

Exponent Laws

EXPONENT LAWS CAN ONLY BE USED WHEN LIKE BASES EXIST

1. Complete each table.
2. Examine the relationship between the exponents in the original expression and the exponent in the expression as a single power. State a rule for this relationship.

DEVELOPING THE MULTIPLICATION LAW

MULTIPLICATION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER
$a^4 \times a^5$	$a \times a \times a \times a \times a \times a \times a \times a \times a \times a$	a^9
$a^2 \times a^4$	$(aa)(aaaa)$	a^6
$a^5 \times a$	$(aaaaa)a$	a^6
$a^6 \times a^2$	$(aaaaaa)(aa)$	a^8
$a^4 \times a^3$	$(aaaa)(aaa)$	a^7

Rule:

- ① Keep the like base
- ② Add exponents

DEVELOPING THE DIVISION LAW

DIVISION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER
$a^5 \div a^4$	$\frac{a \times a \times a \times a \times a}{a \times a \times a \times a}$	a^1
$a^4 \div a^2$	$\frac{aaaa}{aa}$	a^2
$a^5 \div a$	$\frac{aaaaa}{a}$	a^4
$a^6 \div a^2$	$\frac{aaaaaa}{aa}$	a^4
$a^4 \div a^3$	$\frac{aaaa}{aaa}$	a^1

Rule:

- ① Keep the like base (record in numerator)
- ② Subtract the exponents

DEVELOPING THE POWER OF A POWER LAW

DIVISION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER
$(a^4)^5$	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$	a^{20}
$(a^5)^2$	$(a^5)(a^5)$	a^{10}
$(a^2)^3$	$(a^2)(a^2)(a^2)$	a^6
$(a^3)^4$	$(a^3)(a^3)(a^3)(a^3)$	a^{12}
$(a^6)^2$	$(a^6)(a^6)$	a^{12}

Rule: (1) Keep the base
(2) Multiply the exponents

Example 1

Use the exponent laws to simplify.

a. $(x^3)(x^8)$
 x^{11}

c. $(x^2)^4$
 x^8

e. $x^4(x^6y^3)$
 $x^{10}y^3$

b. $x^9 \div x^3$
 $= \frac{x^9}{x^3}$
 $= x^6$

d. $(x^3y^2)(xy^5)$
 $= x^4y^7$

f. $2x^7y^4 \div x^6y$
 $= \frac{2x^7y^4}{x^6y}$
 $= 2xy^3$

When using these laws with *more than one variable* apply the **distributive property**.

POWER OF A PRODUCT LAW

$$(x^4 y^3)^2 = x^8 y^6$$

POWER OF A QUOTIENT LAW

$$(x^4 \div y^3)^2 = \frac{(x^4)^2}{(y^3)^2} = \frac{x^8}{y^6}$$

POWER OF A SUM/DIFFERENCE
no such Law!
 cannot distribute
 must FOIL

$$(x^4 - y^3)(x^4 - y^3)$$

$$= x^8 - x^4 y^3 - x^4 y^3 + y^6$$

$$= x^8 - 2x^4 y^3 + y^6$$

Example 2

Use the exponent laws to simplify.

a. $(x^2 y^7)^3$
 $= x^6 y^{21}$

c. $\left(\frac{x^4}{y^5}\right)^3$
 $= \frac{x^{12}}{y^{15}}$

b. $(2x^6 y)^4$ *for each base*

$$= 2^4 x^{24} y^4$$

don't multiply BASE with POWER

$$= 16x^{24} y^4$$

d. $\left(\frac{3y^2}{x^6}\right)^3$

$$\frac{3^3 y^6}{x^{18}} = \frac{27y^6}{x^{18}}$$

e. $(x + 2y^3)^2$

$$= (x + 2y^3)(x + 2y^3)$$

$$= x^2 + 2xy^3 + 2xy^3 + 4y^6$$

$$= x^2 + 4xy^3 + 4y^6$$

when adding like terms exponents do not change

USING THE DIVISION LAW TO UNDERSTAND ZERO AND NEGATIVE EXPONENTS

1. Complete the table.
2. Examine the relationship between the exponents in the original expression and the exponent in the expression as a single power. State a rule for zero and negative exponents.
3. Test the rules to see if they hold true.

