect – DIAGNOSTIC TEST mark __________.

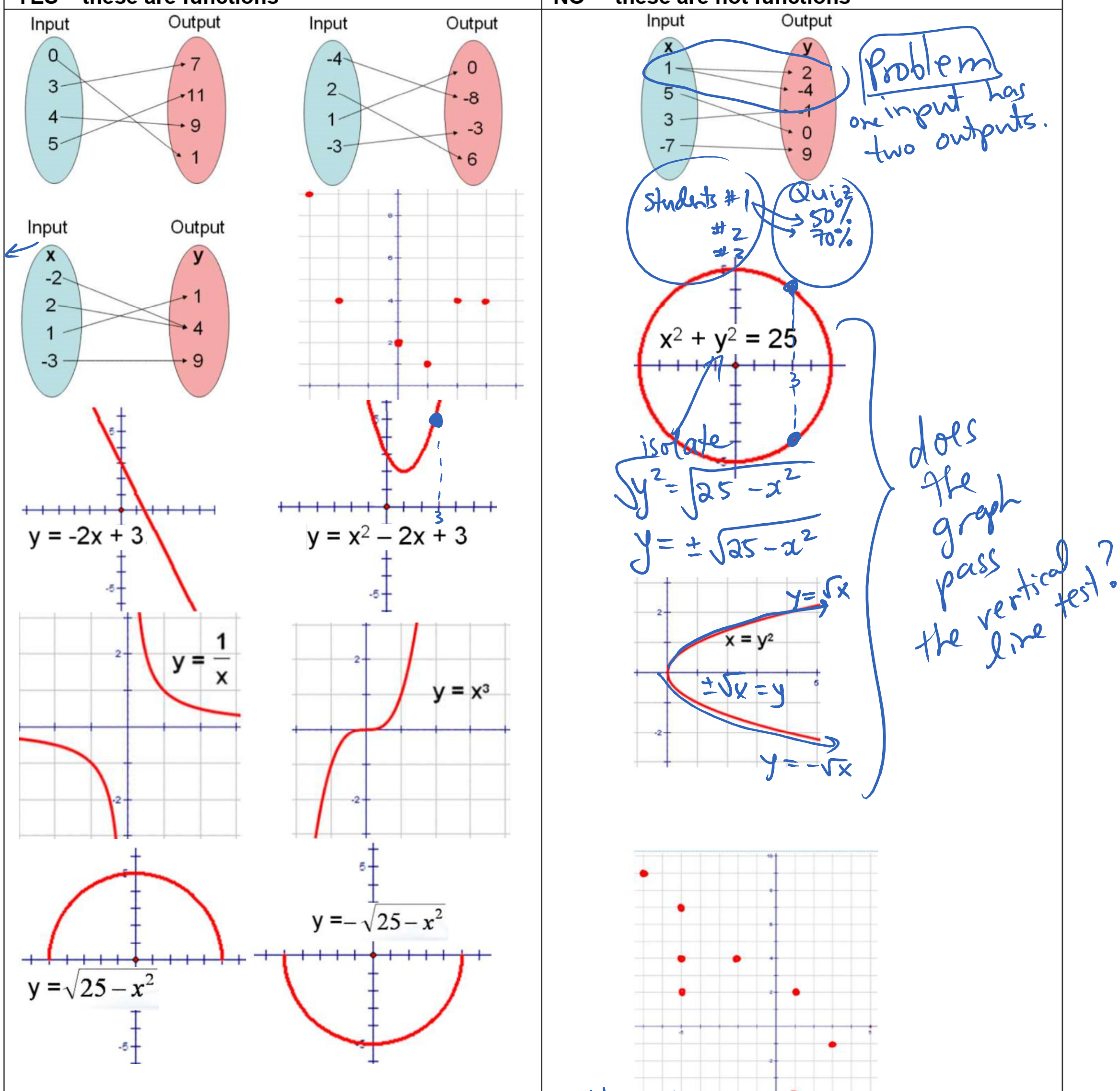
Relations and Functions

1. Mathematics is a study of relationships. These relationships when written with variables are called **relations**. Sometimes a problem may relate variables that have interdependence. We usually call _x_ variable as independent and the _y_ variable the dependent. The independent variable can also be called the _input_ and dependent variable the _output_. This terminology is often used when the relations are **functions.**

2. What is a function? Look at the 'yes' and 'no' examples to determine what makes something a function or not.

x	y
-2	4
2	4
1	1
-3	9

YES – these are functions | **NO – these are not functions**

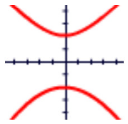


3. What is a function? A function is a relation that has only one output (y) for every input (x). Equations of functions should not have an even power on the y (since isolating y^2 gives $+\sqrt{\ }$ and $-\sqrt{\ }$)

4. How does the **vertical line test** help you determine whether something is a function or not?
If the graph touches any vertical line more than once anywhere, the graph is not a function.

5. Determine if each of the following are functions or not.

a.

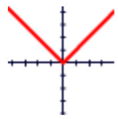


Not a function.

b.

NO

c

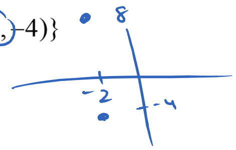


YES

d. $\{(2,5),(3,5),(4,10),(5,0)\}$

YES

e. $\{(-3,9),(-2,8),(-1,2),(-2,-4)\}$



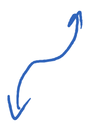
NO

f. $y' = 3-6x$ odd

line

YES

g. $y = x^3 - 2x^2 + 1$



YES

h. $x^2 + 2xy + y^2 = 8$

NO

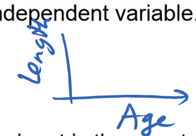
i. $y = x^{-1}$

YES

j. $x - 6 = y^4 + x^2$

NO

k. Sara asked each of her extended family members to measure his/her foot length. Then she graphed the relationship between foot length and age, using the age as an independent variable.



NO since age 15 → length #1, length #2

l. The input is the street address; the output is the postal code.

