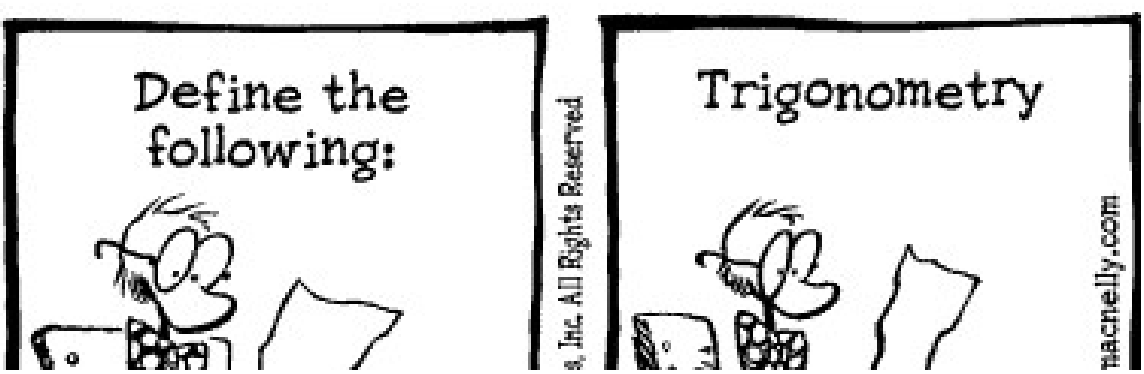
MBF 3C1 Name: ______________________________

# UNIT 5
## Trigonometry

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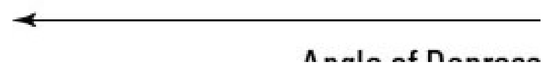
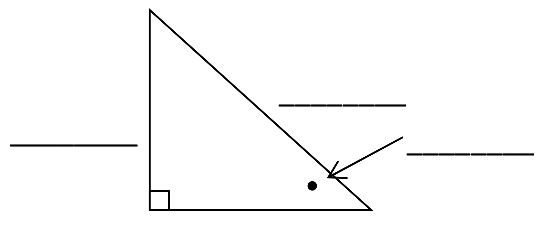
# Trigonometry Essentials

**VOCABULARY ESSENTIALS**

1. An angle of ____________________ is an angle that falls from the horizontal; also known as the angle of declination.
2. An angle of ____________________ rises from the horizontal; also known as the angle of inclination.

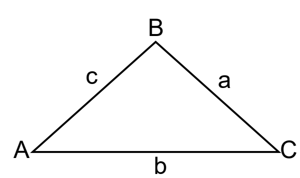


3. ____________________ is a symbol often used to represent a missing angle.
4. A ____________________ triangle has one 90° angle.
5. Any triangle that is not a right triangle is an ____________________ triangle.
6. An ____________________ triangle has three acute angles.
7. An ____________________ triangle has one obtuse angle.
8. A right angle is ____________________.
9. An acute angle is ____________________.
10. An obtuse angle is ____________________.
11. The sum of all the angles in a triangle is ____________________.
12. The ____________________ is the longest side in a right triangle, across from the right angle.
13. The side labelled ____________________ is across from the angle of focus in a right triangle.
14. The side labelled ____________________ is attached to the angle of focus in a right triangle.
15. The ____________________ is the angle given or the angle to be found in a right triangle.



**TRIANGLE ESSENTIALS**

16. To properly label a triangle, use small letters to represent the sides and capital letters to represent the angles.
17. The sides and angles opposite to each other should be labelled with the same letter.
18. In any triangle, the largest side is always across from the largest angle, the smallest side is always across from the smallest angle, and so on.
19. When a question says to solve a triangle, it means find every missing angle and every missing side.




**TRIGONOMETRY ESSENTIALS**

20. Calculators must be in degree mode.
21. The opposite operations to sin, cos and tan are $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.
22. When answering questions, round sides to 1 decimal, angles to a whole number and trig ratios to 4 decimals.

