

# Dividing Rational Expressions

## I. Model Problems

In this examples we will divide and find the restrictions for rational expressions.

**Example 1: Simplify and find the restrictions of  $\frac{x+2}{x+4} \cdot \frac{x+2}{x-5}$ .**

Find the restrictions. Neither denominator can equal zero. The numerator of the divisor cannot equal zero.

$$\begin{array}{r} x + 4 = 0 \\ -4 \quad -4 \\ \hline x = -4 \\ x - 5 = 0 \\ +5 \quad +5 \\ \hline x = 5 \\ x + 2 = 0 \\ -2 \quad -2 \\ x = -2 \\ x \neq -4, -2, 5 \end{array}$$

Change division to multiply by the reciprocal.

$$\frac{(x+2)}{(x+4)} \cdot \frac{(x-5)}{(x+2)}$$

Cancel common factors. You can use parentheses to help see the factors.

$$\frac{\cancel{(x+2)}}{(x+4)} \cdot \frac{(x-5)}{\cancel{(x+2)}}$$

**Answer:  $\frac{x-5}{x+4} x \neq -4, -2, 5$**

## II. Practice Problems

Simplify and find the restrictions.

$$1. \frac{x}{y} \div \frac{a}{b}$$

$$2. \frac{4x^2}{y} \div \frac{5x}{3y^2}$$

$$3. \frac{16x^3y^2}{7x} \div \frac{8xy^2}{4}$$

4.

$$5. \frac{9y^4}{7x^3} \div \frac{3x}{5y}$$

$$6. \frac{x+2}{3x} \div \frac{x+2}{5x}$$

$$7. \frac{x(x+1)}{(y+2)} \div \frac{4x(x+1)}{(y+3)}$$

$$8. \frac{x^2+1}{x-1} \div \frac{x^2+1}{x-1}$$

$$9. \frac{3xy^3(x-3)}{y-2} \div \frac{4xy(x+3)}{y-2}$$

$$10. \frac{(x+3)(y-2)}{(x+4)} \div \frac{(x+4)(y-2)}{x+3}$$

$$11. \frac{x+3}{x+4} \div (x+3)$$

$$12. \frac{(x+3)(y-7)}{(x+2)(x-4)} \div \frac{\cancel{y-7}}{x^2+2x+1}$$

$$13. \frac{(x-8)(x-7)}{(x+7)(x-9)} \div \frac{(x-8)(x+7)}{(x-9)(x-7)}$$

$$14. \frac{(x+4)^2 \cdot x^2 - 16}{(x+1) x^2(2x+3)}$$

$$15. \frac{x^2+7x+10}{x^2+x-6} \div \frac{x^2+7x+10}{x^2-4}$$

$$16. \frac{x+3}{x+1} \div \frac{(x-5)(x+3)}{(x+2)(x-5)} \div \frac{x+2}{(x-5)(x+1)}$$

$$17. \frac{x+4}{x+5} \div \frac{x^2+7x+12}{x^2+6x+5} \div \frac{x^3-7x+12}{x^2+7x+12}$$

$$18. \frac{x-2}{x-6} \div \frac{x^2-x-2}{x^2+2x+1} \div \frac{x+1}{x^2-8x+12}$$

$$19. \frac{x+3}{x+4} \div \frac{x^2+5x+6}{x^2-x-20} \div \frac{x-5}{x^2+6x+8}$$

$$20. \frac{x+4}{x+5} \div \frac{x^2+5x+6}{x^2+7x+10} \div \frac{x+1}{x-1}$$

$$21. \frac{x^2-y^2}{x+5} \div \frac{x+y}{x^2+10x+25}$$

### III. Challenge Problems

1. Find the student's error.

$$\frac{(4x-7)(3x+5)}{(2x+1)(5x+9)} \div \frac{3x+5}{5x+9} \div (2x+1), x \neq -\frac{9}{5}, -\frac{5}{3}, -\frac{1}{2}$$

