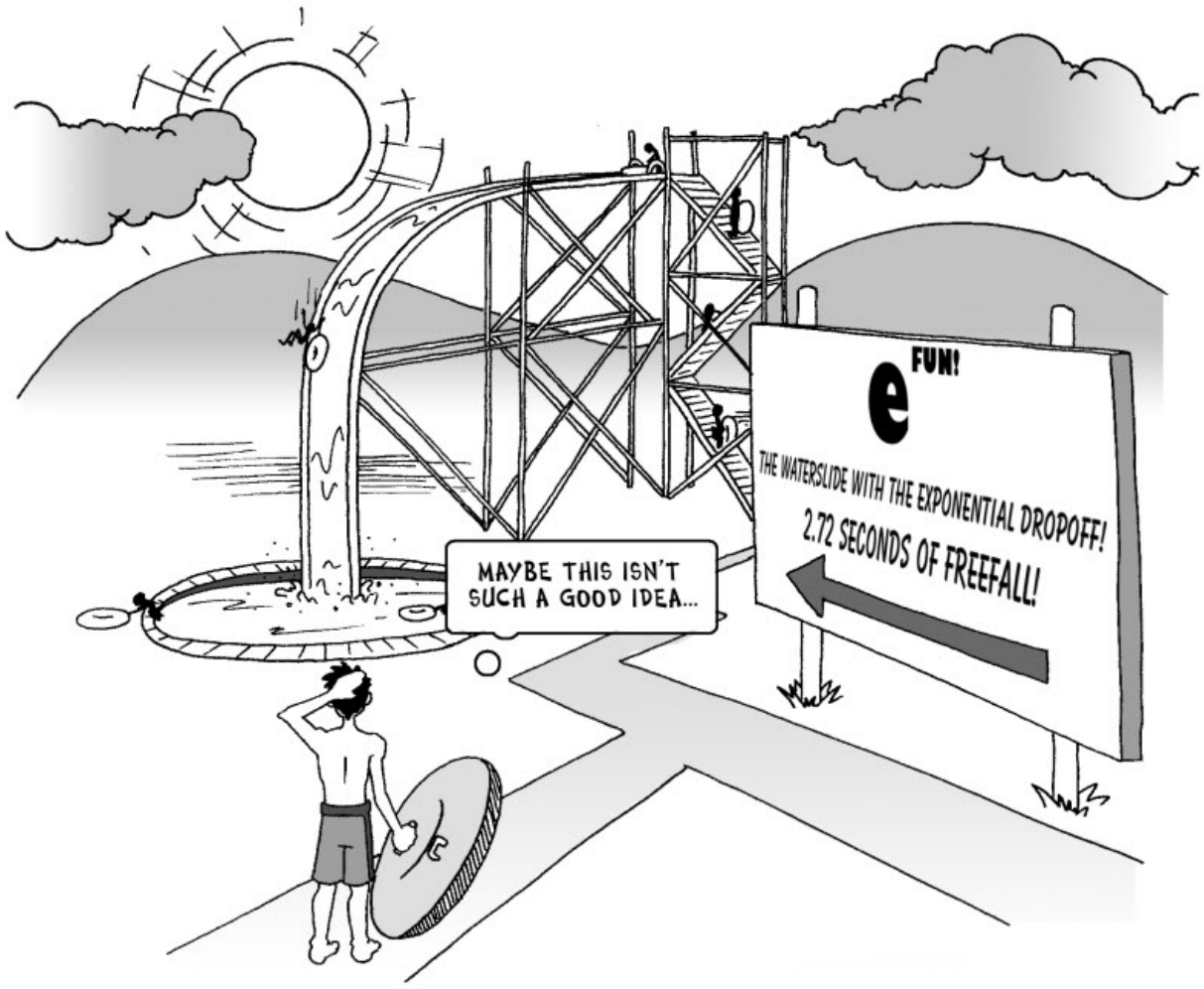
MBF 3C1 Name: _____

UNIT 3

Exponential Relations



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Exponent Laws

EXPONENT LAWS CAN ONLY BE USED WHEN LIKE BASES EXIST

1. Complete each table.
2. Examine the relationship between the exponents in the original expression and the exponent in the expression as a single power. State a rule for this relationship.