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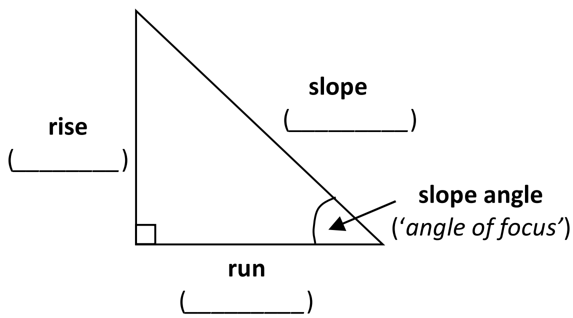
# The Primary Trigonometric Ratios

The primary trigonometric ratios are used to find *side lengths* or *angle measures* in ________________________________.



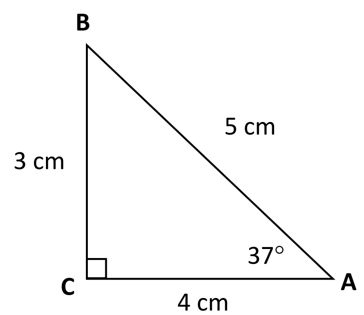

**Slope Angle** – the angle opposite to the rise and adjacent to the run; considered the angle of focus

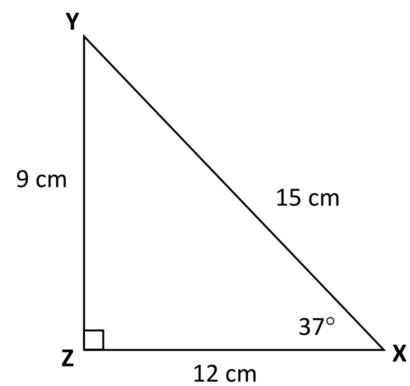
**Opposite Side** – across from the slope angle

**Adjacent Side** – adjacent to the slope angle

**Hypotenuse** – the longest side of a right triangle across from the right angle

Complete the table below using the triangles provided. Round answers to a whole degree (no decimals).




| Triangle | $\frac{\textbf{opposite}}{\textbf{hypotenuse}}$ | sin A | $\frac{\textbf{adjacent}}{\textbf{hypotenuse}}$ | cos A | $\frac{\textbf{opposite}}{\textbf{adjacent}}$ | tan A |
|---|---|---|---|---|---|---|
| **ΔABC** | | | | | | |
| **ΔXYZ** | | | | | | |

1. What do you notice about the ratios of lengths of sides and the trigonometric ratios in both triangles?

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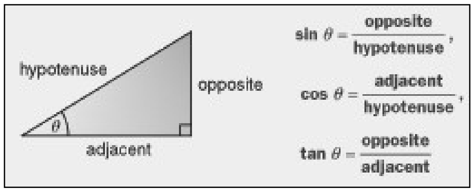
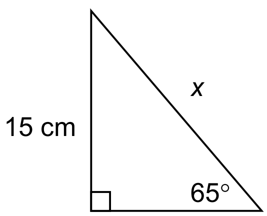
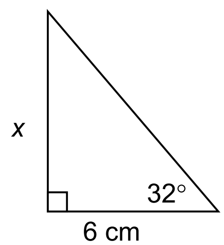
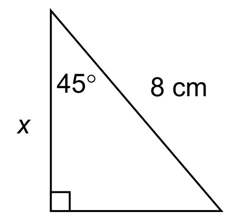
What are the primary trigonometric ratios?





Trigonometric ratios can be used to calculate a side of a right triangle if ____________________ and ___________________ are known.

**Example 1**

| | | |
|---|---|---|
| a.   | b.   | c.   |
| | | |

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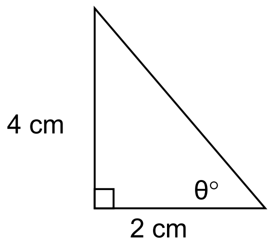
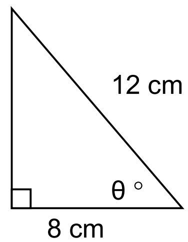
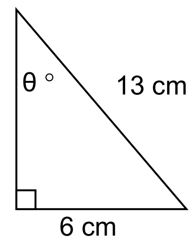
Trigonometric ratios can be used to calculate an angle of a right triangle if ______________________ are known.

