S14ExponentialsNOTES

July 9, 2014 2:47 PM

ExponentialsNOTES

Exponentials Unit 4

2^x OR x^2 ← quadratic not exponential.

exponential power is a variable!

Tentative TEST date_______________

Big idea/Learning Goals

This unit introduces you to a new type of function – the exponential function. There are many relationships in real life that either grow or decay at a constant rate, for example: population growth, radioactive decay, inflation, spread of viruses, or processing power of technological gadgets. These types of relationships can be modelled with an exponential function. Before you learn about properties of this function, you must review exponent laws from gr.9-10 and learn a new exponent law that allows you to work with rational (fraction) exponents. Keep in mind that this year you will not know what the inverse of the exponential is. This is something that you will learn in the gr.12 advanced functions course.

Corrections for the textbook answers:
Sec 4.2 #18 c) b^{m+4n}
Sec 4.3 #2 d) –7776 # 3d) 1/7 #18a) 1/6 and – 1/6
Sec 4.5 #1c) neither, too hard if HA is not given 1st ratios will not be the same… #2a) decrease
Sec 4.7 #10c) P = 2^t #14b) P=$10^{10}(0.01)^n$ 5 applications to reach 1 germ, so 6 to kill all
Review #5f f^(1/5)

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

finish Quad + Day 6

Date	pages	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
	2-3	Review of Exponent Laws & Section 4.2 & Handout #17-30		
Day 7	4-5	Rational Exponents & Simplify Expressions with Exponents Section 4.3 & Handout Section 4.4 & Handout Handout 1 #17-32	Handout 2 #15-22	
	6-7	Solve by SAMDEB-Matching Bases-Trial and Error Three Handouts Handout 1 # 9-18	Handout 2 #13-24	Handout 3 #5,7,9,11
		QUIZ no calculators		
	8-11	Introduction to Exponential Functions Section 4.5 & two Handouts Handout 1 ALL		
	12-13	Transformations of Exponentials Section 4.6 & Handout ALL		
Day 8	14-15	Application of Exponentials Section 4.7 & Handout ALL		
		REVIEW		

+ start Trig

Reflect – previous TEST mark __________, Overall mark now__________.

Review of Exponent Laws

1. Why do you think exponent notation was invented?

$(3)(3)(3)(3)(3)(3) = 3^6$

Exponents are used as shorthand to indicate how many times is the base multiplied by itself.

2. Summarize the 5 laws you learned in grade 9-10.

$a^m \cdot a^n = a^{m+n}$ Keep base!

$2^1 a^{-n} = \frac{2}{a^n}$ only move the base with negative power.

$\frac{a^m}{a^n} = a^{m-n}$ Write the result in numerator.

$(a^m)^n = a^{mn}$ → distribution over mult/div.

$(2^1 a^2 b^3)^4 = 2^4 a^8 b^{12}$
$= 16a^8 b^{12}$

$a^0 = 1$

3. There will be several ways to simplify expressions, depending on what rule you start applying first. Final answers should still match no matter what route you take. To make things easier try to use the ___BEDM~~AS~~___ law first and ___neg power law___ law last.

outer exponent

4. Apply the laws to the following examples as you simplify the questions. Leave everything as exact numbers, with positive exponent answers.

a. $(4x)^2 \times 4x^2$

$= 64x^4$

b. $(3d^{-3})^3 \times 3d^{-2}$

$= 3^3 d^{-9} \cdot 3^1 d^{-2}$

$= 3^4 d^{-11}$

$= \frac{81}{d^{11}}$

c. $4(-2x^5y^0)^{-2} \times (2x^{-1}y^2)^{-3}$

$= 4(-2)^{-2} x^{-10} y^0 (2)^{-3} x^3 y^{-6}$

$= \frac{4(-2)^{-2} x^{-7} y^{-6} (2)^{-3}}{1}$

$= \frac{4}{(-2)^2 x^7 y^6 (2)^3}$

$= \frac{\cancel{4}}{\cancel{4} x^7 y^6 8}$

$= \frac{1}{8x^7y^6}$

Apply the laws to the following examples as you simplify the questions. Leave everything as exact numbers, with positive exponent answers.

