

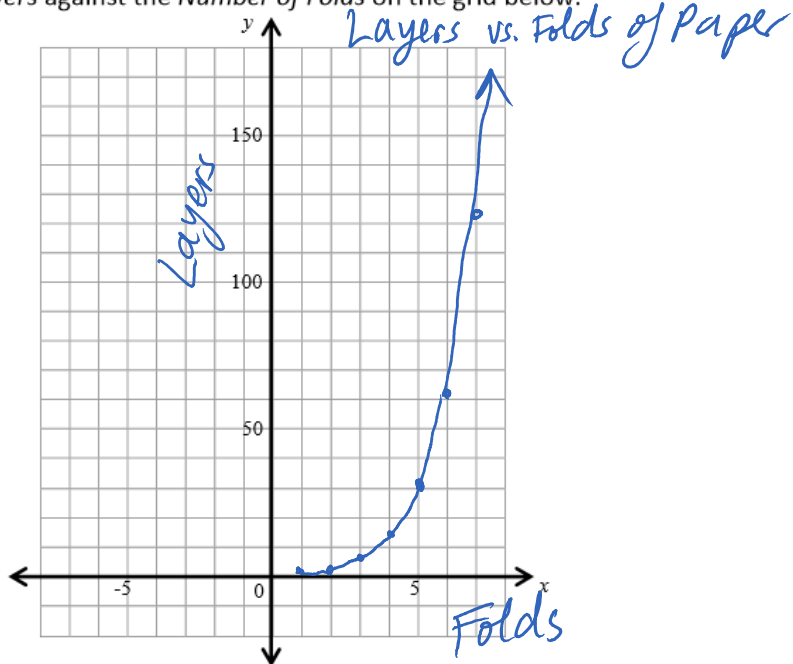
Characteristics of Exponential Relations

INVESTIGATION: Modeling Growth

1. Fold a piece of paper in half and count the number of layers. Record your answer in the table below.
2. Fold the paper in half again. Record the number of folds and the number of layers.
3. Continue this process until you can no longer fold the paper.
4. Use the pattern to extend the table to 10 folds.

NUMBER OF FOLDS	NUMBER OF LAYERS	1 st diff	2 nd diff	1 st ratios
1	2	$4-2=2$	$4-2=2$	$4 \div 2 = 2$
2	4	$8-4=4$	$8-4=4$	$8 \div 4 = 2$
3	8	8	8	$16 \div 8 = 2$
4	16	16	16	2
5	32	32	⋮	↓
6	64	64	⋮	
7	128	⋮		
8	256	⋮		
9	512			
10	1024			

5. Plot the Number of Layers against the Number of Folds on the grid below.



6. Describe how the number of layers changes as the number of folds increases.

Doubles each time, (grows exponentially - fast)

7. Calculate the first and second differences in the table. What do you notice?

Differences are not constant, they grow just like y-values do (see table)

8. Do you notice a pattern in the table? Describe it. Can you create an equation to represent the relation?

x-value pattern goes up by 1
 y-value pattern multiply by 2 each time
 $y = 2^x$ when $x=1$ $y=2$
 $x=2$ $y=4$
 \vdots
 \ddots

9. Calculate the common ratio in the table [$y_2 \div y_1$, $y_3 \div y_2$, $y_4 \div y_3$, etc.]. What do you notice?

1st ratios are constant (see table)

SUMMARIZE the differences between *linear*, *quadratic* and *exponential* relations.

Type of Mathematical Model	Linear Relations	Quadratic Relations	Exponential Relations
EQUATION	$y = mx + b$ degree 1 on variable base	$y = ax^2 + bx + c$ degree 2	$y = b^x$ or $y = ab^x$ variable in exponent
TABLE OF VALUES	1st differences constant	2nd differences constant	1st ratios constant
GRAPH	straight line ↘	U-shaped curve ↻	one side almost horizontal, other side grows/decays fast

Example 1

Examine each equation. Determine the **type of relation** it represents (linear, quadratic or exponential).

$y = -5^x$	$y = x^2 + 5x - 14$	$y = 7x - 1$
Type: <u>exponential</u>	Type: <u>quadratic</u>	Type: <u>linear</u>
Reason: <u>variable in exponent</u>	Reason: <u>degree 2</u>	Reason: <u>degree 1</u>

Example 2

Complete each table. Determine whether the relationship is exponential. Give a **reason** for your answer.

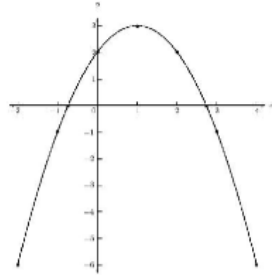
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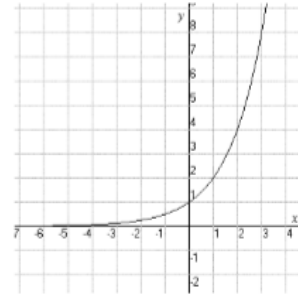
Example 3

Examine each graph. Determine the **type of relation** it represents. Give a **reason** for your answer.



Type: quadratic

Reason: U-shaped



Type: exponential

Reason: flat on one side + growth on the other.