Exponentials Unit 4

Tentative TEST date_______________

Big idea/Learning Goals

This unit introduces you to a new type of function – the exponential function. There are many relationships in real life that either grow or decay at a constant rate, for example: population growth, radioactive decay, inflation, spread of viruses, or processing power of technological gadgets. These types of relationships can be modelled with an exponential function. Before you learn about properties of this function, you must review exponent laws from gr.9-10 and learn a new exponent law that allows you to work with rational (fraction) exponents. Keep in mind that this year you will not know what the inverse of the exponential is. This is something that you will learn in the gr.12 advanced functions course.

Corrections for the textbook answers:
Sec 4.2 #18 c) b^{m+4n}
Sec 4.3 #2 d) –7776 # 3d) 1/7 #18a) 1/6 and – 1/6
Sec 4.5 #1c) neither, too hard if HA is not given 1st ratios will not be the same… #2a) decrease
Sec 4.7 #10c) $P = 2^t$ #14b) $P=10^{10}(0.01)^n$ 5 applications to reach 1 germ, so 6 to kill all
Review #5f f^(1/5)

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pages	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
	2-3	Review of Exponent Laws & Section 4.2 & Handout		
	4-5	Rational Exponents & Simplify Expressions with Exponents Section 4.3 & Handout Section 4.4 & Handout		
	6-7	Solve by SAMDEB-Matching Bases-Trial and Error Three Handouts		
		QUIZ no calculators		
	8-10	Introduction to Exponential Functions Section 4.5 & two Handouts		
	11-12	Transformations of Exponentials Section 4.6 & Handout		
	13-14	Application of Exponentials Section 4.7 & Handout		
		REVIEW		

Reflect – previous TEST mark ___________, Overall mark now___________.

Review of Exponent Laws

1. Why do you think exponent notation was invented?

2. Summarize the 5 laws you learned in grade 9-10.

3. There will be several ways to simplify expressions, depending on what rule you start applying first. Final answers should still match no matter what route you take. To make things easier try to use the ________________________________ law first and ____ law last.

4. Apply the laws to the following examples as you simplify the questions. Leave everything as exact numbers, with positive exponent answers.

a. $(4x)^2 \times 4x^2$

b. $(3d^{-3})^3 \times 3d^{-2}$

c. $4(-2x^5y^0)^{-2} \times (2x^{-1}y^2)^{-3}$

Apply the laws to the following examples as you simplify the questions. Leave everything as exact numbers, with positive exponent answers.

d. $\dfrac{\left(-3c^{4}\right)^{-2}}{c^{-1}\times\left(3c^{-2}\right)^{-2}}$

e. $\left(\dfrac{\left(-2a^{-2}\right)^{3}a^{3}}{4a^{-4}}\right)^{-3}$

f. $\dfrac{\left(-2xy^{3}\times 3x^{-3}y^{-2}\right)^{3}}{6x^{0}y^{-1}}$

g. $\left(5k^{3}\right)^{-1}-5k^{-1}$

h. $\left(-2x^{3}+x^{-1}\right)^{2}$

i. $\dfrac{2x^{-1}}{x^{-1}+x}$

Rational Exponent Law

5. In the examples below what is the power you should raise the base on each side to get the exponent to dissappear? Conclude, how do radicals and exponents relate to each other?

$A = l^2$ $V = x^3$

$\sqrt{A} = l$ $\sqrt[3]{V} = x$

6. Write the expression in another way

$b^{\frac{2}{3}}$

7. What is the **rational exponents** rule?

8. Rewrite the following in a different notation. Simplify if possible.
(HINT if brackets or exponents are not there – insert them)

a. $6^{\frac{2}{7}}$ b. $\sqrt[4]{2x^5}$ c. $\sqrt[3]{(3a)^5}$ d. eg. $\sqrt{16x^7}$