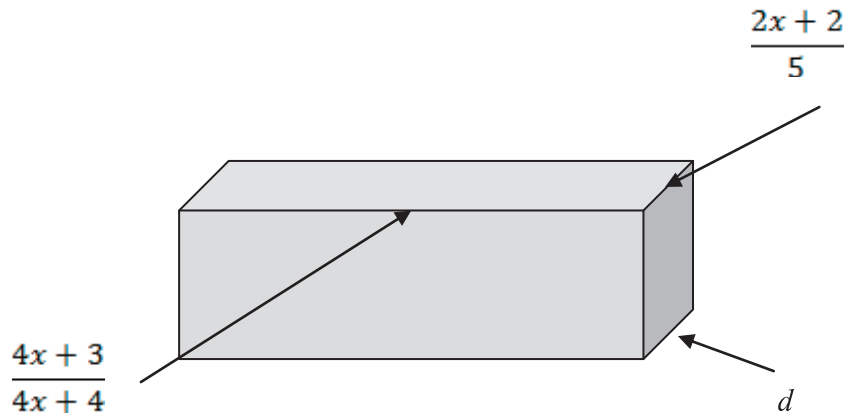
$$\frac{(4x-7)(3x+5)}{(2x+1)(5x+9)} \cdot \frac{5x+9}{3x+5} \cdot \frac{(2x+1)}{1}$$

$$\frac{(4x-7)(\cancel{3x+5}) \cdot \cancel{5x+9} \cdot (\cancel{2x+1})}{(\cancel{2x+1})(\cancel{5x+9}) \cdot \cancel{3x+5} \cdot 1}$$

$$4x-7$$

$$4x-7, x \neq -\frac{9}{5}, -\frac{5}{3}, -\frac{1}{2}$$

2. ~~Find~~ <sup>If</sup> the volume of the prism below is  $\frac{3x+8}{10}$ . Find the depth of the prism.



#### IV. Answer Key

1.  $\frac{xb}{ya}, y \neq 0, a \neq 0, b \neq 0$
2. ~~12x~~,  $x \neq 0, y \neq 0$   $\frac{12x}{5y}$
3.  $\frac{8x^4}{7}, x \neq 0, y \neq 0$
- 5 ~~15y^5~~,  $x \neq 0, y \neq 0$   $\frac{15y^5}{7x^4}$
- 6 ~~5~~,  $x \neq -2, 0$ ,  $\frac{5}{3}$
- 7 ~~y+3~~,  $x \neq -1, 0; y \neq -3, -2$   $\frac{y+3}{4(y+2)}$
- 8 ~~1~~,  $x \neq 1$
- 9 ~~3y^2(x-3)~~,  $x \neq -3, 0; y \neq 0, 2$   $\frac{3y^2(x-3)}{4(x+3)}$
- 10 ~~12x~~,  $x \neq -4, -3; y \neq 2$   $\frac{(x+3)^2}{(x+4)^2}$
- 11 ~~1~~,  $x \neq -4, -3$   $\frac{1}{x+4}$
- 12 ~~12x~~,  $x \neq -1, -2, 4, 7$   $\frac{(x+3)(x+1)^2}{(x+2)(x-4)}$
- 13 ~~13x~~,  $x \neq \pm 7, \pm 9, 8$   $\frac{(x-7)^2}{(x+7)^2}$
- 14 ~~14x~~,  $x \neq \pm 4, -1, -3, -2$   $\frac{x^2(x+4)(2x+3)}{(x+1)(x-4)}$
- 15 ~~15x~~,  $x \neq \pm 2, -3, -5$   $\frac{x+2}{x+3}$
- 16 ~~16x~~,  $x \neq \pm 5, -3, -2, -1$   $\frac{(x+4)(x+1)}{(x-3)(x-4)}$
- 17 ~~17x~~,  $x \neq -5, \pm 4, \pm 3, -1$   $\frac{(x+4)(x+1)}{(x-3)(x-4)}$
- 18 ~~18x~~,  $x \neq -1, 2, 6$
- 19 ~~19x~~,  $x \neq -4, -3, -2, 5$
- 20 ~~20x~~,  $x \neq -5, -3, -2, -1, 1$   $\frac{(x+4)(x-1)}{(x+3)(x+1)}$
- 21 ~~21x~~,  $(x+5)(x-y), x \neq -5; x \neq -y$

#### Challenge Problems

1. The reciprocal of  $2x + 1$  is  $\frac{1}{2x+1}$ .
2.  $\frac{3x+8}{4x+3}$