DEVELOPING THE **MULTIPLICATION LAW**

MULTIPLCATION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER
$a^4 \times a^5$	$a \times a \times a \times a \times a \times a \times a \times a \times a$	a^9
$a^2 \times a^4$		
$a^5 \times a$		
$a^6 \times a^2$		
$a^4 \times a^3$		
Rule:		

DEVELOPING THE **DIVISION LAW**

DIVISION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER
$a^5 \div a^4$	$\frac{a \times a \times a \times a \times a}{a \times a \times a \times a}$	a^1
$a^4 \div a^2$		
$a^5 \div a$		
$a^6 \div a^2$		
$a^4 \div a^3$		
Rule:		

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DEVELOPING THE **POWER OF A POWER LAW**

DIVISION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER
$(a^4)^5$	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$	a^{20}
$(a^5)^2$		
$(a^2)^3$		
$(a^3)^4$		
$(a^6)^2$		
Rule:		

Example 1

Use the exponent laws to simplify.

a. $\left(x^3\right)\left(x^8\right)$

b. $x^9 \div x^3$

c. $\left(x^2\right)^4$

d. $\left(x^3 y^2\right)\left(xy^5\right)$

e. $x^4\left(x^6 y^3\right)$

f. $2x^7 y^4 \div x^6 y$

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When using these laws with *more than one variable* apply the **distributive property**.

POWER OF A PRODUCT LAW

$(x^4y^3)^2$

POWER OF A QUOTIENT LAW

$(x^4 \div y^3)^2$

POWER OF A SUM/DIFFERENCE

$(x^4 - y^3)^2$

Example 2

Use the exponent laws to simplify.

a. $(x^2y^7)^3$

b. $(2x^6y)^4$

c. $\left(\frac{x^4}{y^5}\right)^3$

d. $\left(\frac{3y^2}{x^6}\right)^3$

e. $(x+2y^3)^2$

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USING THE DIVISION LAW TO UNDERSTAND ZERO AND NEGATIVE EXPONENTS

1. Complete the table.
2. Examine the relationship between the exponents in the original expression and the exponent in the expression as a single power. State a rule for zero and negative exponents.
3. Test the rules to see if they hold true.

DIVISION	EXPAND & SIMPLIFY	EXPRESSION AS A SINGLE POWER	USE EXPONENT LAWS
$a^3 \div a$	$\frac{a \times a \times a}{a}$	a^2	a^2
$a^3 \div a^2$			
$a^3 \div a^3$			
$a^3 \div a^4$			
$a^3 \div a^5$			
$a^3 \div a^6$			
$a^3 \div a^7$			

ZERO LAW

Rule:

Test:

	EXPAND & SIMPLIFY	USE DIVISION LAW
$x^2 \div x^2$		
$2^3 \div 2^3$		
$5^4 \div 5^4$		

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NEGATIVE EXPONENT LAW

Rule:		
Test:		
	EXPAND & SIMPLIFY	USE DIVISION LAW
$x^2 \div x^5$		
$2^3 \div 2^4$		
$5^4 \div 5^6$		

Example 3

Use the exponent laws to simplify. Write each answer with a positive exponent.

a. $x^{-8} \times x^8$ b. $x^{10} \div x^{12}$ c. $\left(x^9\right)^0$ d. $x^{-4} \times x^{-2}$

.

e. $\frac{2x}{x^3}$ f. $\frac{x^0}{4x^{-3}}$ g. $(5x^2y^5)^2 \bullet (5^{-2}x^{-6}y)$ h. $\frac{(9x^2y^5)^2}{3^3x^6y^{14}}$

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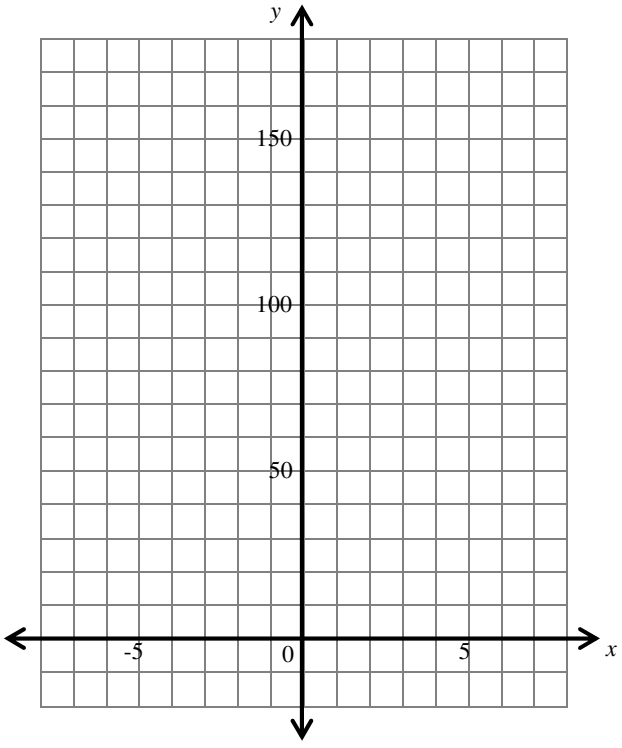
Characteristics of Exponential Relations