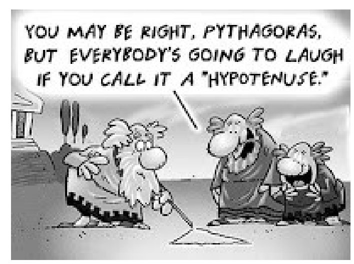
Use $\tan^{-1}$, $\sin^{-1}$ or $\cos^{-1}$ to find a missing angle

**Example 2**

| a. | b. | c. |
|---|---|---|
|  |  |  |
| | | |

To apply the primary trig ratios:

1. draw a diagram if one is not provided
2. determine the __________________ (the angle given or missing)
3. label the triangle with ________________, ________________, and ________________
4. determine which ________________ is to be used
5. solve for the missing __________ or __________




**Example 3**

A construction engineer determines that a straight road must rise vertically 45 m over a 250 m distance measured along the surface of the road (this represents the hypotenuse of the right triangle). Calculate the angle of elevation of the road.

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# The Sine Law

<table>
<tr><th>DRAW AN ACUTE TRIANGLE.<br>*Each angle should be less than 90°*</th><th colspan="6">COMPLETE THE CHART FOR THE TRIANGLE.</th></tr>
<tr><td rowspan="5"></td><td>**Angle**</td><td>**Angle Measure**</td><td>**Sine of Angle**</td><td>**Length of Opposite Side**</td><td colspan="2">**Ratios**</td></tr>
<tr><td></td><td>Measure each angle using a protractor. Be as accurate as possible.</td><td>Calculate the sine of each angle using a calculator.</td><td>Measure the length of each side using a ruler. Be as accurate as possible.</td><td colspan="2">Calculate each of the following ratios using a calculator.</td></tr>
<tr><td>$\angle$A</td><td></td><td></td><td>$a =$</td><td>$\frac{a}{\sin A} =$</td><td>$\frac{\sin A}{a} =$</td></tr>
<tr><td>$\angle$B</td><td></td><td></td><td>$b =$</td><td>$\frac{b}{\sin B} =$</td><td>$\frac{\sin B}{b} =$</td></tr>
<tr><td>$\angle$C</td><td></td><td></td><td>$c =$</td><td>$\frac{c}{\sin C} =$</td><td>$\frac{\sin C}{c} =$</td></tr>
<tr><td>**DESCRIBE ANY RELATIONSHIPS YOU NOTICE IN THE TABLES.**</td><td colspan="6"></td></tr>
</table>

The **SINE LAW**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

can be used to calculate an unknown:

- **side** when *two angles* and *any side* are given
- **angle** when *two sides* and *an opposite angle* are given

When using the sine law, start with the unknown angle or side and then create the appropriate ratio to solve.

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**Example 1**

Find the measure of *c*.


**Example 2**

Find the measure of *C.*


**Example 3**

Two ships are located 15 nautical miles apart. The angle of Ship 1 to the entrance of the port is 55° with respect to Ship 2. Ship 2's angle to the entrance to the port is 45° with respect to Ship 1. Which ship is closer to the port entrance? How far is this ship form port? Round your answer to the nearest tenth.

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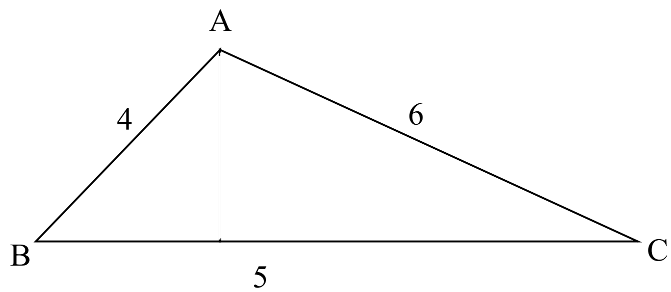
# The Cosine Law

Can the following triangle be solved using the sine law? Why?

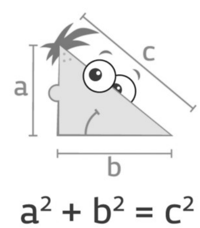



Use the Pythagorean Theorem to help solve the triangle. *(Hint: Don't simplify exponents.)*

Find an equation to determine the value of *h* in the left triangle.

