S14FinanceNOTES

July 9, 2014 2:58 PM

FinanceNOTES

see below

Finance Unit 8

Tentative TEST date_______________

Big idea/Learning Goals

In this unit you will study the applications of linear and exponential relations within financing. You will understand the different formulas you must use for simple interest and compound interest. **Simple interest** means that the interest grows by a *constant* amount each year. **Compound interest** means that the interest grows by a an *increasing* amount each year, because the interest is calculated on the amount deposited as well as on the interest already earned so far. The value of the monetary amount of a single deposit at specific time at the bank is related to sequences, whereas the monetary amount of regular deposits or payments is related to series. The formulas developed here will look different from the formulas you've learned in the sequences and series unit, however they are the same formulas, different letters are used just to signify the constants that relate to finance.

Corrections for the textbook answers:
Sec 8.2 #6 44.8 yrs
Sec 8.2 #9 Plan A $1139.99
Sec 8.4 #8 t=5 years almost 8 months n=22.5

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pages	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-5	Compare Simple and Compound Interest Practice Simple and Compound Interest Section 8.1 & 8.2 & 8.3	8.1 #1,2,3,4,11 8.2 #5,6,7 8.3 #7,8,9,10	
	6-8	Present and Future Value of an Annuity Practice Annuities Section 8.4 (skip #10 or use technology) Section 8.5 (skip #12 or use technology)	8.4 #4,6,8,9 8.5 #4,6,8,11	
	9-11	Mix of Questions - what formula to use? Handout		
	12	Mortgages		
	13-15	Using Technology Section 8.6		
		REVIEW		

Day 15 + start review

Compare Simple and Compound Interest

1. Suppose you put $1000 in a bank that earns 5% **simple interest** per year.

a. Fill in the table.

Term #	Time	Simple Interest	Final Amount	Differences
1	0	NA	1000	
2	1yr	5% (original) 0.05(1000) = $50	$1050	+50
3	2yr	0.05(1000) = $50	$1100	+50
4	3yrs	50	$1150	+50
5	4yrs	50	$1200	+50

b. Look at the differences and determine what type of function this is?

linear

c. To what type of sequence does this table correspond?

arithmetic

d. Find the general term of the final amount column.

$t_n = a + d(n-1)$

$t_n = 1000 + 50(n-1)$

$A = P + Prt$

e. The simple interest formula is the same as the formula above, you just have to modify the letters to use the following

P=principal or initial amount deposited, present value
A=final amount or future value
r = annual interest rate (% written as a decimal)
t = time in years
I = interest earned in $

$\boxed{A = P + \underbrace{Prt}_{\text{interest earned}}}$

$\boxed{I = Prt}$

$\boxed{A = P + I}$

f. How much will you have after 15 years?

$t=15$

$A = 1000 + 1000(0.05)(15)$

$A = 1000 + \$750$

$A = \$1750$

g. How much of the final amount is the interest?

$I = \$750$

2. Suppose you put $1000 in a bank that earns 5% **compound interest** per year, compounded annually

Term #	Time	Interest	Final Amount	Ratios
1	0	NA	1000	$\frac{next}{prev} = \frac{1050}{1000}$
2	1yr	0.05(previous) = $50	$1050	x 1.05
3	2yr	0.05(1050) = $52.50	$1102.50	x 1.05
4	3yr	0.05(1102.50) = $55.13	1157.63	x 1.05
5	4yr	0.05(1157.63) = 57.88	$1215.51	x 1.05

a. Fill in the table.

b. Look at the ratios and determine what type of function this is?

exponential

c. To what type of sequence does this table correspond?

geometric

d. Find the general term of the final amount column.

$t_n = ar^{n-1}$

$t_n = 1000(1.05)^{n-1}$

$A = P(1+rate)^{\#\text{ of compounding periods}}$

e. The compound interest formula is the same as the formula above, you just have to modify the letters to use the following

P=principal or initial amount deposited, present value
A=final amount or future value
C= # of compounding periods in a year
r = annual interest rate (% written as decimal)
i = periodic interest rate (% written as decimal)
t = time in years
n = total # of compounding periods
I = interest earned in $

$A = P(1+i)^n$ $\quad n = Ct \quad i = \frac{r}{C}$

$(I = A - P)$

f. How much will you have after 15 years?