INVESTIGATION: ***Modeling Growth***

1. Fold a piece of paper in half and count the number of layers. Record your answer in the table below.
2. Fold the paper in half again. Record the number of folds and the number of layers.
3. Continue this process until you can no longer fold the paper.
4. Use the pattern to extend the table to 10 folds.

NUMBER OF FOLDS	NUMBER OF LAYERS			
1				
2				
3				
4				
5				
6				
7				

5. Plot the *Number of Layers* against the *Number of Folds* on the grid below.



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6. Describe how the number of layers changes as the number of folds increases.

7. Calculate the first and second differences in the table. What do you notice?

8. Do you notice a pattern in the table? Describe it. Can you create an equation to represent the relation?

9. Calculate the common ratio in the table [$y_2 \div y_1$, $y_3 \div y_2$, $y_4 \div y_3$, etc.]. What do you notice?

SUMMARIZE the differences between ***linear***, ***quadratic*** and **exponential** relations.

Type of Mathematical Model	Linear Relations	Quadratic Relations	Exponential Relations
EQUATION			
TABLE OF VALUES			
GRAPH			

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Example 1

Examine each equation. Determine the **type of relation** it represents (linear, quadratic or exponential).

$y = -5^x$	$y = x^2 + 5x - 14$	$y = 7x - 1$
Type: _____	Type: _____	Type: _____
Reason: _____	Reason: _____	Reason: _____

Example 2

Complete each table. Determine whether the relationship is exponential. Give a **reason** for your answer.

x	3^x	y	1^{st} Ratios
-2			
-1			
0			
1			
2			

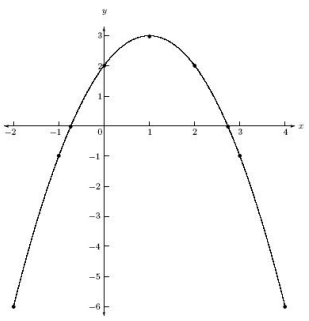
Reason: _____

x	$\left(\frac{1}{2}\right)^x$	y	1^{st} Ratios
-2			
-1			
0			
1			
2			

Reason: _____

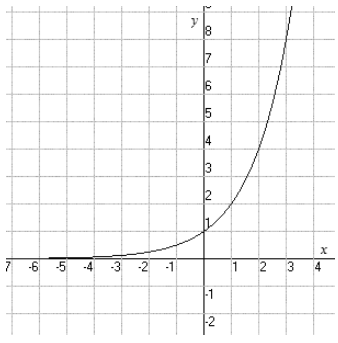
Example 3

Examine each graph. Determine the **type of relation** it represents. Give a **reason** for your answer.



Type: _____

Reason: _____



Type: _____

Reason: _____

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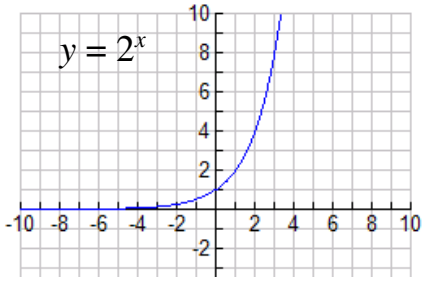
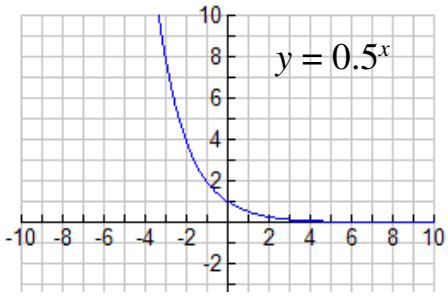
Exponential Relationships

The generalization for an exponential relation is _____, where *b* represents the _____.

Compare the two types of exponential relationships in the chart below.