d. $\dfrac{\left(-3c^{4}\right)^{-2}}{c^{-1}\times\left(3c^{-2}\right)^{-2}}$

$= \dfrac{(-3)^{-2}c^{-8}}{c^{-1}\cdot(3)^{-2}c^{4}}$

$= \dfrac{(-3)^{-2}c^{-8--1-4}}{(3)^{-2}}$

$= \dfrac{(-3)^{-2}c^{-11}}{(3)^{-2}} = \dfrac{3^{2}}{(-3)^{2}c^{11}}$

$= \dfrac{9}{9c^{11}}$

$\dfrac{1}{c^{11}}$

e. $\left(\dfrac{\left(-2a^{-2}\right)^{3}a^{3}}{4a^{-4}}\right)^{-3}$

$= \left(\dfrac{(-2)^{3}a^{-6}a^{3}}{4a^{-4}}\right)^{-3}$

$= \left(\dfrac{(-8)a^{-6+3--4}}{4}\right)^{-3}$

$= \left(-2a^{1}\right)^{-3}$

$= (-2)^{-3}a^{-3}$

$\dfrac{1}{-8a^{3}}$

f. $\dfrac{\left(-2xy^{3}\times 3x^{-3}y^{-2}\right)^{3}}{6x^{0}y^{-1}}$

$= \dfrac{\left(-6x^{-2}y\right)^{3}}{6y^{-1}}$

$= \dfrac{(-6)^{3}x^{-6}y^{3}}{6y^{-1}}$

$y^{3}y^{1}$

y^{3--1}

$= \dfrac{-216y^{4}}{6x^{6}}$

$= \dfrac{-36y^{4}}{x^{6}}$

g. $\left(5k^{3}\right)^{-1}-5k^{-1}$

$= 5^{-1}k^{-3} - 5k^{-1}$

$= \dfrac{1}{5k^{3}} - \dfrac{5\cdot 5k^{2}}{k\cdot 5k^{2}}$

$= \dfrac{\left(1-25k^{2}\right)}{5k^{3}}$

h. $\left(-2x^{3}+x^{-1}\right)^{2}$

$= \left(-2x^{3}+x^{-1}\right)\left(-2x^{3}+x^{-1}\right)$

FOIL

$= 4x^{6} - 2x^{2} - 2x^{2} + x^{-2}$

$= 4x^{6} - 4x^{2} + \dfrac{1}{x^{2}}$

$\therefore$ LCD

not all over x^{2}!!

i. $\dfrac{2x^{-1}}{x^{-1}+x}$

not allowed if there's +/-

$= \dfrac{\frac{2x^{-1}}{1}}{\frac{x^{-1}}{1}+\frac{x}{1}}$

$= \dfrac{\frac{2}{x}}{\left(\frac{1}{x}+x\right)}$ LCD

$= \dfrac{2}{x} \div \left(\dfrac{1}{x} + \dfrac{x\cdot x}{1\cdot x}\right)$

$= \dfrac{2}{x} \div \left(\dfrac{1+x^{2}}{x}\right)$

$= \dfrac{2}{x}\cdot\left(\dfrac{x}{1+x^{2}}\right)$

$= \dfrac{2\cancel{x}}{\cancel{x}\left(1+x^{2}\right)} = \dfrac{2}{1+x^{2}}$

Rational Exponent Law

5. In the examples below what is the power you should raise the base on each side to get the exponent to dissapear? Conclude, how do radicals and exponents relate to each other?

$A = l^2$ $\quad (\quad)^{\frac{1}{2}} = \sqrt{\quad}$

$\sqrt{A} = l$

$V = x^3$ $\quad (\quad)^{\frac{1}{3}} = \sqrt[3]{\quad}$

$\sqrt[3]{V} = x$

6. Write the expression in another way

$b^{\frac{2}{3}} = \left(b^{\frac{1}{3}}\right)^2$

$= \left(\sqrt[3]{b}\right)^2 = \sqrt[3]{b^2}$

7. What is the **rational exponents** rule?

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ OR

Flower Power

m

n Roots

8. Rewrite the following in a different notation. Simplify if possible.
(HINT if brackets or exponents are not there – insert them)

a. $6^{\frac{2}{7}}$

$= \sqrt[7]{6^2}$

$\cancel{\sqrt[7]{6}}$

make the root # small

b. $\sqrt[4]{2x^5}$ $\quad (2x^5)^{\frac{1}{4}}$

$= 2^{\frac{1}{4}} x^{5/4}$

c. $\sqrt[3]{(3a)^5}$

$= \sqrt[3]{3^5 a^5}$ OR $(3a)^{5/3}$

$= 3^{5/3} a^{5/3}$

d. Eg. $\sqrt[2]{16x^7}$

$= 16^{\frac{1}{2}} x^{7/2}$

OR $(16x^7)^{\frac{1}{2}}$

9. Simplify. Keep answers as exact reduced fractions and don't leave answers with negative exponents.

a. $-49^{\frac{1}{2}}$

$= \cancel{\sqrt{-49}}$ OR $-\sqrt{49}$

no brackets on base.

$= -7$

b. $(-125)^{\frac{2}{3}}$

$= \sqrt[3]{(-125)^{-2}}$

$= (-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$

c. Eg. $\left(\frac{36}{49}\right)^{-1.5} \rightarrow -\frac{3}{2}$

$= \sqrt[2]{\left(\frac{36}{49}\right)^{-3}} = \left(\frac{6}{7}\right)^{-3} = \frac{6^{-3}}{7^{-3}}$

$= \frac{7^3}{6^3} = \frac{343}{216}$

d. $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\left(\frac{8}{27}\right)^{-1}} = \left(\frac{2}{3}\right)^{-1} = \frac{2^{-1}}{3^{-1}} = \frac{3}{2}$

e. $8^{\frac{2}{3}} \times 8^{\frac{1}{3}}$

$= 8^{\frac{2}{3} + -\frac{1}{3}}$

$= 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

f. $\frac{8 \times 8^{\frac{2}{3}}}{\sqrt[3]{8}}$ $\quad 8^{-\frac{1}{3}}$

$= 8^{1 + \frac{2}{3} - \frac{1}{3}}$

$= 8^{4/3} = \sqrt[3]{8^4} = 2^4 = 16$

10. Simplify the following. Give a reason $\underbrace{\text{why you can't cancel } x^6 \text{ or divide 512 with 4.}}_{\text{not the same roots.}}$