$i = \frac{0.05}{1}$ C=annual. $n = 1(15)$

$A = 1000\left(1 + \frac{0.05}{1}\right)^{15}$

$A = \$2078.93$

g. How much of the final amount is the interest?

$I = 2078.93 - 1000$

$I = 1078.93$

Practice Simple and Compound Interest

When you read a word problem, ask yourself the following questions:

- Is this simple or compound interest?
  (Usually if the word "compounded" isn't there it is simple interest.)
- Given a monetary amount, is it the present value, or the future value, or the interest earned?
  P A I
- Is the time given in years?
  (If not, convert using the following : $1year = 12months = 52weeks = 365days$)

Finally, if the question is compounded, it will tell you the frequency of compounding. Here is what you must know:

C=1 annually
C=2 semi-annually (twice in a year)
C=4 quarterly
C=12 monthly
C=6 bimonthly
C=24 semi-monthly (twice in a month)
C=26 bi-weekly (every other week)
C=52 weekly
C=365 daily

3. All of the questions so far will be about a **single deposit** being made. Summarise all the formulas you will have to use for **simple and compound interest** that involves single deposits.

Simple

$A = P + Prt$

$I = Prt$

Compound

$A = P(1+i)^n$

$i = \frac{r}{c}$ $n = ct$

4. An investment of $4000 is invested at 5% for 130 weeks. What is the final amount? How much of this is interest?

P $r = 0.05$ $t = \frac{130}{52}$ $A = ?$ $I = ?$

$A = P + Prt$

$A = 4000 + 4000(0.05)\left(\frac{130}{52}\right)$

$A = \$4500$ ← final amount

$I = \$500$ interest earned.

5. Suppose you spent $1200 on your credit card. The credit card charges 19.5% compounded "monthly." If you forget to pay it for 3 months, how much would you owe in total? How much of that is interest?

$t = \frac{3}{12}$ yr $P = 1200$ $A = ?$ $I = ?$ $r = 0.195$ $C = 12$

$A = P(1+i)^n$ $i = \frac{r}{c}$ $n = ct$

$A = 1200\left(1 + \frac{0.195}{12}\right)^3$ $i = \frac{0.195}{12}$ $n = \cancel{12}\left(\frac{3}{\cancel{12}}\right)$

$n = 3$

$A = \$1259.46$ ← total on the bill

$I = \$59.46$ ← interest

6. What principal is needed to have \$100 in interest in 2 years invested at 2.5% interest?

$P = ?$ $I = 100$ $t = 2$ $r = 0.025$

$$I = Prt$$
$$100 = P(0.025)(2)$$
$$\$2000 = P$$

original deposit.

7. How much needs to be invested today to have \$25000 in 10 years, at 6% per year, compounded quarterly.

$P = ?$ A t $r = 0.06$ $c = 4$

$$A = P(1+i)^n \qquad i = \frac{r}{c} \qquad n = ct$$
$$25000 = P\left(1 + \frac{0.06}{4}\right)^{40} \qquad i = \frac{0.06}{4} \qquad n = 4(10)$$
$$n = 40$$
$$\$13781.56 = P$$

invest today

8. How long would it take for \$2500 to grow to \$2700 at an interest of 4.5%?

$t = ?$ P A $r = 0.045$

$$A = P + Prt$$
$$2700 = 2500 + 2500(0.045)t$$
$$200 = 112.5t$$
$$1.8_{\text{yrs.}} = t$$

5

Present and Future Value of an Annuity

When the question is not about a single deposit but about many regular (same) deposits or payments you must use a different set of formulas. The term **annuity** describes a *series* of regular deposits or payments. You will use the following examples to develop the annuity formulas for present value of an annuity and future value of an annuity.