Exponential Growth	**Exponential Decay**
	
When *b* is **greater than 1**, the relation is _____ because the graph moves **up to the right**. (The graph increases slowly and then more rapidly.)	When *b* is **between 0 and 1**, the relation is _____ because the graph moves **down to the right**. (The graph decreases rapidly and then more rapidly.)
The value of *b* is considered the **growth factor**.	The value of *b* is considered the **decay factor**.
What will the x-intercept be? To find the *x*-intercept, let *y* = ____ and solve:	
What will the y-intercept be? To find the *y*-intercept, let *x* = ____ and solve:	

MBF 3C1 Name: _____

Example 1

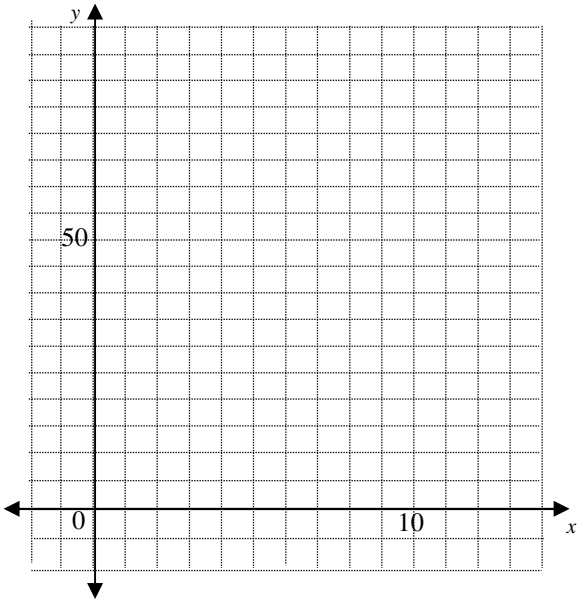
Use the table of values to answer the questions below.

x	***y***	**1st ratios**
0	1	
1	3	
2	9	
3	27	
4	81	

a. Is the relationship is exponential? Give a reason for your answer.

b. State the equation for the relationship.

c. Graph the relationship.



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Example 2

A small garden centre propagates tulip bulbs. The gardener begins with 100 bulbs. Each bulb produces several new bulbs. Three of these are kept for the next round. Write an equation to represent the number of new bulbs.

Round	Number of New Bulbs
0	
1	
2	
3	
n	

1. Start with the initial number of bulbs: ____
2. Multiply by the growth factor for each round: ___
3. Do you notice a pattern? Will it help you to create an equation for the situation?

The generalization for exponential growth and decay is _____.

- a = _____
- b = the _____ (common ratio)
- y = the value after x periods of exponential growth/decay

So equation for this question is: _____

Example 3

Suppose you have _____ in the bank and you deposit it into a savings account that pays _____ interest per year.

a) Find the equation that models this exponential growth

b) Find the amount of money you will have after 3 years.

MBF 3C1 Name: _____

Solving Problems with Exponential Relations

Example 1

To determine the equation of an exponential relationship given a rate of growth or decay as a percentage:

	a. A principal of $500 is invested at 8% per year, compounded annually. Write an exponential equation to represent the relationship.	b. A new car costs $20 000. It's value decreases 16% per year after it is purchased. Write an exponential equation to represent the relationship.
1. Start with the generalization for an exponential relation.		
2. Sub in the initial amount, __.		
3. Sub in the common ratio, __. ★ for special words: • double use _____, • for half-life use _____, • for triple use _____, etc ★ for Percent: • convert the percent to a decimal • for growth, _____ • for decay, _____		

Example 2

The population of Alberta between 1987 and 2005 can be modelled by an exponential equation. The population in 1987 was 2.4 million and the growth rate was 1.7%.

a. Write an equation to model the situation.

b. Use your model to calculate the population in 1985.

c. Use your model to calculate the population in 2012.

MBF 3C1 Name: _____

EXTRA: Exponent Laws

1. Write as a single power.

a) $2^2 \times 2^3$ b) $3^1 \times 3^4$ c) $(-2)^2 \times (-2)^4$

d) $4^5 \times 4^2$ e) $(1.02)^2 \times (1.02)^5$ f) $(-3)^3 \times (-3)^5$

g) $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^5$ h) $\left(-\frac{1}{3}\right)^4 \times \left(-\frac{1}{3}\right)^2$ i) $\left(\frac{2}{5}\right)^5 \times \left(\frac{2}{5}\right)^2$