$\frac{\sqrt[3]{512x^6}}{\sqrt{4x^6}}$

$= \frac{\sqrt[3]{512}\sqrt[3]{x^6}}{\sqrt{4}\sqrt{x^6}}$ awkward $x^{6/3}$ $\quad x^{6/2}$

$= \frac{8x^2}{2x^3}$

$= \frac{4}{x}$

Simplify Expressions with Exponents

1. Explain why these are not equivalent:

$\sqrt[3]{27x^4}$ and $(27x)^{\frac{4}{3}}$

power of 4 is only on x (no brackets)

2. Simplify the following. Keep answers as exact reduced fractions and and don't leave answers with negative exponents.

- Inside Brackets 1st
- outer exponent
- roots
- neg powers.

a. $\left(8x^6y^9\right)^{\frac{1}{3}}\left(27x^{-12}y^2\right)^{-\frac{1}{3}}$

$= 8^{\frac13}x^2y^3\,27^{-\frac13}x^4y^{-2/3}$

$= \sqrt[3]{8}\,x^6\,\sqrt[3]{27}^{\,-1}\,y^{7/3}$

$= 2x^6y^{7/3}3^{-1} = \frac{2x^6y^{7/3}}{3}$

$3 + \frac{3}{3\times1} + \frac{-2}{3}$

$\frac{9-2}{3}$

b. $\left(\frac{64m^{15}}{343}\right)^{-\frac{2}{3}}$

$= \frac{64^{-2/3}m^{-10}}{343^{-2/3}} = \frac{343^{2/3}}{64^{2/3}m^{10}} = \frac{\sqrt[3]{343^2}}{\sqrt[3]{64^2}\,m^{10}}$

$= \frac{7^2}{4^2m^{10}} = \frac{49}{16m^{10}}$

c. $\left(256a^{12}b^{20}\right)^{\frac{3}{4}}$

$64a^9b^{15}$

d. $\left(3a^{\frac{3}{2}}\right)\left(-7a^{\frac{1}{5}}\right)$

$-21a^{\frac{17}{10}}$

$= (3)(-7)\,a^{3/2}a^{1/5}$

$= -21a^{17/10}$

$\frac{5\cdot3}{5\cdot2} + \frac{1\cdot2}{5\cdot2}$

$= \frac{15+2}{10}$

e. $\left(8x^{\frac{3}{4}}y^2\right)^{-\frac{1}{3}}$

$\frac{3}{4}\left(-\frac{1}{3}\right)$

$= 8^{-\frac13}x^{-\frac14}y^{-2/3}$

$= \frac{1}{8^{\frac13}x^{1/4}y^{2/3}}$

$\frac{1}{2x^{1/4}y^{2/3}}$

* don't flip exponent!!

f. $\frac{25x^{\frac{1}{3}}}{5x^{\frac{1}{4}}}$

$5x^{\frac{1}{12}}$

g. $\frac{\left(\sqrt[3]{4a^2}\right)^2}{\sqrt[6]{4a^2}}$

$= \frac{\left(4^{1/3}a^{2/3}\right)^2}{4^{1/6}a^{2/6}} = \frac{4^{2/3}a^{4/3}}{4^{1/6}a^{2/6}}$

$= 4^{\frac23-\frac16}a^{\frac43-\frac26}$

$= 4^{\frac12}a^1$

$\sqrt{4}\,a$

$2a$

$2a$

$\frac{2\cdot2}{2\cdot3}-\frac16$, $\frac{4-1}{6}$, $\frac36$

$\frac{2\cdot4}{2\cdot3}-\frac26$, $\frac{8-2}{6}$, $\frac66$

h. $\left[(x-9)^{18}\right]^{\frac{1}{9}}$

$= (x-9)^{18/9}$

$= (x-9)^2$

FOIL

$x^2-18x+81$

not for exponentials.

Solve Equations by SAMDEB-Matching Bases-Trial&Error methods

Solve Equations by SAMDEB-Matching Bases-Trial&Error methods

When the variable is **on the base**, not in the exponent, to solve it you must isolate it by using SAMDEB

$x^2, x^3 \ldots$

When the variable is **in the exponent**, not on the base, to solve it you must Match BASES to equate powers

$2^x, 5^x \ldots$

When the variable is **in the exponent** and bases **cannot** be matched you must use trial + error

* next year this will be with $\boxed{\log}$ button

1. Match the method to each given question. Then solve.

a. $2^{2x} = 8$ MATCHING bases METHOD.

$2^{2x} = 2^3$

ignore base if it appears once on each side!