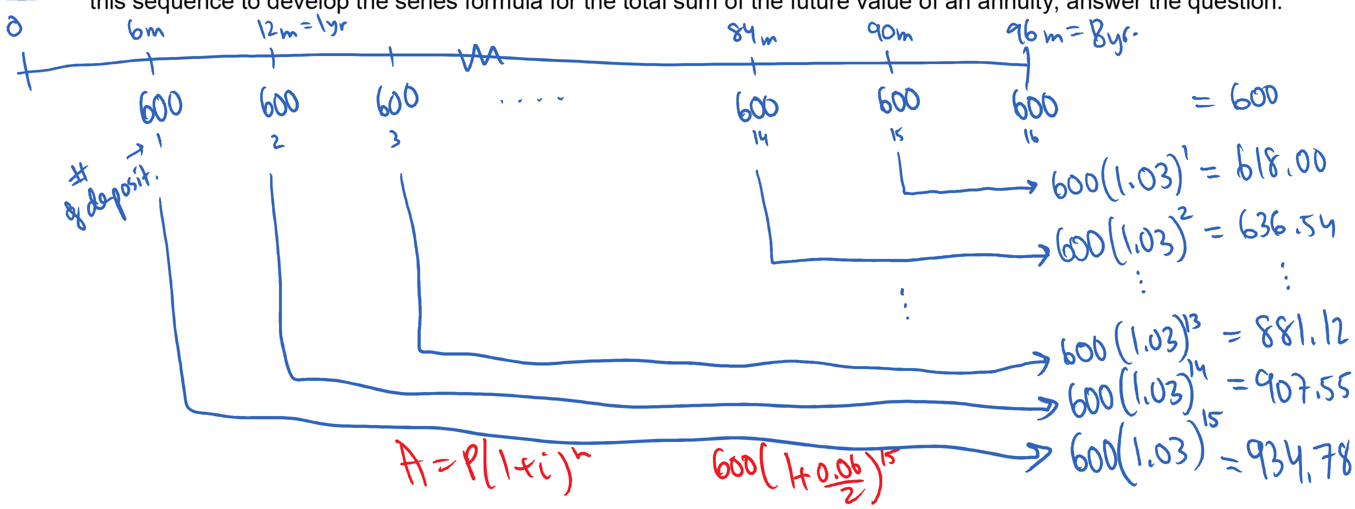
A. "Mario deposits $600 (← without interest) at the end of every 6 months into a savings account paying 6% compounded semi-annually. He does this for 8 years. What is the amount of annuity and what is the total (amount will have interest.) interest earned?" **FV**

B. "Steve is buying a new Harley motorcycle. His monthly payments are $650 with interest within the payments which was charged at 3% compounded monthly. What is the cash price (no interest) of the motorcycle (has interest) if he makes payments for 4 years?" **PV**

Annuity formulas are derived from the sum formula of a geometric series. Identify which of the above questions deal with present value and which with future value.

PV= present value or discounted value (big amount of $ in the present, without interest) ← payments have interests
FV= future value or accumulated value (big amount of $ at a future date, with interest) ← deposits have interest in
R = regular deposit/payment (smaller amount of $ deposited/paid many times over a period of time)
For present value – R is with interest. For future value – R is without interest

1. For problem A, draw a timeline with the deposits made, find the sequence of monetary values in the future, analyze this sequence to develop the series formula for the total sum of the future value of an annuity, answer the question.



Sum of all terms (geo)

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{16} = \frac{600\left((1.03)^{16} - 1\right)}{(1.03 - 1)}$$

$n = ct$
$2(8)$

$i = \frac{r}{c}$

Comparison

$FV = \$12094.13$ ← with interest

$R \times n = 600(16)$
$= \$9600$ ← his money

$I = \$2494.13$

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{600\left[\left(1+\frac{0.06}{2}\right)^{16} - 1\right]}{\left(\frac{0.06}{2}\right)}$$

$= \$12094.13$

applying interest to P

neg. power removes interest from A.

2. Recall the compound interest formula, isolate the formula for P, write it in two ways. One of these notations will be useful in the development of the present value of the annuity.

has interest → $A = P(1+i)^n$

↑ isolated

$$P = \frac{A}{(1+i)^n} = A(1+i)^{-n}$$

3. For problem B, draw a timeline with the payments made, find the sequence of monetary values at the present, analyze this sequence to develop the series formula for the total sum of the present value of an annuity, answer the question.

B. "Steve is buying a new Harley motorcycle. His monthly payments are $650 with interest within the payments which was charged at 3% compounded monthly. What is the cash price of the motorcycle if he makes payments for 4 years?"

