

Finding the vertex of a Quadratic

To find the **vertex** of a quadratic when it is in the **factored form** $y=a(x - r)(x - t)$ you must:

- ① Find zeros
- ② Find a.o.f.s = $\frac{\text{add zeros}}{2} = h$
- ③ Find opt. val = sub a.o.f.s into equation = k
- ④ vertex = (h, k)

To find the **vertex** of a quadratic when it is in the **standard form** $y=ax^2 + bx + c$ you must:

- ① Factor
- ② See steps above

To find the **vertex** of a quadratic when it is in the **vertex form** $y=a(x - h)^2 + k$ you must:

$\begin{matrix} \rightarrow & \uparrow & \uparrow \\ \text{ignore} & \text{switch} & \text{keep} \\ \text{vertex} = (h, k) \end{matrix}$

Factored Form	Zeros or x-int	x (axis of symm)	y (optimal val or max/min value)	Vertex and vertex Form	Convert to standard form	y-int
$y = -3(x-2)(x-8)$	$x-2=0$ $x=2$ $x-8=0$ $x=8$	$\frac{2+8}{2}$ $= \frac{10}{2}$ $= 5$	$-3(5-2)(5-8)$ $-3(3)(-3)$ $= 27$	vertex = (5, 27) $y = -3(x-5)^2 + 27$	$-3(x^2 - 8x - 2x + 16)$ $-3x^2 + 24x + 6x - 48$ $-3x^2 + 30x - 48$	$y\text{-int} = -48$

Standard Form	y-int	Zeros or x-int	x (axis of symm)	y (optimal val or max/min value)	Vertex and vertex form
$y = x^2 - 3x - 18$	-18	$y = (x-6)(x+3)$ $x-6=0$ $x=6$ $x+3=0$ $x=-3$	$\frac{6+(-3)}{2}$ $= \frac{3}{2}$ $= 1.5$	$(1.5)^2 - 3(1.5) - 18$ $= -20.25$	vertex = (1.5, -20.25) $y = 1(x-1.5)^2 - 20.25$

Vertex Form	Vertex	Is the vertex a max or a min?	Convert to standard form
$y = -3(x-2)^2 + 5$	vertex = (2, 5)	$a = -3$ opens down \therefore MAX	$y = -3(x-2)(x-2) + 5$ $y = -3(x^2 - 2x - 2x + 4) + 5$ $y = -3x^2 + 6x + 6x - 12 + 5$ $y = -3x^2 + 12x - 7$

Example

The flight of a baseball is modelled by $y = -4.9x^2 + 9.8x + 14.7$ where x is the time, in sec, and y is the height, in m, above the ground.

- a. What is the height of the ball 0.5 seconds after it was hit?

$$y = -4.9(0.5)^2 + 9.8(0.5) + 14.7$$

$$y = 18.375$$

\therefore height was about 18.4 m

- b. What is the height of the ball when it was hit?

initial y-int

\therefore 14.7m height when it was hit

- c. How long does it take for the ball to reach the ground?

$$y = -4.9 \left(\frac{-4.9x^2 + 9.8x + 14.7}{-4.9} \right)$$

zeros

$$y = -4.9(x^2 - 2x - 3)$$

$$y = -4.9(x-3)(x+1)$$

zeros: $x-3=0$
 $x=3$

$x+1=0$
 $x=-1$ time can't be neg.

\therefore lands on the ground in 3sec.

- d. Find the maximum height.

at vertex. (optimal value)

$$a.o.s = \frac{\text{add zeros}}{2} = \frac{3 + -1}{2} = \frac{2}{2} = 1$$

$$\text{opt. value} = \text{sub a.o.s} = -4.9(1)^2 + 9.8(1) + 14.7$$

$$= 19.6$$

\therefore max height is 19.6m