YES address → code

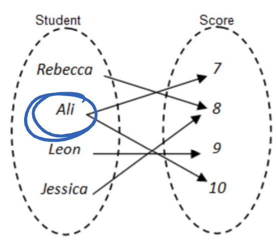
m. The input is the postal code, the output is the street address.

NO code → apt #1, apt #2

n. A vending machine produces pop, gum, chocolate bars, etc. depending on the button pressed.

YES button → item

o. A relation is shown in the arrow diagram. The input is a student's name and the output is their score out of 10 on a math quiz.



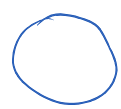
NO

6. True or False? "All functions are relations." Explain if true and give a counterexample if false.

T. all functions are still a relationship between variable.

7. True of False? "All relations are functions." Explain if true and give a counterexample if false.

F ex.



$x^2+y^2=25$

Like: All integers are real #'s vs. All real #'s are integers

T

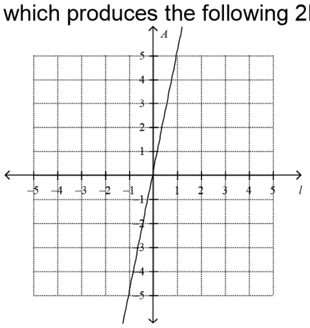
F ex. π, $\sqrt{2}$, $5/4$

Function Notation

1. Formulas use several variables to describe a relationship. For example $A = lw$ has three variables. If this was graphed as a relationship between variables, it will produce a three dimensional graph – these are studied at universities in the 2nd year. For grade 11, we only look at the relationship between TWO variables.

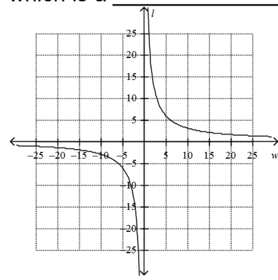
$y = 5x$

Let's say width is given $w = 5$ then $A = 5l$ is a linear relationship, which produces the following 2D graph:



Let's say area was given $A = 30$ then $30 = lw$ or $l = \frac{30}{w}$ → $y = \frac{30}{x}$ which is a rational relationship, with a graph:

parent $\boxed{y = \frac{1}{x}}$



2. Let's say you were not given the value of the constant at all. But you were told length is a constant, then $A = lw$ is a linear relationship and the input is w and the output is A

$A = \#w$

In the above question, the way the formula is written $A = lw$, it is not clear what is the input and what is the output unless you explicitly write it . Math is all about shorthand, so function notation was invented.

3. What is **function notation**? ☆ Brackets here, do NOT mean to multiply

Replace output y with $\underbrace{\boxed{\text{name}(\text{input})}}_{\text{output}}$ ex. $f(x) = y$
$h(t) = h$
$d(t) = d$

4. You have seen function notation for specific functions, but you just didn't know it. Indicate what is the name of the function, the input and the output of each of the following

a. $\cos(\theta)$. → name is cosine → input is θ

b. $\sin^{-1}(0.5)$ → name is sine inverse
arcsin(0.5) → input is 0.5 → output is $\sin^{-1}(0.5) = 30°$

c. $\sqrt[3]{(9)}$ → name cube roote → input is 9 → output is $\sqrt[3]{9} \doteq 2.08....$

d. $((2x))^2$ → name is squared → input is $2x$ → output is $(2x)^2 = 4x^2$

5. Explain why it is incorrect to write the following, give corrected versions.

a. $\cos = \frac{1}{2}$ θ missing input.

b. $\tan x = 0.5234$ ✓
$x = 0.5234 \tan^{-1}$ ✗
$x = \tan^{-1}(0.5234)$
$\tan^{-1} 0.5234$

6. When can function notation be used? Explain by using this example: "Pythagorean theorem with one side as output and hypotenuse as input."

$a^2 + b^2 = c^2$

output (a), input (c)

to use function notation → isolate output
→ replace it with function notation.

$\sqrt{a^2} = \pm\sqrt{c^2 - b^2}$

$a = +\sqrt{c^2 - b^2}$ or $a = -\sqrt{c^2-b^2}$

name(input)

$a(c) = \sqrt{c^2 - b^2}$ or $a(c) = -\sqrt{c^2-b^2}$

7. What could be confusing when using function notation? Explain by using this example:

Brackets may or may not mean to multiply

Evaluate $m(3) - n$

a) if m = 2 and n = 6

$m(3) - n$

$= (2)(3) - 6 = 0$

multiply since m is #.

b) if $m(x) = x^2$ and n = 6

$m(3) - n$

$= (\ )^2 - 6$

$= 3^2 - 6 = 9 - 6 = 3$

brackets do not mean to multiply since m is a function

8. Find the following for the functions $f(x) = 2x^4 - 3x^2$, $h(a) = \sqrt{a-5}$, $g(k) = \dfrac{6k}{k+1}$

a. $f(2)$ not multiplication but replace $x = 2$

$f(2) = 2(\ )^4 - 3(\ )^2$

$f(2) = 2(2)^4 - 3(2)^2$ BEDMAS

$f(2) = 32 - 12$

$\boxed{f(2) = 20}$ → $x = 2$, $y = 20$ pt. $(2, 20)$

b. $h(69)$

$h(69) = \sqrt{(\ ) - 5}$

$h(69) = \sqrt{(69) - 5}$

$h(69) = \sqrt{64}$

$h(69) = 8$

c. $f(\sqrt{5}) - h(30)$

$= \left[2(\sqrt{5})^4 - 3(\sqrt{5})^2\right] - \left[\sqrt{(30) - 5}\right]$

$= \left[2(25) - 3(5)\right] - \sqrt{25}$

$= 50 - 15 - 5$

$f(\sqrt{5}) - h(30) = 30$

d. $f(-3) + 4g(3)$

$\left[2(-3)^4 - 3(-3)^2\right] + 4\left[\dfrac{6(3)}{(3)+1}\right]$

$= 2(81) - 27 + \cancel{4}\left(\dfrac{18}{\cancel{4}}\right)$

$= 162 - 27 + 18$

$\therefore f(-3) + 4g(3) = 153$

e. $2g\left(\dfrac{1}{2}\right)f(-1) = -4$

$2\left[\dfrac{6(\frac{1}{2})}{\frac{1}{2} + \frac{1 \times 2}{1 \times 2}}\right]\left[2(-1)^4 - 3(-1)^2\right]$