0, 1m, 2m, 3m, ..., 47m, 48m

650, 650, 650, ..., 650, 650

$648.38 $= 650(1.0025)^{-1}$

$646.76 $= 650(1.0025)^{-2}$

$645.15 $= 650(1.0025)^{-3}$

⋮

$578.03 $= 650(1.0025)^{-47}$

$576.58 $= 650(1.0025)^{-48}$

$r = 1.0025$

1st term

a

$650\left(1+\frac{0.03}{12}\right)^{-48}$ $P = A(1+i)^{-n}$

want sum of geo: $S_n = \frac{a(r^n - 1)}{r-1}$

$$S_{48} = \frac{650(1.0025)^{-48}\left[(1.0025)^{48} - 1\right]}{1.0025 - 1}$$

$$= \frac{650\left[(1.0025)^0 - (1.0025)^{-48}\right]}{0.0025}$$

$$= \frac{650\left[1 - (1.0025)^{-48}\right]}{0.0025}$$

Comparison

PV = 29 366.15 ← cash price of motorcycle

$R \times n = 650(48)$

= $31 200 ← he paid with interest.

$I = \$1833.85$

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{650\left[1-\left(1+\frac{0.03}{12}\right)^{-48}\right]}{\left(\frac{0.03}{12}\right)}$$

= $29 366.15

7

FV → $\bar{w}$ interest } PV - no interest.

R → no interest } R - $\bar{w}$ interest

Practice Annuities

4. Jasmine wants to save money for retirement in an annuity. She plans to make equal monthly deposits, at the end of each month for 25 years in a trust account that has a guaranteed interest rate of 9% compounded monthly. She wants to have $500 000 in the account at the end of the 25 years. What amount must be her monthly deposit? What is the interest that she earned by using the trust fund?

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$i = \frac{r}{c}$, $i = \frac{0.09}{12}$

$n = ct$, $n = (12)(25)$, $= 300$

$$500\,000 = \frac{R\left[\left(1+\frac{0.09}{12}\right)^{300} - 1\right]}{\left(\frac{0.09}{12}\right)}$$

$$500\,000 = R(1121.121937\ldots)$$

$$\$445.98 = R$$

∴ each month she need to deposit \$445.98

445.98×300
$= 133794$

interest = \$366 205.

5. Sonia purchases a new vehicle for $27000 at 8.2% compounded quarterly and makes payments at the end of ~~every~~ 3 months. She has two choices for the term: 5 years or 8 years. Find the monthly payment for each term. How much would Sonia save in interest by selecting the shorter term?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

5yr

$$27000 = \frac{R\left[1-\left(1+\frac{0.082}{4}\right)^{-20}\right]}{\left(\frac{0.082}{4}\right)}$$

$$27000 = R(16.2728091.6)$$

$$\$1659.21 = R$$

compare: $1659.21(20)$
$= 33184.20$

$I = 6184.20$

8yr.

$$27000 = \frac{R\left[1-\left(1+\frac{0.082}{4}\right)^{-32}\right]}{\left(\frac{0.082}{4}\right)}$$

$$27000 = R(23.2987\ldots)$$

$$1158.86 = R$$

Compare: $1158.86(32)$
$= 37083.52$

$I = 10083.52$

∴ she saves \$3899.32 in shorter term

6. Paul is 16 years old and decides to deposit $2000 at the end of each year that pays 9% compounded annually. Determine Paul's age when his annuity is worth $30 000.

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$i = \frac{r}{c} = \frac{0.09}{1}$