$h^2 + x^2 = 4^2$

$h^2 = 4^2 - x^2$




Find an equation to determine the value of *h* in the right triangle.

$h^2 + (5-x)^2 = 6^2$

$h^2 = 6^2 - (5-x)^2$

Put the equations together since $h^2 = h^2$.

$$4^2 - x^2 = 6^2 - (5-x)^2$$
$$4^2 - x^2 = 6^2 - (5^2 - 5x - 5x + x^2)$$
$$4^2 - x^2 = 6^2 - (5^2 - 10x + x^2)$$
$$4^2 - x^2 = 6^2 - 5^2 + 10x - x^2$$
$$4^2 - x^2 + 5^2 - 10x + x^2 = 6^2$$
$$4^2 + 5^2 - 10x = 6^2$$
$$4^2 + 5^2 - 10(4\cos B) = 6^2$$
$$c^2 + a^2 - 2ac\cos B = b^2$$

$$\cos B = \frac{x}{4}$$
$$4\cos B = x$$

The **cosine law**

$$c^2 = a^2 + b^2 - 2ab\cos C$$

can be used to calculate an unknown:

- **side** when *two sides* and a *contained angle* (the angle between two given sides) are given
- **angle** when *three sides* are given

When using the cosine law, the unknown angle or side will either be the first or last variable in the formula.

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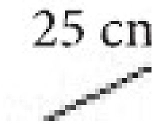
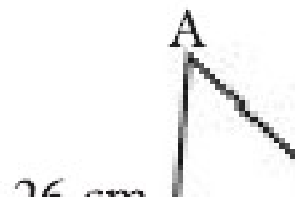
**Example 1**

Find *b*.



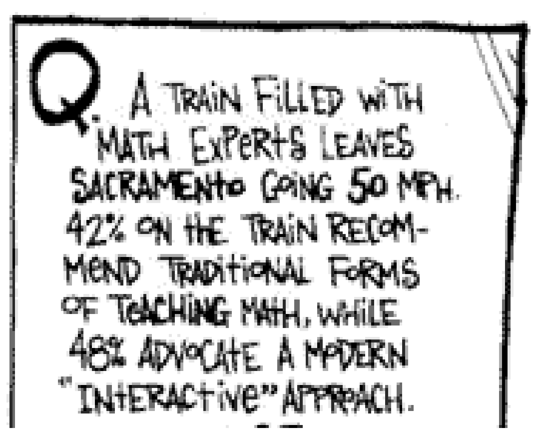

**Example 2**

Find *A.*




**Example 3**

Two hikers set out in different directions from a marked tree on the Bruce Trail. The angle formed between their paths measures 50°. After 2 hours, one hiker is 6 km from the starting point and the other is 9 km from the starting point. How far apart are the hikers, to the nearest tenth of a kilometre?



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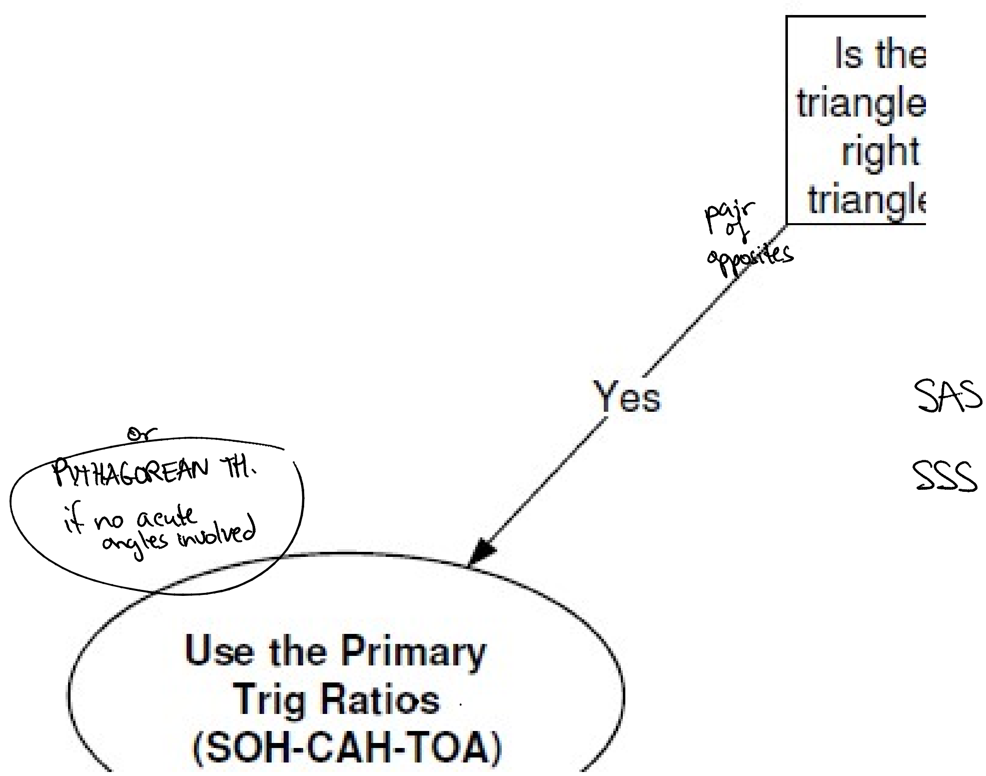
# Solving Problems with Trigonometry