$= 2\left[\dfrac{3}{\left(\frac{3}{2}\right)}\right]\left[2(1) - 3(1)\right]$

$= 2\left(\cancel{3} \times \left(\dfrac{2}{\cancel{3}}\right)\right)(-1) = -4$

f. $g(3)^{h(6)}$

$= \left[\dfrac{6(3)}{3+1}\right]^{\sqrt{6-5}}$

$= \left(\dfrac{18}{4}\right)^{\sqrt{1}} = \left(\dfrac{9}{2}\right)^1$

9. Isolate the output variable and record the answer in function notation.

a. $2x-6y=18$

$2x-6f(x)=18$ Isolate 1st.

$\frac{-6y}{-6}=\frac{18}{-6}-\frac{2x}{-6}$

$y=-3+\frac{1}{3}x$

$y=-3+\frac{x}{3}$

$\therefore f(x)=-3+\frac{x}{3}$

b. $3x=\frac{(y+1)}{(y-2)}$

$3x(y-2)=1(y+1)$

$3xy-6x=y+1$

$3xy-y=6x+1$

common factor

$y(3x-1)=6x+1$

$y=\frac{6x+1}{3x-1}$ $\therefore f(x)=\frac{6x+1}{3x-1}$

* Goal is to get y to appear once to isolate it.

c. $4+\sqrt{2y-3x}=10-4$ BEDMAS

$(\sqrt{2y-3x})^2=(6)^2$

$2y-3x=36$

$\frac{2y}{2}=\frac{36}{2}+\frac{3x}{2}$ or $y=\frac{36+3x}{2}$

$y=18+\frac{3}{2}x$

$\therefore f(x)=18+\frac{3}{2}x$

10. Write down the formula, in function notation for the perimeter of a rectangle, considering perimeter as input and length as output.

$P=2l+2w$

input output.

$\frac{P-2w}{2}=\frac{2l}{2}$

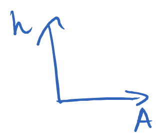
$\therefore l(P)=\frac{P-2w}{2}=L$

OR $f(x)=\frac{x-2w}{2}$

11. Write down the formula, in function notation for the area of the triangle, considering area as input and height as output.

$A=\frac{bh}{2}$ output

input



$2A=bh$

$\frac{2A}{b}=h$

$\therefore h(A)=\frac{2A}{b}$

12. Write down the area of a circle as a function of its perimeter.

$A=\pi r^2$ $P=2\pi r$ circumference

$\frac{P}{2\pi}=r$ sub.

$A=\pi\left(\frac{P}{2\pi}\right)^2$

$A=\frac{\cancel{\pi}P^2}{4\pi^{\cancel{2}}}$

$A=\frac{P^2}{4\pi}$ $\therefore A(P)=\frac{P^2}{4\pi}$

Goal (1.) only A's and P's in ONE equation

(2.) A(P) = area as a function of perimeter

P(A) = perimeter as a function of area. (isolate P)

all possible independent values of x
all resulting dependent y values from the domain

"set of ..." $\{ \ldots \}$
"such that" s.t. or $|$
"element of/part of" $\in$
"less than" $<$
ex. $x < 3$ OR $3 > x$

"or", union $\cup$
"and", intersection $\cap$

$\mathbb{N}$ $\mathbb{W}$ $\mathbb{Z}$ $\mathbb{Q} \leftarrow \mathbb{R} \rightarrow \bar{\mathbb{Q}}$

$\mathbb{N}$ = natural #s/counting #'s $= \{x \mid x = 1, 2, 3, \ldots\}$
$\mathbb{W}$ = whole #'s $= \{x \mid x \in \mathbb{N} \cup x = 0\}$
$\mathbb{Z}$ = integers
$\mathbb{Q}$ = rational #'s or fractions
$\bar{\mathbb{Q}}$ = irrational #'s (non repeating, non terminating decimals)

$\mathbb{R}$ - real #s
$\mathbb{I}$ - imaginary #'s.

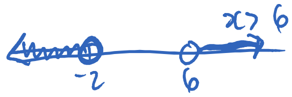
include $x=3$ closed/shaded in.
not include $x=5$ open circle

$\{y \mid y \in \mathbb{R}, -3 \le y < -2, -1 \le y \le 1\}$

shading? use $\mathbb{R}$

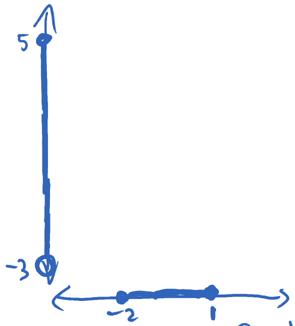
$\{x \mid x \in \mathbb{R}, x < -2, x = 1, 3 \le x < 5\}$
OR $\{x \mid x \in \mathbb{R}, x < -2$ or $x = 1$ or $x \geq 3$ and $x < 5\}$

$x > 6$ or $x < -2$



* can use the "nested" notation only if shading is occuring between the numbers.

Not a function.



Yes it is a function

$D = \{x \mid x \in \mathbb{R}, -2 \leq x \leq 1\}$

$R = \{y \mid y \in \mathbb{R}, 5 \geq y > -3\}$

OR $\{y \in \mathbb{R} \mid \ldots$

Yes

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \geq -3\}$

NO

$D = \{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$

$R = \{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

$D = \{x \in \mathbb{R} \mid -5 \leq x < 2\}$

$R = \{y \in \mathbb{R} \mid -2 \leq y \leq 3\}$ YES.