$n = ct = 1(t)$

$$30\,000 = \frac{2000\left[(1+0.09)^t - 1\right]}{(0.09)}$$

×0.09

÷2000

+1

$$2.35 = (1.09)^t$$

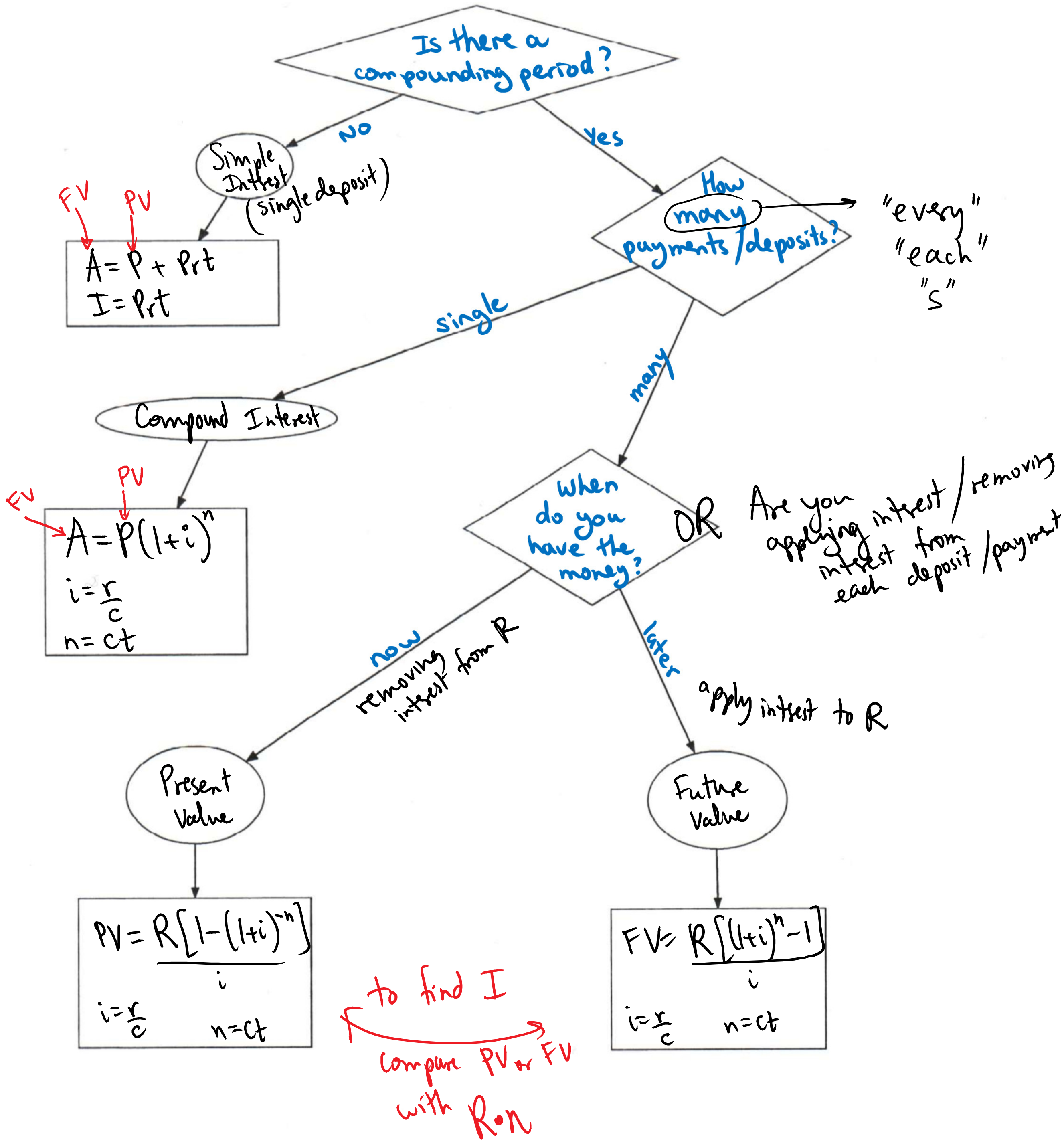
trial + error

$10_{yrs} \doteq t$

∴ He will be 26 yrs. old.

8

Calculating Interest Flowchart



Mix of Questions – what formula to use?

1. Jamail wants to have $31000 in 4.5 years. How much should he invest now at 5.68% compounded semi-annually?

once

$$A = P(1+i)^n$$

ANS : 24093.68

2. Abbas invested a certain amount at a rate of 5% over 40 weeks. If he received $49.96 in interest, determine his initial amount of investment.

$$A = P + Prt \quad \text{or} \quad I = Prt$$

ANS : 1298.96

3. Joanna was awarded $48000 and she has selected to invest it in an annuity which will pay her 6.8% per annum compounded quarterly for 1.5 years. How large is each payment if she is to receive a payment ~~every 3 months~~?

many payments.