9. Simplify. Keep answers as exact reduced fractions and don't leave answers with negative exponents.

a. $-49^{\frac{1}{2}}$ b. $(-125)^{-\frac{2}{3}}$ c. eg. $\left(\frac{36}{49}\right)^{-1.5}$

d. $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ e. $8^{\frac{2}{3}} \times 8^{-\frac{1}{3}}$ f. $\frac{8 \times 8^{\frac{2}{3}}}{\sqrt[3]{8}}$

10. Simplify the following. Give a reason why you can't cancel x^6 or divide 512 with 4.

$$\frac{\sqrt[3]{512x^6}}{\sqrt{4x^6}}$$

Simplify Expressions with Exponents

1. Explain why these are not equivalent:

   $\sqrt[3]{27x^4}$ and $(27x)^{\frac{4}{3}}$

2. Simplify the following. Keep answers as exact reduced fractions and and don't leave answers with negative exponents.

   a. $\left(8x^6y^9\right)^{\frac{1}{3}}\left(27x^{-12}y^2\right)^{-\frac{1}{3}}$

   b. $\left(\dfrac{64m^{15}}{343}\right)^{-\frac{2}{3}}$

   c. $\left(256a^{12}b^{20}\right)^{\frac{3}{4}}$

   d. $\left(3a^{\frac{3}{2}}\right)\left(-7a^{\frac{1}{5}}\right)$

   e. $\left(8x^{\frac{3}{4}}y^2\right)^{-\frac{1}{3}}$

   f. $\dfrac{25x^{\frac{1}{3}}}{5x^{\frac{1}{4}}}$

   g. $\dfrac{\left(\sqrt[3]{4a^2}\right)^2}{\sqrt[6]{4a^2}}$

   h. $\left[(x-9)^{18}\right]^{\frac{1}{9}}$

Solve Equations by SAMDEB-Matching Bases-Trial&Error methods

When the variable is **on the base**, not in the exponent, to solve it you must isolate it by using ________________________

When the variable is **in the exponent**, not on the base, to solve it you must ____________________________________

When the variable is **in the exponent** and bases **cannot** be matched you must use ______________________________

1. Match the method to each given question. Then solve.

   a. $2^{2x} = 8$

   b. $x^{\frac{4}{5}} = 81$

   c. $3^{x} = 12$

2. Practice SAMDEB method:

   a. $x^{-\frac{1}{2}} = \frac{9}{16}$

   b. $\sqrt[3]{x+5} = 4$

   c. $37 = 1 + (6x)^{\frac{2}{3}}$

There are several useful constants that are used for math
π is called _________ and π = _________________ is used with anything circular
e is called _______________________ and e = _________________ is used with exponential continuous growth/decay

3. Practice trial & error method:

   a. $6^{x} + 5 = 10$

   b. $2 \cdot e^{x} = 17$

   c. $3(2)^{2m-1} + 2 = 14$

4. Practice matching bases method:

a. $3^{2x-5} = 1$

b. $5^{4-x} = 5^{x}$

c. $7^{2x} \cdot 7^{3-x} = 49^{x+5}$

d. $4^{x} \cdot \frac{1}{16} = 2^{3x+6}$

e. $\left(\frac{1}{5}\right)^{-3x} \cdot 25^{x-1} = \frac{1}{125}$

f. $3^{x+3} - 3^{x} = 234$

Introduction to Exponential Functions

1. Recall the shapes of the functions you've learned so far (linear, quadratic, square root, cubic, cube root, rational, absolute value).

2. How can you recognize an exponential from a table?

3. What type of function is each of the below tables?

a.

x	y
10	7
12	9
14	11
16	13
18	15
20	17

b.

x	y
1	2
2	5
3	10
4	17
5	26
6	37

c.

x	y
1	3
2	9
3	27
4	81
5	243
6	729

d.

x	y
1	16
2	8
3	4
4	2
5	1
6	0.5

e.