"asymptote"

$D = \{x \in \mathbb{R} \mid x \neq 2\}$

OR

$\{x \in \mathbb{R},\ x < 2,\ x > 2\}$

$R = \{y \in \mathbb{R} \mid y \neq 3\}$

1. can't sq. root negative numbers → Solve radicand ≥ 0 (only parts under root)
2. can't divide by zero → Solve denominator $\neq 0$

$x^2 + 3x + 2 \neq 0$

$(x+1)(x+2) \neq 0$

$x + 1 \neq 0$ or $x + 2 \neq 0$

$D = \{x \in \mathbb{R} \mid x \neq -1,\ x \neq -2\}$

no denom
no sq. roots $D = \{x \in \mathbb{R}\}$

$b^3 - 4b \neq 0$

$b(b^2 - 4) \neq 0$

$b(b+2)(b-2) \neq 0$

$D: \{b \in \mathbb{R} \mid b \neq 0, \pm 2\}$

$4 - 2x \geq 0$ (−4, −4)

$-2x \geq -4$

$\frac{-2x}{-2} \geq \frac{-4}{-2}$

*flip sign if mult or div by neg.

$D = \{x \in \mathbb{R},\ x \leq 2\}$

$x - 4 > 0$ not equal to.

$x > 4$

$\therefore D: \{x \in \mathbb{R} \mid x > 4\}$

Transformations REVIEW of Quadratics

1. Vertex form of a quadratic tells you more than just the vertex. What do the constants represent in terms transfomations for $y = a(x-h)^2 + k$?

a → reflect in x-axis if a is neg. $(a<0)$
→ vertical stretch OR compression
$a>1$ or $a<-1$ } $|a|>1$; $|a|<1$

h → horizontal shift/translation
if $h<0$ left
if $h>0$ right } but switch sign when pull out of bracket.

ex. $y = 3(x+5)^2 - 6$
$h = -5$

k → vertical shift.
if $k>0$ up
if $k<0$ down

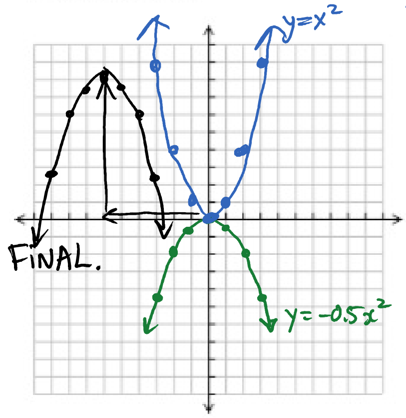
2. State the transformations of $y = -0.5(x+6)^2 + 8$ in the order that you would apply them.

stretch/compress/reflect 1st
shifts last.

x = same, $y \cdot a$ { $a = -0.5$ → reflection in x-axis → vertical compression

{ $h = -6$ → shift left.
$k = 8$ → shift up.

3. Sketch $y = -0.5(x+6)^2 + 8$ showing all steps using different colours.



$y = x^2$ parent
$y = -0.5x^2$
$y = -0.5(x+6)^2 + 8$

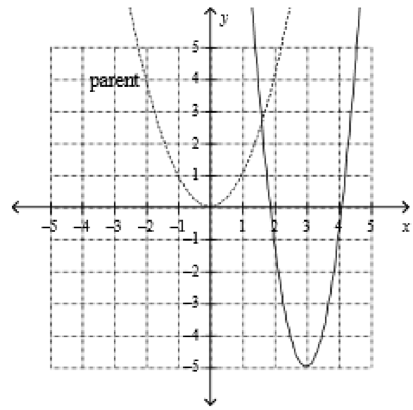
* if a is not there $a = 1$ ex. $y = (x-1)^2 + 5$
1

if h/k are not there
$h = 0$ or $k = 0$

$y = x^2 + 5$
$(x-0)^2$

If the quadratic is written in vertex form but x has a coefficient on it, like $y=(6-2x)^2-5$, it can be rewritten in two ways.

Method 1: Factor out the coefficient and keep it in the square function.
Method 2: Factor out the coefficient and pull it out of the square function completely:



Method 1	Method 2
$y=(6-2x)^2-5$	$y=(6-2x)^2-5$
$y=(-2x+6)^2-5$	$y=(-2x+6)^2-5$
$y=1(-2(x-3))^2-5$	$y=(-2(x-3))^2-5$
a k d c	$y=(-2)^2(x-3)^2-5$
	$y=4(1x-3)^2-5$
	a k d c

Both equations will yeild the same graph as shown

Method 2 gives a result that you know how to deal with since it is in the form $y=a(x-h)^2+k$. However when you study other types of functions, you might not be able to pull out the coefficient out of the function. So look at method 1 and think about what could the coefficient on x inside the square function could mean?

K - horizontal stretch/reflect/compress

4. You will now study a new transformations form: $y=a(k(x-d))^2+c$. What do you think the constants represent here?

a ⇝ SAME as before

k ⇝ horizontal reflection (in y-axis) if $k<0$
horizontal stretch $|k|<1$ OR compression $|k|>1$ *opposite !!

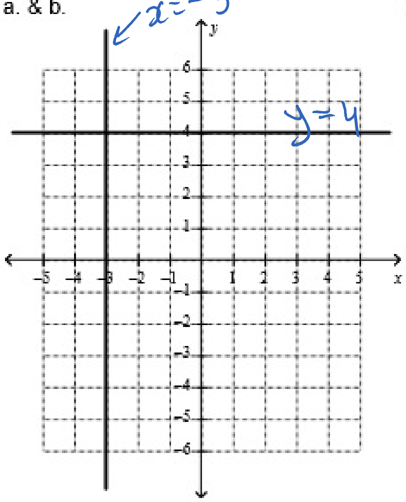
d ⇝ shifts left/right

c ⇝ shifts up/down

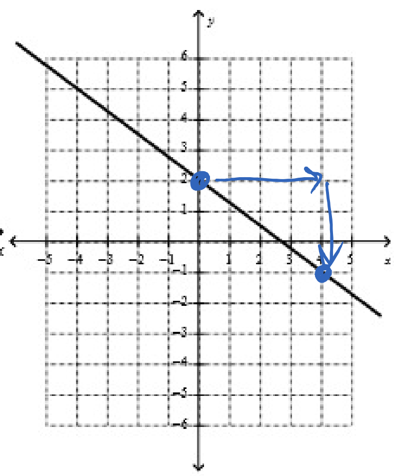
Parent/Basic Functions and their Properties

1. This course is a preparation course for the Advanced Functions course in grade 12. You will learn several new functions. Before we get into those, review the relations you already know. For the following, state the equations of the graphs drawn.

a. & b.