$\frac{2x}{2} = \frac{3}{2}$ $x = \frac{3}{2}$

b. $x^{\frac{4}{5}} = 81$ SAMDEB

$\left(\sqrt[5]{x^4}\right)^5 = (81)^5$

$\sqrt[4]{x^4} = \sqrt[4]{81^5}$

$x = 3^5 = 243$

c. $3^x = 12$ Trial + error

try $3^3 = 27$ big

$3^2 = 9$ small

$3^{2.5} = 15.588\ldots$ big

$3^{2.25} = 11.844$

$\therefore x \doteq 2.25$ OR $x \doteq 2.27$

2. Practice SAMDEB method:

a. $x^{-\frac{1}{2}} = \frac{9}{16}$

$\frac{1}{x^{1/2}} = \frac{9}{16}$

$\frac{1}{\sqrt{x}} = \frac{9}{16}$

$\frac{16}{9} = \frac{9\sqrt{x}}{9}$

$\left(\frac{16}{9}\right)^2 = \left(\sqrt{x}\right)^2$

$\frac{256}{81} = x$

b. $\left(\sqrt[3]{x+5}\right)^3 = (4)^3$

$x + 5 = 64$ -5

$x = 59$

c. $37 = 1 + (6x)^{\frac{2}{3}}$

$(36)^3 = \left(\sqrt[3]{(6x)^2}\right)^3$

$\sqrt{36^3} = \sqrt{(6x)^2}$

$\frac{6^3}{6} = \frac{6x}{6}$

$6^2 = x$

$36 = x$

e^x e^1

There are several useful constants that are used for math

π is called pi and π = 3.14159 2654... is used with anything circular

e is called Euler's constant and e = 2.718 28182... is used with exponential continuous growth/decay

3. Practice trial & error method:

a. $6^x + 5 = 10$

isolate

$6^x = 5$

try

$6^1 = 6$ too big

$6^0 = 1$ too small

$6^{0.5} \doteq 2.4$ too small

$6^{0.9} \doteq 5.015$

$\therefore x \doteq 0.9$ by trial + error.

b. $2 \cdot e^x = 17$

$2(2.718)^x = 17$

isolate

$2.718^x = 8.5$

try

$2.718^2 = 7.387$ too small

$2.718^{2.2} \doteq 9.022$ too big.

$x \doteq 2.1$

c. $3e^{\boxed{2m-1}} + 2 = 14$

let $x = 2m-1$

$3e^x + 2 = 14$ -2

isolate

$3e^x = 12$

$2.718^x = 4$

try

$x \doteq 1.4$

$2m - 1 = 1.4$ $+1$

$2m = 2.4$

$m = 1.2$

4. Practice matching bases method:

a. $3^{2x-5} = 1$

$3^{2x-5} = 3^0$

ignore bases

$2x - 5 = 0$

$2x = 5$

$x = \frac{5}{2}$

check:

LS	RS
$3^{2(5/2)-5}$	$= 1$
$3^{5-5} = 3^0$	

✓

b. $5^{4-x} = 5^x$

ignore base

$4 - x = x$

$\frac{4}{2} = \frac{2x}{2}$

$2 = x$

LS	RS
5^{4-2}	$= 5^2$

c. $7^{2x} \cdot 7^{3-x} = 49^{x+5}$

$(a^m)^n = a^{mn}$

$\underbrace{7^{2x} \cdot 7^{3-x}}_{\text{combine using}} = (7^2)^{x+5}$

combine using $a^n \cdot a^m = a^{n+m}$

$7^{2x+3-x} = 7^{2x+10}$

ignore bases

$2x + 3 - x = 2x + 10$

$x + 3 = 2x + 10$

$-7 = x$

d. $4^x \cdot \frac{1}{16} = 2^{3x+6}$

* can't reduce if powers are different!

power law

$\frac{(2^2)^x}{1} \cdot \frac{1}{2^4} = 2^{3x+6}$

$\frac{2^{2x}}{2^4} = 2^{3x+6}$

$\frac{a^m}{a^n} = a^{m-n}$

$2^{2x-4} = 2^{3x+6}$

ignore base

$2x - 4 = 3x + 6$

$-4 - 6 = 3x - 2x$

$-10 = x$

e. $\left(\frac{1}{5}\right)^{-3x} \cdot 25^{x-1} = \frac{1}{125}$

$(5^{+3x})(\boxed{5^2}^{\,x-1}) = (5^{-3})$

$5^{3x} \cdot 5^{2x-2} = 5^{-3}$

$5^{5x-2} = 5^{-3}$

ignore bases

$\therefore 5x - 2 = -3$

$5x = -1$

$x = \frac{-1}{5}$

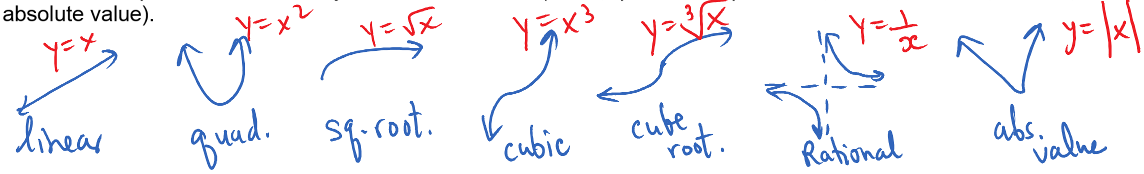
f. $3^{x+3} - 3^x = 234$

HARD

Adv. Functions? Look online.

Introduction to Exponential Functions → Graphs.

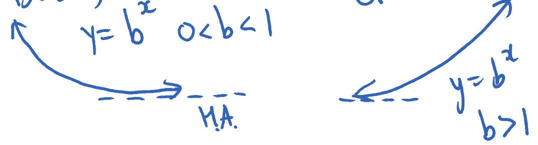
1. Recall the shapes of the functions you've learned so far (linear, quadratic, square root, cubic, cube root, rational, absolute value).