PV
no interest.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

ANS : 8482.69

4. How much money must be invested now at 6.6% ~~per annum~~ compounded monthly to provide for monthly payments of $400 for 3 years?

annual

R w/ interest $

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

ANS : 13031.63

5. Bal invested $5400 for 3.5 years at a rate of 6% compounded semi-annually. How much money will he have at the end of his investment?

once!

$$A = P(1+i)^n$$

ANS: 6641.32

6. Suzanna wants to have $23000 in four years. How much would she have to deposit every month for the next 4 years at a rate of 10% compounded monthly in order to have enough money.

many.

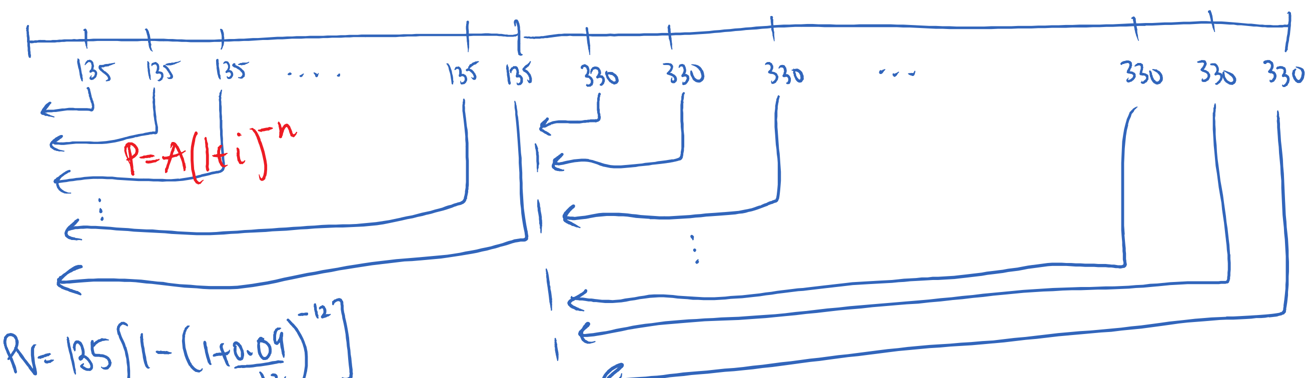
$$FV = \frac{R[(1+i)^n - 1]}{i}$$

ANS: 391.67

7. How much does Claudia need to invest now in order to provide for monthly payment of $135 in the first year and monthly payments of $330 for the next two years if interest is 9% compounded monthly.

many

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$



$$PV = \frac{135\left[1-\left(1+\frac{0.09}{12}\right)^{-12}\right]}{\left(\frac{0.09}{12}\right)}$$

$$PV = \frac{330\left[\left(1-\left(1+\frac{0.09}{12}\right)^{-24}\right)\right]}{\left(\frac{0.09}{12}\right)}$$

PV= 1543.71

bring to present (1yr.)

PV= 7223.42

$$7223.42\left(1+\frac{0.09}{12}\right)^{-12}$$

$= 6603.92$

∴ Total at present time = $8147.63

Mortgages

Mortgage	the amount of money borrowed to purchase a house
Amortization Period	amount of time to repay the loan usually 10 to 25 years
Term	how long the interest rate is "locked" for. Length of term can change from 6 months to years

In Canada mortgage payment are usually done monthly, but according to Canadian Law interest rates can only be compounded semi-annually. Thus we need to change the interest rate from semi-annual to a monthly rate.

To do this use: $(1+s)^2=(1+m)^{12}$ Where m = monthly rate
s = semi annual rate } like i

Ex. You just purchased a \$250,000 home. You make a \$50,000 deposit and mortgage the rest at a rate of 5% compounded semi-annually, amortized for 25 years.

a) What is the amount of the mortgage? \$200 000

b) What is the equivalent monthly rate of interest?

$$(1+s)^2=(1+m)^{12}$$ solve for m

$$\left(\left(\ \right)^2\right)^{\frac{1}{12}}$$

$$\sqrt[12]{\left(1+\frac{0.05}{2}\right)^2}=\sqrt[12]{(1+m)^{12}}$$

$$(1.025)^{1/6}=1+m$$

$$(1.025)^{1/6}-1=m=0.004123915\ldots$$ ← use for "i" in PV

c) Calculate the monthly mortgage payment

$$PV=\frac{R\left[1-(1+i)^{-n}\right]}{i}$$

n = ct
= 12(25)
= 300
* c is monthly now

$$200\,000=\frac{R\left[1-(1+0.004123915)^{-300}\right]}{0.004123915}$$

$$200\,000=R\left[171.938\ldots\right]$$

$$\$1163.21=R$$

d) How much did you pay for the house?