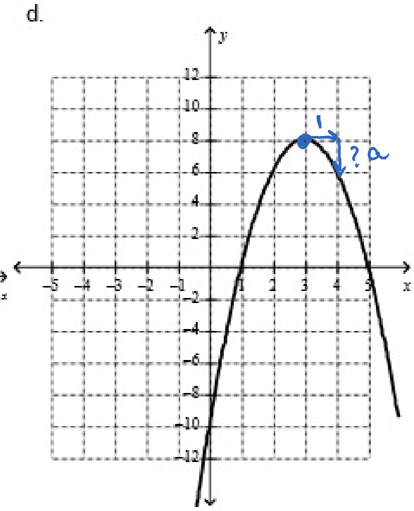


c.



$y=mx+b$
$y=\frac{-3}{4}x+2$

d.

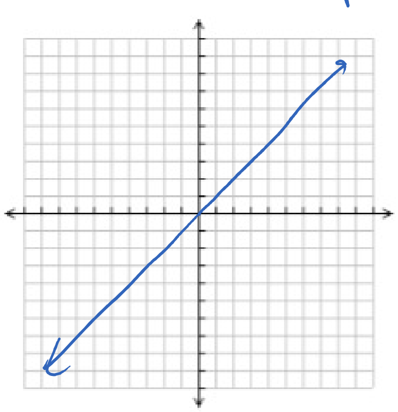


$y=a(x-h)^2+k$
$y=-2(x-3)^2+8$

There are many lines and parabolas possible. However each set belongs to a particular **family** or type. For each family of functions, there is a **parent** or a basic function.

2. For the functions above, write the **parent** equations for linear and quadratic functions, create a table of values and a sketch. State the general equations for linear and quadratic functions and then state the domain and range for these general equations.

Linear parent $y=x$

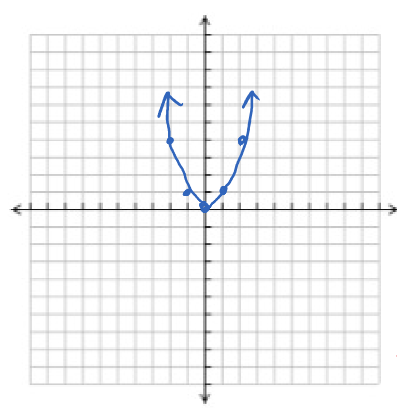


$D=\{x\in\mathbb{R}\}$
$R=\{y\in\mathbb{R}\}$

x	y
-2	-2
0	0
5	5

general: $y=mx+b$
$D=\{x\in\mathbb{R}\}$
$R=\{y\in\mathbb{R}\}$
not $y=a(k(x-d))+c$

Quadratic parent
$y=x^2$



x	y
-2	4
-1	1
0	0
1	1
2	4

$D=\{x\in\mathbb{R}\}$
$R=\{y\in\mathbb{R}, y\geq 0\}$

general: $y=a(x-h)^2+k$
not $y=a(k(x-d))^2+c$

11

Transformed: $y = a\,f\big(k(x-d)\big)+c$

$\sqrt{\ }$ $\quad (\ )^2 \quad \sin(\ )\ \ldots$

3. Develop the table of values and a sketch for each of the following new parent functions. State the transformed version of the equation and then state the domain and range for the transformed equation.

a. Square Root Function $y=\sqrt{x}$

(a+c are vertical transformations outside the function)

x	y
0	0
1	1
4	2
9	3

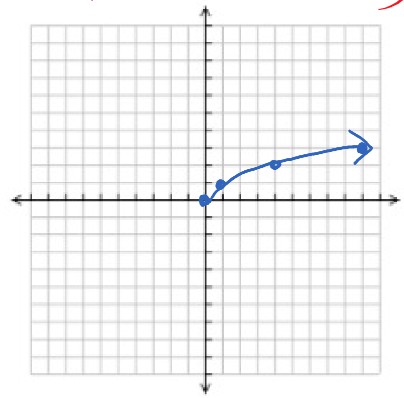
$y = a\sqrt{k(x-d)}\ +c$

$y = 1\sqrt{1(x-0)}\ +0$

ex. $y=-2\sqrt{3(x+5)}\ -10$

$D=\{x\in\mathbb{R}\mid x\geq 0\}$ from d

$R=\{y\in\mathbb{R}\mid y\geq 0\}$ from c



b. Rational Function $y=\dfrac{1}{x}$

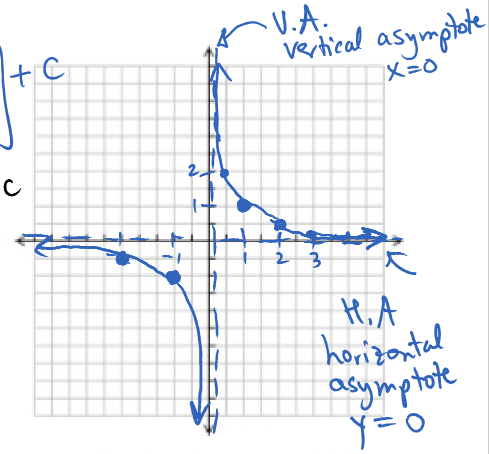
x	y
-2	-0.5
-1	-1
0	Does not exist (asymptote)
1	1
2	0.5
3	$0.\overline{3}$
0.5	2

transformed: $y=a\left[\dfrac{1}{k(x-d)}\right]+c$

OR $y=\dfrac{a}{k(x-d)}+c$

$D=\{x\in\mathbb{R}\mid x\neq 0\}$

$R=\{y\in\mathbb{R}\mid y\neq 0\}$



4. In the last question, lines $x=0$ and $y=0$ are called **asymptotes** for that function. Explain, what do you think are asymptotes?