2. What makes an equation exponential? How can you recognize an exponential from a graph? (use technology to see) How can you recognize an exponential from a table?

- To be exponential - variable must be in the exponent
  ex. 2^x is exponential , x^2 is not (quad.)

- Decay $y=b^x$ $0<b<1$ — Growth $y=b^x$ $b>1$

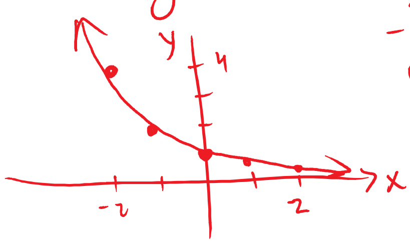


- 1st differences = next y − prev y. → LINEAR
- 2nd differences → QUAD.
- 3rd differences → CUBIC
- 1st ratios = next y ÷ prev. y → EXPONENTIAL

3. Give an example of a base that creates exponential growth, and one for decay. Sketch these parent graphs. Are these the only two parent exponential graphs? Use technology to discuss why base can't be negative or one.

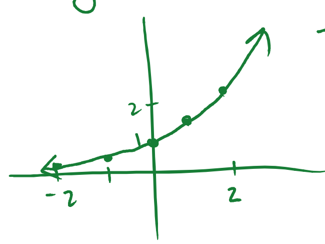
ex. decay $y=0.5^x$

x	y
−2	$0.5^{-2}=4$
−1	$0.5^{-1}=2$
0	$0.5^{0}=1$
1	$0.5^{1}=0.5$
2	$0.5^{2}=0.25$



ex. growth $y=1.5^x$

x	y
−2	$1.5^{-2}\doteq 0.\overline{4}$
−1	$1.5^{-1}=0.\overline{6}$
0	$1.5^{0}=1$
1	$1.5^{1}=1.5$
2	$1.5^{2}=2.25$



No, These are not the only exponential graphs
There are ∞ many choices for the base
as long as $b\neq 0$ $b\neq 1$ $b\neq$ neg.

no continuous graph!

$(a^n)^m = a^{nm}$ $a^n \cdot a^m = a^{n+m}$

4. Write down the full transformed form for an exponential and then the simplified version. Explain the what the constant control. Use the following example to show why simplified is algebraically the same.

$y = -4(2)^{3(x-1)} + 5 \rightarrow y = -\frac{1}{2}(8)^x + 5$

a = −4, k = 3, d = 1, c = 5, parent: $y = 2^x$

a = $-\frac{1}{2}$, k = 1, d = 0 } nothing, c = 5, parent: $y = 8^x$

How to simplify:

$y = -4(2)^{3x-3} + 5$

$y = -4(2)^{3x}(2)^{-3} + 5$

$y = -4(2)^{3x}\left(\frac{1}{8}\right) + 5$

$y = -\frac{1}{2}(2^3)^x + 5$

$y = -\frac{1}{2}(8)^x + 5$

$-4\left(\frac{1}{8}\right) = \frac{-4}{8} = \frac{-1}{2}$

5. Make a note about HA and y-int for parent shapes. How can you find the domain, range, HA and y-int if the parent shape is transformed? Find these for the above example.

can see from the sketch.

For parent graphs: • HA y = 0 x-axis • y-int (0,1)

For transformed: • HA y = c *always • y-int (0, a) *only if k=1, d=0 } simplified form only.

6. What type of function is each of the below tables?

a.

x	y	1st diff.
10	7	+2
12	9	+2
14	11	+2
16	13	+2
18	15	+2
20	17	+2

∴ Linear

b.

x	y	1st diff.	2nd
1	2	+3	+2
2	5	+5	+2
3	10	+7	+2
4	17	+9	
5	26	+11	
6	37		

∴ quadratic

c.

x	y
1	3
2	9
3	27
4	81
5	243
6	729

×3, ×3, ×3

1st ratios: 9/3 = 3, 27/9 = 3, 81/27 = 3

∴ exponential growth HA y=0

d.

x	y
1	16
2	8
3	4
4	2
5	1
6	0.5

× ½, × ½

1st ratios: 8/16 = ½, 4/8 = ½

∴ exponential decay. HA y=0

e.

X	y
1	2.2
3	-0.5
5	-5.8
~~6~~	~~-10.1~~
7	-16.1
9	-36.3
11	-76.0

1st ratios: $\frac{-0.5}{2.2} = 0.227$, $\frac{-5.8}{-0.5} = 11.6$ ⋮

look like not exponential!

HA y=5

f.
same as table e, just subtract 5 from each y value

shift up/down ∴ HA y = c must be given this for tables.

x	y
1	-2.8
3	-5.5
5	-10.8
7	-21.1
9	-41.3
11	-81

1st ratios: $\frac{-5.5}{-2.8} = 1.96\ldots$, $\frac{-10.8}{-5.5} = 1.96\ldots$, $\frac{-21.1}{-10.8} = 1.96\ldots$

HA y=0 ∴ exponential.

9

Parent: $\boxed{y = b^x}$ Simplified Transformed Form $\boxed{y = ab^x + c}$

$y = ab^{k(x-d)} + c$ full transf.

7. Find the equations that will model each of the following.

a. b. Make a note of the shortcut to find the 'b' if HA is y=0

Transformed Form

$y = a\, b^{k(x-d)} + c$ full transf.