X	y
1	2.2
3	-0.5
5	-5.8
6	-10.1
7	-16.1
9	-36.3
11	-76.0

f.
same as table e, just
subtract 5 from each y value

4. What makes an equation exponential? Give examples

5. How can you recognize an exponential from a graph? (use technology to see then discuss why base can't be negative or one.)

6. Give an **example** of a base that creates exponential growth, and one for decay. Sketch these parent graphs. What is the HA and y-int for each one of the parent graphs? What is the domain and range?

7. For the parent graph $y = b^x$ write down the full transformed form and then the simplified version.

8. Identify the parent function and transformation constants for each. Show how to get the full transformed form into simplified one.

$y = -4(2)^{3(x-1)} + 5 \rightarrow y = -\frac{1}{2}(8)^x + 5$

Parent y=	y=
a=	a=
k=	k=
d=	d=
c=	c=

9. Practice changing the form:

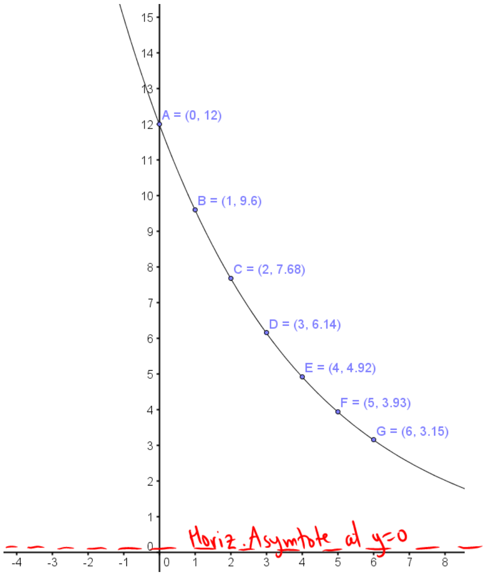
a) $y = 5(2)^{2x-4}$

b) $y = 0.5(8)^{\frac{2-x}{3}} - 4$

10. For the simplified version, explain what the constants will mean in terms of graph:

11. Find the equations that will model each of the following.

a.

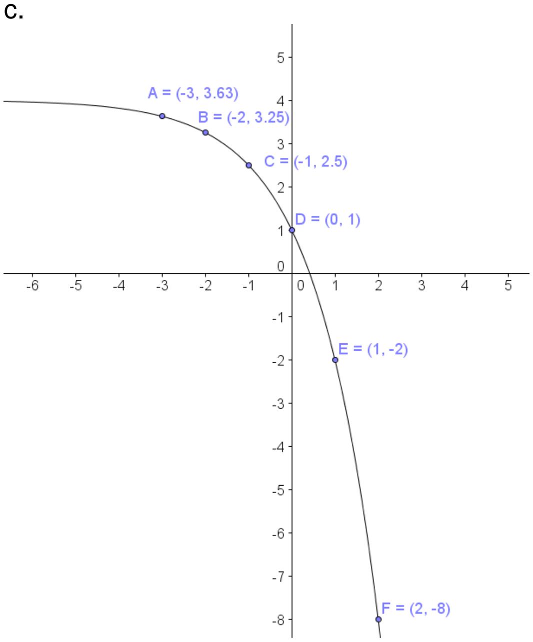


b.

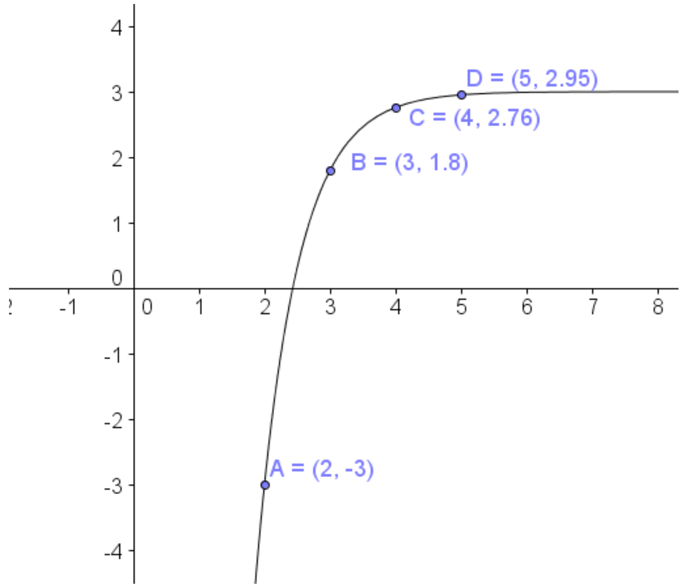
x	y
-1	168
0	84
1	42
2	21
3	10.5
4	5.25
5	2.625