Asymptotes indicate what the graph is approaching.

* V.A. cannot be crossed by a graph

H.A. can (you'll see in Adv. Functions)

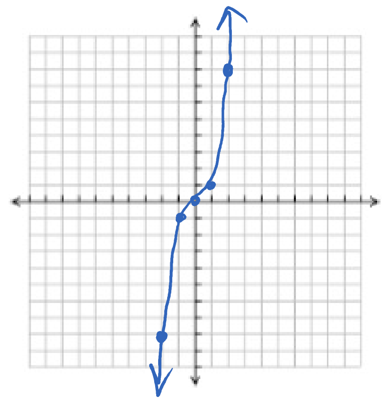
5. Show a table of values, a sketch, a transformed version of the equation, and domain & range

a. Cubic Function $y = x^3$

x	y
−2	−8
−1	−1
0	0
1	1
2	8

$y = a\left(k(x-d)\right)^3 + c$

$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R}\}$



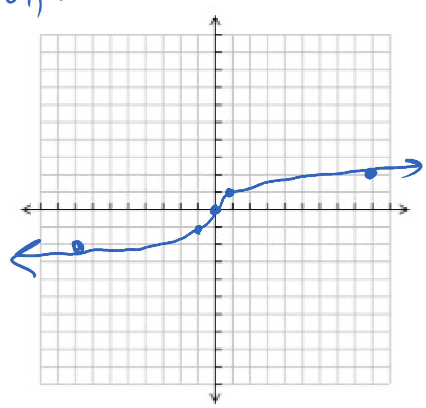
b. Cube Root Function $y = \sqrt[3]{x}$

perfect sq: 1, 4, 9, 16, 25, ...
perfect cubes: 1, 8, 27, 64, ...

x	y
0	0
1	1
8	$\sqrt[3]{8} = 2$
−1	1
−8	−2

$y = a\sqrt[3]{k(x-d)} + c$

$D = \{x \in \mathbb{R}\}$
$R = \{x \in \mathbb{R}\}$



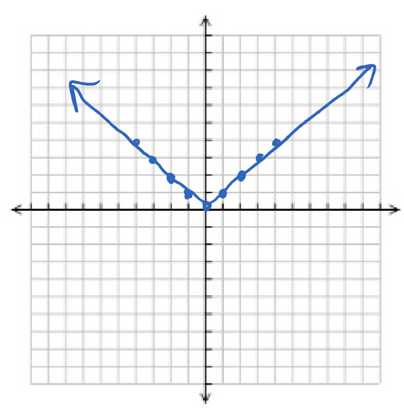
c. Absolute Value Function $y = |x|$

x	y
−2	$\lvert -2 \rvert = 2$
−1	1
0	0
1	1
2	2

drop all negatives in the output.

$y = a\left|k(x-d)\right| + c$

$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R} \mid y \geq 0\}$



Transformations of New Functions

$y = af(k(x-d)) + c$ * d is not visible unless k is factored out (is there a bracket between k and x?)

Identify the parent function, the transformations that were applied, domain&range, and sketch showing all steps using different colours.

1. $y = -3\sqrt{0.5x+2} - 5 = -3\sqrt{0.5(x+4)} - 5$

parent: $y = \sqrt{x}$

transformations: $a = -3$ → reflected in x-axis, vertical stretch } $y \cdot a$

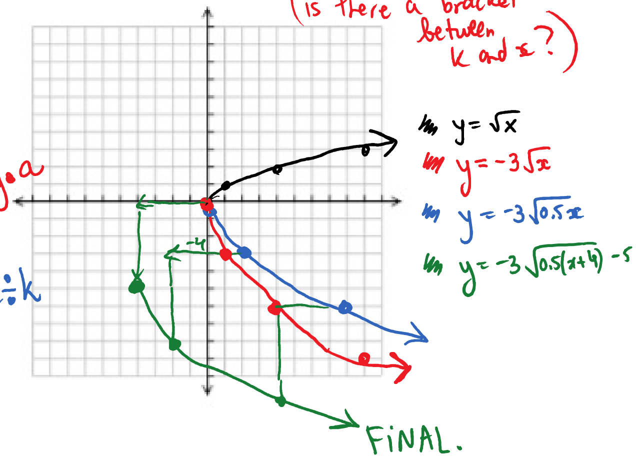
$k = 0.5$ → not reflected, horiz. stretch } $x \div k$

$d = -4$ → shift left

$c = -5$ → shift down

$D = \{x \in \mathbb{R} \mid x \geq -4\}$

$R = \{y \in \mathbb{R} \mid y \leq -5\}$



2. $y = \dfrac{5}{0.25x+1}$ Now try the short cut

$\dfrac{5}{0.25(x+4)}$

work with table of values 1st, then plot only final graph.

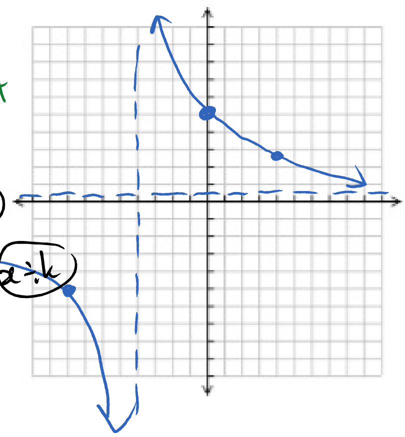
parent: $y = \dfrac{1}{x}$

transf: $a = 5$ → vertical stretch $y \cdot a$

$k = 0.25$ → horizontal stretch $x \div k$

$d = -4$ → shift left.