7. Find the equations that will model each of the following.

a.

x	y
3	17.49783
6	71.6182
9	314.7336
12	1406.839
15	6312.711
18	28350.5

Horizontal asymptote at y=2. c

$y = ab^x + 2$

$17.5 = ab^3 + 2$ pt. (3, 17.5)

$\boxed{15.5 = ab^3}$ ①

$71.6 = ab^6 + 2$ pt. (6, 71.6)

$\boxed{69.6 = ab^6}$ ②

$\frac{②}{①}$ $\frac{69.6 = ab^6}{15.5 = ab^3}$

$\sqrt[3]{4.49} = \sqrt[3]{b^3}$

$1.65 = b$

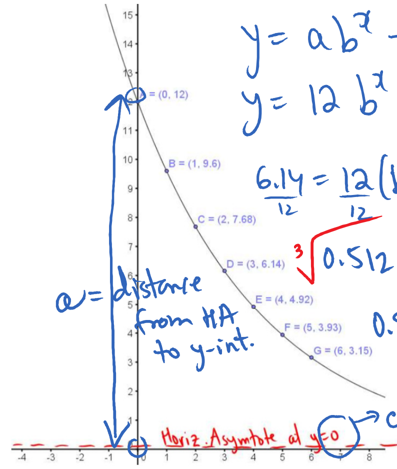
sub in ①

$15.5 = a(1.65)^3$

$3.45 = a$

$\boxed{\therefore y = 3.45(1.65)^x + 2}$

b. Make a note of the shortcut to find the 'b' if HA is y=0.



a = distance from HA to y-int.

Horiz. Asymptote at y=0 → c

$y = ab^x + c$

$y = 12\, b^x + 0$ pt. (3, 6.14)

$\frac{6.14}{12} = \frac{12(b)^3}{12}$

$\sqrt[3]{0.512} = \sqrt[3]{b^3}$

$0.8 = b$

$\boxed{\therefore y = 12(0.8)^x + 0}$

c.

x	y
-1	168
0	84
1	42
2	21
3	10.5
4	5.25
5	2.625

y-int

distance a=84

Horizontal asymptote at y=0. = c

$y = 84(b)^x + 0$ pt. (1, 42)

$42 = 84(b)^1$

$\frac{1}{2} = b$

$\boxed{\therefore y = 84\left(\frac{1}{2}\right)^x + 0}$

d.

$y = ab^x + 3$ pt. (2, -3)

$-3 = ab^2 + 3$

$\boxed{-6 = ab^2}$ ①

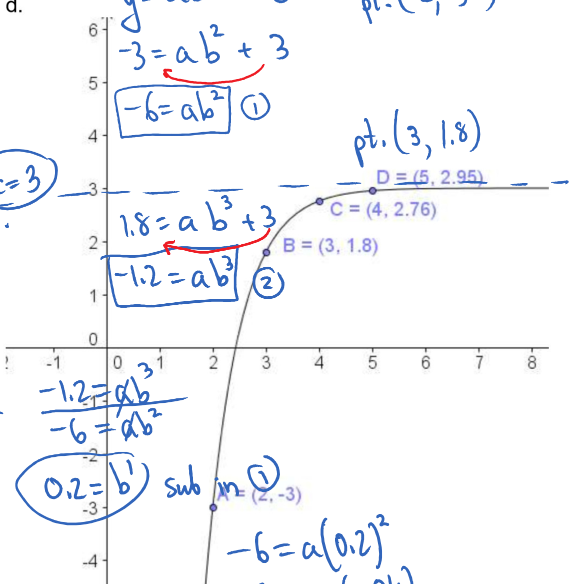
pt. (3, 1.8)

c=3

HA

$1.8 = ab^3 + 3$

$\boxed{-1.2 = ab^3}$ ②



$\frac{②}{①}$ $\frac{-1.2 = ab^3}{-6 = ab^2}$

$0.2 = b^1$ sub in ①

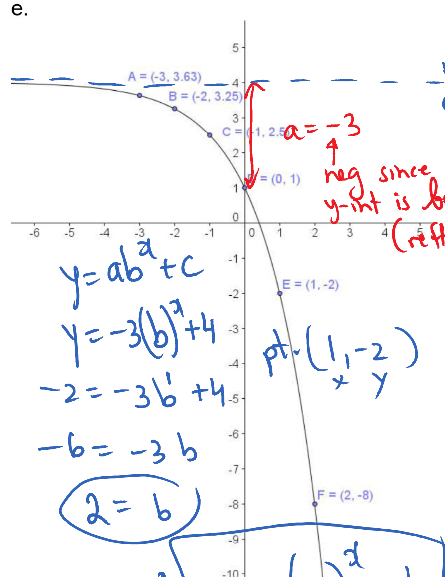
$-6 = a(0.2)^2$

$-6 = a(0.04)$

$-150 = a$

$\boxed{\therefore y = -150(0.2)^x + 3}$

e.



HA c=4

a = -3

neg since y-int is below HA (reflected)

$y = ab^x + c$

$y = -3(b)^x + 4$ pt. (1, -2) x y

$-2 = -3b^1 + 4$

$-6 = -3b$

$2 = b$

$\boxed{\therefore y = -3(2)^x + 4}$

Transformations of Exponentials

1. Clarify how to separate a simplified exponential 2^{2x-4}. This is often useful for solving exponentials that involve addition/subtraction of same base, as well as for sketching.