Steps to solving trigonometry problems:

1. state _____________________ and draw a _____________________
2. choose the appropriate _____________________
    - For right triangles,
        - use the ___________________: ___________, ___________, and ___________
            → if an acute angle is involved
        - use the ____________________________
            → if NO acute angle is involved
    - For non-right triangles,
        - the ___________________: _____________________ can be used when
            → one pair of opposite side and angle must be given plus one more piece of information (the 3rd angle can be found from subtracting two angles from 180°)
        - the __________________: _____________________ can be used when
            → two sides and a contained angle are given
            → three sides are given

*Use the following flowchart to help you decide which formula to use:*




3. _____________________for the missing variable
4. make sure the answer _____________________in the context of the question
5. write a _____________________

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**Example 1**

From the top of the Niagara Escarpment, Juan sees a car below at an angle of depression of 40°. He is approximately 100 m above the car. How far is the car from the base of the escarpment? Round your answer to the nearest metre.

**Example 2**

Micah is standing on the ground between two buildings on the opposite sides of a park. The top of the first building is 152 m from Micah, at an angle of elevation of 38°, while the top of the second building is 175 m from Micah, at an angle of elevation of 53°. How far apart are the tops of the two buildings? Round your answer to the nearest metre.

**Example 3**

Sam is on a hiking trip. On the first section of the hike, he walks 5 km from the Loon Campsite to the Owl Campsite. Then, he turns 68° and hikes 7 km to the Eagle Campsite. He then returns to the Loon Campsite. What is the distance from the Eagle campsite to the Loon campsite, to the nearest kilometre?

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# Trigonometry: Group Task

*Instructions:*

1. In a group of 3 or 4, cut out the diagrams and formulas. Match each diagram and formula to the correct word problem. Glue your answers.
2. Cut out the problem strips. Divide the problems evenly between the group members. Each group member is to complete a full solution for their questions in the space provided.
3. Each person should get one other member of the group to check their answers to make sure there are no mistakes. Make any necessary corrections.
   ** Make sure your solution includes a final statement.**
4. Staple the problems together **in order**. Put all group member names on the cover sheet. Hand in.