Horizontal asymptote at y=0.

c.



d.



e.

x	y
3	17.49783
6	71.6182
9	314.7336
12	1406.839
15	6312.711
18	28350.5

Horizontal asymptote at y=2.

Transformations of Exponentials

1. Clarify how to separate a simplified exponential 2^{2x-4}. This is often useful for solving exponentials that involve addition/subtraction of same base, as well as for sketching.

2. Solve $3^{x+1}-3^{2x}=-648$

3. Rrewrite in simplified form then state the parent and transformations. Sketch.

a. $y=0.5^{-x}+6$

b. $y=-4(0.8)^{2x-4}$

c. $y=0.5(2.5)^{\frac{2-x}{3}}-4$

4. Rewrite in simplified form, state the parent function and then sketch.

a. $y = 2(3.5)^{-x+6} - 3$

b. $y = -3(1.6)^{\frac{x-4}{2}}$

c. $y = -0.5(4)^{6+3x} + 8$

5. Describe what transformations must be done, and in what order, for the function $y = 2(4.5)^x + 1$ be become $y = 4.5^{6-3x}$

Application of Exponentials

1. Most real life word problems of growth or decay have a horizontal asymptote at y=0. What is the equation usually used for **word problems?** Explain the significance of EACH letter in the context of a word problems.

2. Summarize how to find the 'b' in the equation.

3. Clarify the differences between growth **facto**r and growth **rate** for $f(x)=125(1.32)^x$

4. Assign variables and set up the models for the following word problems.

a. The value of the $250 thousand cottage increases by 0.1% every 3 weeks.

b. The 40 grams of radioactive matter within a mass decays at 2% every 30 seconds.

c. The 200 fruit fly population doubles every 5 days.

5. Assign variables and set up the models for the following word problems. Then solve the question.

a. A certain strain of yeast cell doubles under certain conditions every 20 minutes. If there were 350 initially, how many will there be in 3 hours?

b. How long will it take for a 1 gram sample of polonium-210 to lose $\frac{3}{4}$ of its radioactivity if its half life is 140 days?

c. For a biology experiment, the number of cells present is 1000. After 4 hours the count is estimated to be 256 000. What is the doubling period of the cells?

Solve the following problems:

6.
a). A drug's effectiveness decreases as time passes. Each hour the 250mg drug is only 95% effective as the previous hour. How effective is the drug after 150 minutes?

b) How long will it take for the dose to reach the low level of 52mg?

7.
a) Carbon-14 has a half life of 5730 years. Determine the % of original carbon left after 1000 years.
(If no initial amount is given, assume 100% is the initial amount)

b) Some pre-historic cave paintings were discovered in a cave in France. If the paint contained 48% of the original carbon-14, estimate the age of the painting.

8.
Health officials found traces of radium-F beneath the local library. After 69 days they observed that a certain amount of the substance decayed to $\frac{1}{\sqrt{2}}$ of its original mass. Determine the half-life of radium-F.

9.
After an oven is turned on, its temperature, *T*, is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$, where *m* represents the number of minutes after the oven is turned on and T represents the temperature of the oven, in degrees Fahrenheit. About how many minutes does it take for the oven's temperature to reach 300°F? Show your work and/or explain how you arrived at an answer.