2. Solve $3^{x+1} - 3^{2x} = 234$

Hard. not on Quiz.

3. Rewrite in simplified form, state the parent function and then sketch.

a.

$y = 0.5^{-x} + 6$

b.

$y = -4(0.8)^{2x-4}$

$y = -4(0.8)^{2x}(0.8)^{-4}$

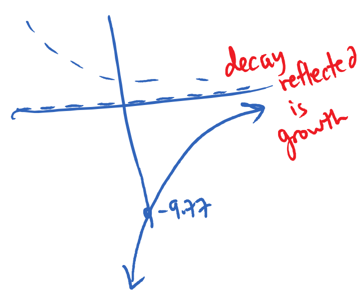
$y = -4(0.8^2)^x(0.8)^{-4}$

$y = -9.77(0.64)^x + 0$

parent: $y = 0.64^x$ decay

$a = -9.77$ - reflect in x-axis - vertical stretch

$\cancel{c = 0}$ HA=0



c.

$y = 0.5(2.5)^{\frac{2-x}{3}} - 4$

$a^{n+m} = a^n \cdot a^m$

simplified form

$y = 0.5(2.5)^{\frac{2}{3} - \frac{1}{3}x} - 4$

$y = 0.5(2.5)^{2/3}(2.5)^{-1/3x} - 4$

$a^{nm} = (a^n)^m$

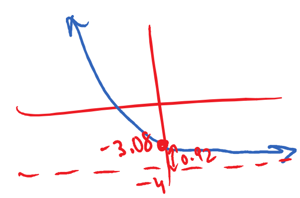
$y = 0.5(2.5)^{2/3}(2.5^{-1/3})^x - 4$

$y = \underset{a}{0.92}\,\underset{b}{(0.74)}^x \underset{c}{-4}$

parent: $y = 0.74^x \rightarrow$ decay

distance between HA + y-int $\rightarrow$ $a = 0.92$ vertical compress

HA $\rightarrow$ $c = -4$ shift down



4. State the parent and transformations of each of the following then rewrite in simplified form and restate the parent and trasnformations. Sketch.

do 1st.

a.

$y = 2(3.5)^{-x+6} - 3$

$y = 2(3.5)^{-x}(3.5)^{6} - 3$

$y = 2\left(3.5^{-1}\right)^{x}(3.5)^{6} - 3$

$y = 3676.5(0.29)^{x} - 3$

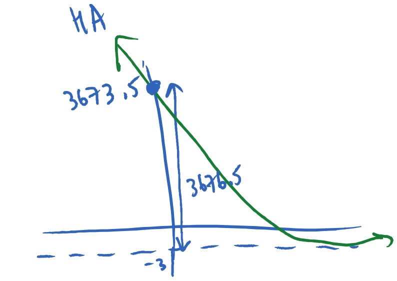
parent: $y = 0.29^{x}$ decay

$a = 3676.5$ vertical stretch

distance HA and y-int.

$c = -3$ shift down

HA



b.

$y = -3(1.6)^{\frac{x-4}{2}}$

$y = -3(1.6)^{\frac{1}{2}x}(1.6)^{\frac{-4}{2}} + \emptyset$

$y = -3\left(1.6^{\frac{1}{2}}\right)^{x}(1.6)^{-2} + \emptyset$

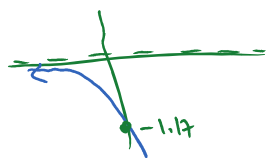
$y = -1.17(1.26)^{x} + \emptyset$

parent: $y = 1.26^{x}$ growth

$a = -1.17$ - reflected in x-axis

- vertical stretch

$c = 0 \leftarrow$ HA



c.

$y = -0.5(4)^{6+3x} + 8$

5. Describe what transformations must be done, and in what order, for the function $y = 2(4.5)^{x} + 1$ be become $y = 4.5^{6-3x}$

Application of Exponentials

$y = ab^x + \cancel{c}$

1. Most real life word problems of growth or decay have a horizontal asymptote at y=0. What is the equation usually used for **word problems?** Explain the significance of EACH letter in the context of a word problems.

$y = ab^{\frac{x}{p}}$

y ⇝ final amount, dependent
x ⇝ independent variable, usually time.
a ⇝ initial amount
* b ⇝ growth/decay FACTOR (not rate)
p - period of growth or decay

2. Summarize how to find the 'b' in the equation.

Words: "double" $b = 2$
"half-life" $b = \frac{1}{2}$

rate: % increase $b = 1 + r$
% decrease $b = 1 - r$
* r must be in decimal not %

3. Clarify the differences between growth **factor** and growth **rate** for $f(x) = 125(1.32)^x$

factor $b = 1.32$ bigger than 1 ∴ growth
$b = 1 + r$
$1.32 = 1 + r$
$r = 0.32$
$r = 32\%$

4. Assign variables and set up the models for the following word problems.

a. The value of the $250 thousand cottage increases by 0.1% every 3 weeks.