$c = 0$



x	y = parent = $\frac{1}{x}$
−2	−0.5
−1	−1
0	V.A.
1	1
2	0.5
H.A.	0

$\frac{x}{k} \pm d$	$y \cdot a \pm c$
$\frac{x}{0.25} - 4$	$y(5) + 0$
~~−12~~	~~2.5~~ → not fit.
−8	−5
−4	V.A.
0	5
4	2.5
H.A.	0

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3. Once you understand how to apply the transformations in the correct order, there are some shortcuts you can take. Summarize the steps to sketch the final function without sketching all the intermediate functions.

Steps for transforming any shape

(1.) Ensure that the function is written with x appearing once!
(ex. $y = x^2 + 2x - 5$ transformations are not visible)

(2.) Ensure that k and x have a bracket between them otherwise "d" is not visible.

(3.) List the parent + transformations

(4.) Use tables

x	$y=$ parent	$\frac{x}{k} \pm d$	$y(a) \pm c$

← Plot this last one only.

4. Write the equation given the parent function and the transformations. Sketch using the shortcut.

a. $f(x) = |x|$, reflected in x-axis, horizontal compression by 2, shift up by 5

$a = -1$

by $\frac{1}{2}$? $c = 5$

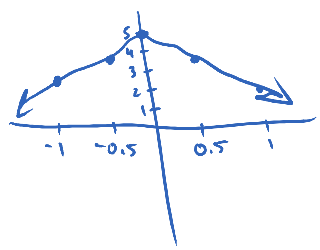
$k = 2$

$k = \frac{1}{2}$

$y = a|k(x-d)| + c$ ($d = \varnothing$)

$y = -1|2x| + 5$

x	$y=\lvert x\rvert$	$\frac{x}{2}$	$y\cdot(-1)+5$
-2	2	-1	3
-1	1	-0.5	4
0	0	0	5
1	1	0.5	4
2	2	1	3



b. $g(x) = x^3$, reflected in y-axis, horizontal stretch by 3, vertical compression by 2, shift left 4.

$k = -?$ $|k| < 1$

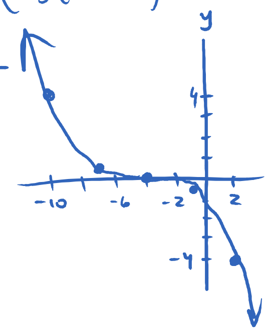
by $\frac{1}{3}$ by $\frac{1}{2}$ $d = -4$

$k = -\frac{1}{3}$ $a = \frac{1}{2}$

$y = a(k(x-d))^3 + c$

$y = \frac{1}{2}\left(-\frac{1}{3}(x+4)\right)^3$

x	$y=x^3$	$x \div -\frac{1}{3} - 4$	$y \cdot \frac{1}{2}$
-2	-8	2	-4
-1	-1	-1	-0.5
0	0	-4	0
1	1	-7	0.5
2	8	-10	4



2 x's transformations are not visible

5. What are the transformations of $y = -3x^2 + 6x + 0$

$\div -3$ $\div -3$

$y = -3(x^2 - 2x + 1 - 1) + 0$

$y = -3(x-1)(x-1) - 1(-3)$

$y = -3(x-1)^2 + 3$

$a = -3$ → reflection in x-axis, → vertical stretch

$k = 1$

$d = 1$ → shift right

$c = 3$ → shift up

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Inverses

1. You have already seen inveses in action. Identify the inverse operations that would be used to solve the following equations.

a. $\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{4}{7}\right)$

$\theta = \cos^{-1}\left(\frac{4}{7}\right) \doteq 55°$

b. $\frac{100}{\pi} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$

$\sqrt{\frac{100}{\pi}} = \sqrt{r^2}$

inverse operation: divide by π

sq. root

2. Before you go on, clarify the differences between the words that sometimes get confused.

OPPOSITE – switch sign
ex. 2 and -2
ex. $x-3$ and $-x+3$

RECIPROCAL – flip fraction
ex. 2 and $\frac{1}{2}$
ex. $x-1$ and $\frac{1}{x-1}$

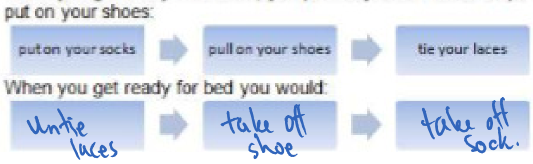
INVERSE – undo operation
ex. $(\ )^2$ and $\sqrt{\ }$
ex. $\sin(\ )$ and $\sin^{-1}(\ )$

3. When finding inverses, it is crucial to remember to undo operations in BEDMAS backwards order. Look at this real life example to understand the importance of order. Then apply it in the mathematical example about temperature. Find the inverse for each of the following.

a.

When you get ready for school, you probably follow these steps to put on your shoes:

When you get ready for bed you would:



BEDMAS → undoing "SAMDEB"

b.
American visitors to Canada would use this formula to convert Celsius to Fahrenheit: $F = \frac{5}{9}C + 32$. What would Canadians use to convert back?

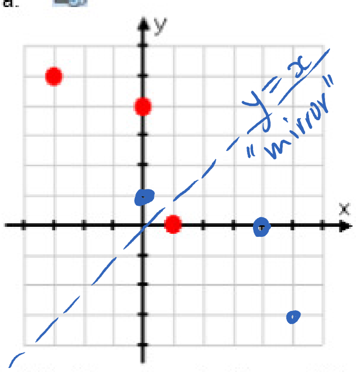
$F - 32 = \frac{5}{9}C$

$\boxed{\frac{9}{5}(F-32) = C}$

4. Notice in both examples the input became the output and vise versa. Sketch the inverse of the following graphs

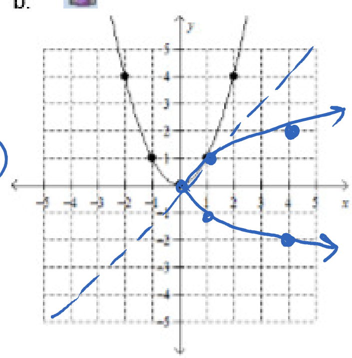
x and y get switched.

a.



f pts.	f^{-1} pts
(-3,5)	(5,-3)
(0,4)	(4,0)
(1,0)	(0,1)

b.



f pts	f^{-1} pts
(-2,4)	(4,-2)
(-1,1)	(1,-1)
(0,0)	(0,0)
(1,1)	(1,1)
(2,4)	(4,2)

5. What type of graph will remain the same when input and output get switched? Why?

$y = x$
or
$y = \frac{1}{x}$

6. The inverse may or may not be a function. What kind of test can you do to ensure that both the graph and its inverse are functions?