R·n
= 1163.21(300) = \$348 963 + 50 000 (deposit)

Total = \$398,963
− 250 000

e) How much interest did you pay?

I = \$148 963

 Date:____________ Name: ________________________

Using Technology – in class with TI-83 Graphing Calculators

Using pages 591-598 in Nelson 2008 textbook which gives TVM solver instructions for a graphing calculator. Answer the following questions

1. Distinguish when you need to use the negative key and when you need to use subtraction key on graphing calculators.

2. Record what buttons must be pressed to get the TVM solver on your calculator screen.

3. What must be done on the calculator to solve for a specific variable?

Use your textbook and the graphing calculator to solve some questions from section 8.6 of the textbook. Record some solutions below.

Excel Assignment – to do at home if have Excel, or at lunch on school computer.

Problem 1 - Saving
Gary wants to try to save money for travelling after he has graduated from university. He has decided to deposit $50 at the end of each month into an account that pays 6% compounded monthly. Create a spreadsheet to track Gary's progress towards achieving his financial goal.

1. Open Microsoft Excel
2. Label five columns with the following headings
   (start at the VERY top left corner – otherwise instructions below will not work)

Period	Balance	Interest	Payment	New Balance

3. Highlight the last 4 columns
   - right click
   - click Format Cells
   - Under the Number tab click on currency
     This automatically will put dollar signs and round your entries to 2 decimal places
4. In the next row, enter the following:

Period	Balance	Interest	Payment	New Balance
1	0	0	50	50

5. In cell A3 type: =A2+1 (begin by typing the equals sign to indicate to Excel you are giving it a formula)
6. In cell B3 type: =E2
7. In cell C3 type: =E2*0.06/12
8. In cell D3 type: =D2
9. In cell E3 type: =B3+C3+D3
10. Highlight from the third row down to show 5 years of payments.
    - then press CTRL D to fill down
11. The first nine periods of Gary's amortization table should look like this: (DON'T type the numbers in, check if the formulas given above will give you these numbers)

Period	Balance	Interest	Payment	New Balance
1	$0.00	$0.00	$50.00	$50.00
2	$50.00	0.25	$50.00	$100.25
3	$100.25	0.50125	$50.00	$150.75
4	$150.75	0.753756	$50.00	$201.51
5	$201.51	1.007525	$50.00	$252.51
6	$252.51	1.262563	$50.00	$303.78
7	$303.78	1.518875	$50.00	$355.29
8	$355.29	1.77647	$50.00	$407.07
9	$407.07	2.035352	$50.00	$459.11

12. How much money will Gary have in his account after 1 year?___________
13. How much money will Gary have in his account after 4 years?__________
14. Gary shopped around and found another bank that offers 7.5%/a compounded monthly. How would you change the spreadsheet to adapt to this change?

____

____

15. With new interest rate in place, calculate the total amount of interest Gary's annuity would earn after 5 years
    - at the bottom of column C type: =sum(C2:C61)

____

____

Problem 2 – Borrowing

Laura borrowed $12 600 at 8%/a compounded quarterly, to buy a used car. She will make blended quarterly payments of $415. Create a spreadsheet to track Laura's progress as she repays her loan.

16. Label five columns with the following headings:

Payment #	Payment	Interest Paid	Principal Paid	Outstanding Balance

17. Format the last four columns for currency
18. In the next row enter the following

Payment #	Payment	Interest Paid	Principal Paid	Outstanding Balance
0				12 600

19. State what needs to be typed in each of these cells in order to complete the spreadsheet

Cell A3	
Cell B3	
Cell C3	
Cell D3	
Cell E3	

20. How much will Laura still owe after the first year of payments? ________________

21. How long will it take Laura to completely repay the loan? ____

22. After 2 years, the bank drops its prime lending rate by 2%/a. State what needs to be typed in which cell in order to properly adjust the spreadsheet to reflect this change. (Keep the original interest rate for the first 2 years!)

____

____

23. How long will it take Laura to repay the loan after this change is made to the interest rate

____

____