$r = 0.001$
$b = 1 + r$
$y = 250000(1.001)^{\frac{x}{3}}$ (x ← weeks, y ← value.)

b. The 40 grams of radioactive matter within a mass decays at 2% every 30 seconds.

$r = 0.02$
$\therefore b = 1 - 0.02$
$y = 40(0.98)^{\frac{x}{30}}$ (y ← grams, x ← sec)
$y = 40(0.98)^{\frac{x}{0.5}}$ (x ← min)

c. The 200 fruit fly population doubles every 5 days.

$b = 2$
$y = 200(2)^{\frac{x}{5}}$ (x ← days, y ← pop.)

5. Assign variables and set up the models for the following word problems. Then solve the question.

a. A certain strain of yeast cell doubles under certain conditions every 20 minutes. If there were 350 initially, how many will there be in 3 hours?

$y = 350(2)^{\frac{x}{20}}$ (y ← cells, x ← minutes)

$y(180) = 350(2)^{\frac{180}{20}}$
$= 350(2)^9$
$= 350(512)$
$= 179\,200$

∴ There will be 179 200 cells in 3 hrs.

b. How long will it take for a 1 gram sample of polonium-210 to lose $\frac{3}{4}$ of its radioactivity if its half life is 140 days?

initial → later final

$b = \frac{1}{2}$

$y = ab^{x/p}$
$y = 1\left(\frac{1}{2}\right)^{\frac{x}{140}}$ (x ← days, y ← grams.)

y = final grams left.
lose 3/4 g

$y = \frac{1}{4}$

$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{x}{140}}$

match bases $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{\frac{x}{140}}$

ignore base

$\frac{2}{1} = \frac{x}{140}$

$\boxed{x = 280 \text{ days}}$

c. For a biology experiment, the number of cells present is 1000. After 4 hours the count is estimated to be 256 000. What is the doubling period of the cells?

initial = a
final = y
$b = 2$
doubling period → ?

$\frac{256000}{1000} = \frac{1000}{1000}(2)^{\frac{4}{p}}$

$256 = 2^{4/p}$ match bases

$2^8 = 2^{4/p}$

ignore base

$\frac{8}{1} = \frac{4}{p}$

$8p = 4$

$p = \frac{1}{2}$

∴ doubles every 0.5 hr.

6. Solve the following problems:

a.

A drug's effectiveness decreases as time passes. Each hour the 250mg drug is only 95% effective as the previous hour. How effective is the drug after 150 minutes?

a mg, $r = 5\%$, x

$p = 60$ min

$b = 1 - r$

$b = 1 - 0.05 = 0.95$

$y = 250(0.95)^{\frac{150}{60}}$

$y = 219.9$ mg.

How long will it take for the dose to reach the low level of 52mg?

$x = ?$ final y

$52 = 250(0.95)^{\frac{x}{60}}$ min

$0.208 = (0.95)^{x/60}$

$\frac{x}{60} = \frac{\log(0.208)}{\log(0.95)}$

$\frac{x}{60} = \frac{30.6}{1}$

$x = 1836$ min.

b.

Carbon-14 has a half life of 5730 years. Determine the % of original carbon left after 1000 years.

(If no initial amount is given, assume 100% is the initial amount)

$b = \frac{1}{2}$, p, x

$y = 100\left(\frac{1}{2}\right)^{\frac{1000}{5730}}$ in %

$y = 88.6\%$ of original carbon

Some pre-historic cave paintings were discovered in a cave in France. If the paint contained 48% of the orinial carbon-14, estimate the age of the painting.

$x = ?$ Final $\% = y$

$48 = 100\left(\frac{1}{2}\right)^{\frac{x}{5730}}$

$0.48 = \left(\frac{1}{2}\right)^{x/5730}$

$\frac{x}{5730} = \frac{\log(0.48)}{\log\left(\frac{1}{2}\right)}$

$\frac{x}{5730} = \frac{1.058}{1}$

$x = 6067$ yrs.

d.

Health officials found traces in radium-F beneath the local library. After 69 days they observed that a certain amount of the substance decayed to $\frac{1}{\sqrt{2}}$ of its original mass. Determine the half-life of radium-F.

x, unknown

$b = \frac{1}{2}$ $p = ?$

use $a = 100\%$ or $a = 1$

then $y = \frac{1}{\sqrt{2}}(100\%)$

or $y = \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} = 1\left(\frac{1}{2}\right)^{\frac{69}{p}}$

$\left(\frac{1}{2}\right)^{1/2} = \left(\frac{1}{2}\right)^{\frac{69}{p}}$

ignore base

$\frac{1}{2} = \frac{69}{p}$

$p = 138$ days

e.

After an oven is turned on, its temperature, T, is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$, where m represents the number of minutes after the oven is turned on and T represents the temperature of the oven, in degrees Fahrenheit. About how many minutes does it take for the oven's temperature to reach 300°F? Show your work and/or explain how you arrived at an answer.

$300 = 400 - 350(3.2)^{-0.1m}$

$\frac{-100}{-350} = \frac{-350}{-350}(3.2)^{-0.1m}$

$0.29 = (3.2)^{-0.1m}$

$\underbrace{-0.1m}_{\text{whole exp}} = \frac{\log(0.29)}{\log(3.2)}$

$-0.1m = -1.06\ldots$

$m = 10.6$ minutes.

14