<table>
<tr>
<td>$\tan\theta = \frac{opp}{adj}$</td>
<td>$\sin\theta = \frac{opp}{hyp}$</td>
<td>$\cos C = \frac{a^2+b^2-c^2}{2ab}$</td>
</tr>
<tr>
<td>$\frac{a}{\sin A} = \frac{b}{\sin B}$</td>
<td>$\sin\theta = \frac{opp}{hyp}$</td>
<td>$a^2 = b^2 + c^2 - 2bc\cos A$</td>
</tr>
<tr>
<td>$\frac{a}{\sin A} = \frac{c}{\sin C}$</td>
<td>$\sin\theta = \frac{opp}{hyp}$</td>
<td></td>
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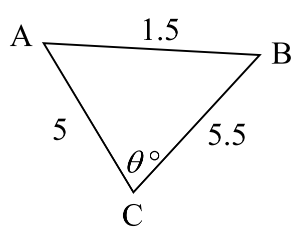
<td>
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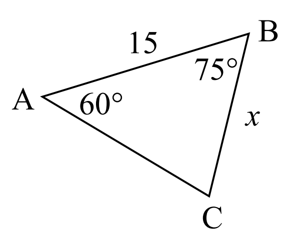
<td>
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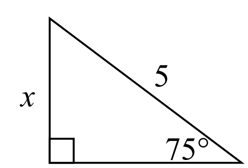
<td>
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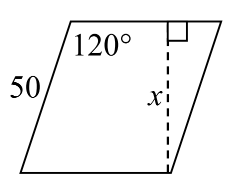
<td>
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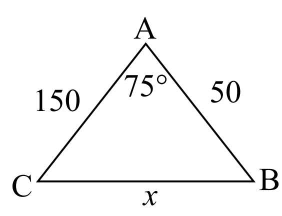
<td>
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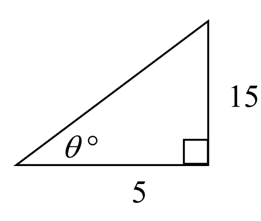
<td>
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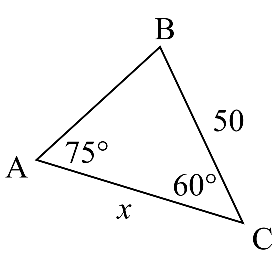
<td>
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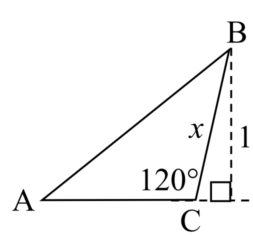
<td>
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<td></td>
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</table>

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| | | |
|---|---|---|
| **1.**<br>A ladder is 5 m long. It is leaning against a wall. The foot of the ladder makes an angle of 75° with the ground. How far up the wall is the top of the ladder? Give the length to the nearest tenth of a metre. | **2.**<br>A cable car rises 15 m for every 5 m it travels horizontally. What angle does the cable make with the level ground? | **3.**<br>Ships A and B at sea are 15 km apart. The port, C, can be seen from the deck of each ship. The angles between the line joining the ships and the lines of sight to the port are 60° and 75°, respectively. How far is ship A from the port? |
| Diagram | Diagram | Diagram |
| Formula | Formula | Formula |
| Solution<br><br>Completed by: __________________<br>Peer edited by: __________________ | Solution<br><br>Completed by: __________________<br>Peer edited by: __________________ | Solution<br><br>Completed by: __________________<br>Peer edited by: __________________ |

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| 4. | 5. | 6. |
| --- | --- | --- |
| Three straight roads join three towns, A, B and C. The road between towns A and C makes an angle of 75° with the road between towns A and B, and an angle of 60° with the road between towns C and B, which is 50 km long. A person drives from A to B to C. Determine the distance between A and C. | In ΔABC the largest angle, C, is 120°. The height of the triangle is 15 cm. Determine the length of the side that connects the height to C. | A rhombus has sides that measure 50 cm and obtuse angles that measure 120°. Find the height of the rhombus. |
| Diagram | Diagram | Diagram |
| Formula | Formula | Formula |
| Solution<br><br>Completed by: ________________<br>Peer edited by: ________________ | Solution<br><br>Completed by: ________________<br>Peer edited by: ________________ | Solution<br><br>Completed by: ________________<br>Peer edited by: ________________ |

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| **7.** A hockey net is 1.5 m wide. A player is 5 m from one goal post and 5.5 m from the other. Within what angle must she keep her shot in order to score a goal? | **8.** A children's playground is triangular. Two sides measure 0.15 km and 50 m. The angle between them is 75°. Find the measure of the third side. |
|---|---|
| Diagram | Diagram |
| Formula | Formula |
| Solution<br><br>Completed by: ________________<br>Peer edited by: ________________ | Solution<br><br>Completed by: ________________<br>Peer edited by: ________________ |

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Trigonometry Group Task

Date: ______________

Names: ______________