DIVISION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER	USE EXPONENT LAWS
$a^3 \div a$	$\frac{a \times a \times a}{a}$	a^2	a^2
$a^3 \div a^2$	$\frac{aaa}{aa}$	$a^{3-2} = a^1$	a
$a^3 \div a^3$	$\frac{aaa}{aaa}$	$a^{3-3} = a^0$	$a^0 = 1$
$a^3 \div a^4$	$\frac{aaa}{aaaa}$	$a^{3-4} = a^{-1}$	$\frac{1}{a}$
$a^3 \div a^5$	$\frac{aaa}{aaaaa}$	$a^{3-5} = a^{-2}$	$\frac{1}{a^2}$
$a^3 \div a^6$	$\frac{aaa}{aaaaaa}$	$a^{3-6} = a^{-3}$	$\frac{1}{a^3}$
$a^3 \div a^7$	$\frac{aaa}{aaaaaaa}$	$a^{3-7} = a^{-4}$	$\frac{1}{a^4}$

pattern
 $\div a$
 $\div a$

ZERO LAW

Rule:

Any base to power zero is ONE

Test:

	EXPAND & SIMPLIFY	USE DIVISION LAW
$x^2 \div x^2$	$\frac{x \cdot x}{x \cdot x} = 1$	$x^{2-2} = x^0$
$2^3 \div 2^3$	$\frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{8}{8} = 1$	$2^{3-3} = 2^0$
$5^4 \div 5^4$	$\frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5} = 1$	$5^{4-4} = 5^0$

NEGATIVE EXPONENT LAW

Rule:	Apply this law last!! Take <u>only</u> the base with negative power and <u>switch</u> it's placement: numerator to denominator (or vice versa) and change power to be positive	
Test:	EXPAND & SIMPLIFY	USE DIVISION LAW
$x^2 \div x^5$	$\frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^3}$	$x^{2-5} = x^{-3}$
$2^3 \div 2^4$	$\frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{2}$	$2^{3-4} = 2^{-1}$
$5^4 \div 5^6$	$\frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{5^2}$	$5^{4-6} = 5^{-2}$

Example 3

Use the exponent laws to simplify. Write each answer with a positive exponent.

a. $x^{-8} \times x^8$
 $= x^{-8+8}$
 $= x^0$
 $= 1$

b. $x^{10} \div x^{12}$
 $= x^{10-12}$
 $= x^{-2}$
 $= \frac{1}{x^2}$

c. $(x^9)^0$
 $= x^{9 \times 0}$
 $= x^0$
 $= 1$

d. $x^{-4} \times x^{-2}$
 $= x^{-4+-2}$
 $= x^{-6}$
 $= \frac{1}{x^6}$

e. $\frac{2x}{x^3}$
 $= 2x^{1-3}$
 $= 2x^{-2}$
 $= \frac{2}{x^2}$

f. $\frac{x^0}{4x^{-3}}$
 $= \frac{1}{4x^{-3}}$
 $= \frac{1x^3}{4}$

g. $(5x^2y^5)^2 \cdot (5^{-2}x^{-6}y)$
 $= 5^2 x^4 y^{10} \cdot 5^{-2} x^{-6} y$
 $= 5^{2+-2} x^{4+-6} y^{10+1}$
 $= 5^0 x^{-2} y^{11}$
 $= \frac{1y^{11}}{x^2}$

h. $\frac{(9x^2y^5)^2}{3^3x^6y^{14}}$
 $= \frac{9^2 x^4 y^{10}}{3^3 x^6 y^{14}}$
not same base
 $= \frac{(3^2)^2 x^{4-6} y^{10-14}}{3^3}$
 $= \frac{3^{4-3} x^{-2} y^{-4}}{3^3}$
 $= \frac{3^1}{x^2 y^4}$