"one-to-one" function

If the graph passes BOTH vertical + Horizontal line tests the original AND the inverse will both be functions.

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7. What do you think will be the notation for the inverse function? Just be careful, what can it be confused with?

Inverse of $f(x)$ is recorded as $f^{-1}(x)$

Be careful not to think of it as exponent. ex. $3^{-1} = \frac{1}{3}$

$\sin^{-1} x \neq \frac{1}{\sin x}$

8. What do you think is the relationship between domain and range for the relation and its inverse?

Domain of $f(x)$ becomes range of $f^{-1}(x)$ and vise versa.

9. What do you think will be the steps to find the inverse algebraically so that function notation can be used?

Steps: (1) Switch x and y ($f(x)$ is y)

SAMDEB → (2) Isolate the output y

To have it appear once you may have to complete sq. or common factor.

(3.) Rename the y using function notation for inverse : $f^{-1}(x)$ ← must be a function (one output)

10. Demonstrate the steps you've outlined on the following $\cancel{f(x)}\ y = 3(x+5)^2 + 2$

inverse : $x = 3(y+5)^2 + 2$

$\frac{x-2}{3} = \frac{\cancel{3}(y+5)^2}{\cancel{3}}$

$\pm\sqrt{\left(\frac{x-2}{3}\right)} = y+5$

$\pm\sqrt{\frac{x-2}{3}} - 5 = y$

$\therefore f^{-1}(x) = +\sqrt{\frac{x-2}{3}} - 5$

OR

$f^{-1}(x) = -\sqrt{\frac{x-2}{3}} - 5$

$\frac{x}{3} - \frac{2}{3}$

$\frac{1}{3}(x-2)$

11. Find the inverse for each of the following

a. $\cancel{g(x)}\ y = \frac{1-x}{2}$

$\frac{x}{1} = \frac{1-y}{2}$ ← inverse

$2x = 1 - 1y$

$\frac{2x - 1}{-1} = \frac{-1y}{-1}$

$-2x + 1 = y$

$\therefore g^{-1}(x) = -2x + 1$

b. $\cancel{h(x)}\ y = -2(15-5x)^3 + 16$

inverse: $x = -2(15-5y)^3 + 16$

$-\frac{1}{2}(x-16)$ ← $\frac{x-16}{-2} = (15-5y)^3$

$\frac{1}{5}(\ \ )$ ← $\frac{\sqrt[3]{\frac{x-16}{-2}} - 15}{-5} = y$

* Bad notation if record fraction inside a fraction

fix : $\frac{1}{5}\sqrt[3]{-\frac{1}{2}(x-16)} + 3 = y$

↑ a ↑ k ↑ d ↑ c

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Inverses can help you find range algebraically:

12. You have seen how to find domain of a function without graphing. Remind yourself how to find domain without graphing.

denom $\neq 0$ radicand ≥ 0

"thing under root"

13. To find range without graphing there are TWO possibilities. What are they?

1. If x appears once then a, k, d, c are visible and Range will come from c for some functions.
2. If x appears more than once, find inverse's domain to get original function's range

14. Find the domain and range of the function, (use inverses to help you, if neccesary).

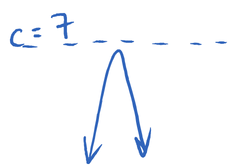
a. $f(x) = -2|3x-6|+7$

$= -2|3(x-2)|+7$

$D = \{x \mid x \in \mathbb{R}\}$ pg. 13

$R = \{y \mid y \in \mathbb{R},\ y \leq \underset{c}{7}\}$

$c = 7$



b. $\overset{y}{g(x)} = \dfrac{-2x}{6-3x} + 4$

to find range do METHOD 2 → $\mathbb{R}$

denom $\neq 0$

$6 - 3x \neq 0$

$-3x \neq -6$

$D = \{x \mid x \in \mathbb{R},\ x \neq 2\}$

inverse : $x = \dfrac{-2y}{6-3y} + 4$

$\dfrac{(x-4)}{1} = \dfrac{-2y}{(6-3y)}$

$(x-4)(6-3y) = -2y$

$6x - 3xy - 24 + 12y = -2y$

$6x - 24 = -2y - 12y + 3xy$

$6x - 24 = -14y + 3xy$

$6x - 24 = y(-14 + 3x)$

$\dfrac{6x-24}{-14+3x} = g^{-1}(x)$

denom $\neq 0$

$-14 + 3x \neq 0$

$3x \neq 14$

$x \neq \dfrac{14}{3}$

$\therefore R = \{y \mid y \in \mathbb{R},\ y \neq \frac{14}{3}\}$

15. Domain of $j(x)$ is $-4 \leq x \leq 6$ and the range is $0 \leq y \leq 10$. A new function $k(x)$ is formed when transformations are applied as follows: $k(x) = -2j(3x+6) - 4$, what is the range of $k^{-1}(x)$?

$3(x+2)$

Domain (x)	Range (y)	$x \div k$ ($x \div 3$)	$y \cdot a$ ($y \cdot (-2)$)	$x - 2$	$y - 4$
-4	0	$-4/3$	0	$-10/3$	-4
⋮	⋮	⋮	⋮	⋮	⋮
6	10	2	-20	0	-24

~~$R_k = \{y \in \mathbb{R},\ -24 \leq y \leq -4\}$~~

$R_{k^{-1}} = \{y \in \mathbb{R},\ -\frac{10}{3} \leq y \leq 0\}$