2. Write as a single power.

a) $2^5 \div 2^3$ b) $\frac{3^4}{3^2}$ c) $(-2)^5 \div (-2)$

d) $4^5 \div 4^2$ e) $(-10)^6 \div (-10)^2$ f) $(-3)^8 \div (-3)^3$

g) $5^{10} \div 5^5$ h) $(-3)^7 \div (-3)^4$ i) $6^6 \div 6^2$

5. Write as a single power.

a) $12^{10} \times 12^5$ b) $\frac{20^{10}}{20^7}$ c) $(-7)^8 \times (-7)^6$

d) $(-8)^8 \div (-8)^6$ e) $(1.09)^{22} \times (1.09)^4$ f) $16^5 \div 16^4$

6. Write as a single power.

a) $x^{30} \times x^{40}$ b) $\frac{x^{15}}{x^{13}}$ c) $(-a)^8 \times (-a)^3$

d) $\frac{(-a)^{18}}{(-a)^{15}}$ e) $m^2 \times m^6$ f) $\frac{r^{16}}{r^{14}}$

7. Write as a single power.

a) $(2^2)^3$ b) $(3^2)^{12}$ c) $(10^4)^2$

d) $(10^2)^4$ e) $[(-2)^4]^3$ f) $[(-5)^2]^6$

8. Write as a single power.

a) $(x^3)^2$ b) $(a^4)^5$ c) $[(-x)^3]^2$

d) $[(-a)^4]^5$ e) $(m^3)^3$ f) $[(-n)^2]^2$

Answer Key:

1. a) 2^5 b) 3^5 c) $(-2)^6$
d) 4^7 e) $(1.02)^7$ f) $(-3)^8$
g) $(\frac{1}{2})^8$ h) $(\frac{1}{3})^6$ i) $(\frac{2}{5})^7$

2. a) 2^2 b) 3^2 c) $(-2)^4$
d) 4^3 e) $(-10)^4$ f) $(-3)^5$
g) 5^5 h) $(-3)^3$ i) 6^4

3. a) c^8 b) a^5 c) e^{11}
d) d^5 e) m^7 f) $(\frac{a}{2})^7$

4. a) c^2 b) a c) e
d) d^3 e) m f) $(-x)^5$

5. a) 12^{15} b) 20^3 c) $(-7)^{14}$
d) $(-8)^2$ e) $(1.09)^{26}$ f) 16

6. a) x^{70} b) x^2 c) $(-a)^{11}$
d) $(-a)^3$ e) m^8 f) r^2

7. a) 2^6 b) 3^{24} c) 10^8
d) 10^8 e) $(-2)^{12}$ f) $(-5)^{12}$

8. a) x^6 b) a^{20} c) $(-x)^6$
d) $(-a)^{20}$ e) m^9 f) $(-n)^4$

MBF 3C1 Name: _____

EXTRA: Zero and Negative Exponents

1. Evaluate.

a) 3^{-1}	**b)** 10^{0}	**c)** $\left(\frac{1}{2}\right)^{0}$
d) 8^{-2}	**e)** 9^{-1}	**f)** 1^{0}
g) 6^{-1}	**h)** 5^{-2}	**i)** 10^{-1}
j) 7^{-2}	**k)** 4^{-1}	**l)** 2^{-2}

2. Evaluate.

a) 10^{-2}	**b)** 4^{-2}	**c)** 2^{-3}
d) $\frac{1}{10^{-2}}$	**e)** $\frac{1}{4^{-2}}$	**f)** $\frac{1}{2^{-3}}$
g) 6^{-2}	**h)** 2^{-5}	**i)** 3^{-4}
j) $\frac{1}{8^{-2}}$	**k)** $\frac{1}{4^{-3}}$	**l)** $\frac{1}{10^{-1}}$

3. Evaluate.

a) $\left(\frac{1}{2}\right)^{-1}$	**b)** $\left(\frac{1}{3}\right)^{-1}$	**c)** $\left(\frac{2}{3}\right)^{-1}$
d) $\left(\frac{3}{4}\right)^{-3}$	**e)** $\left(\frac{5}{9}\right)^{-2}$	**f)** $\left(\frac{3}{2}\right)^{-3}$
g) $\left(\frac{1}{2}\right)^{-5}$	**h)** $\left(\frac{2}{3}\right)^{-4}$	**i)** $\left(\frac{7}{8}\right)^{-2}$

Answer Key

1. a) $\frac{1}{3}$	**b)** 1	**c)** 1	**d)** $\frac{1}{64}$
e) $\frac{1}{9}$	**f)** 1	**g)** $\frac{1}{6}$	**h)** $\frac{1}{25}$
i) $\frac{1}{10}$	**j)** $\frac{1}{49}$	**k)** $\frac{1}{4}$	**l)** $\frac{1}{4}$
2. a) $\frac{1}{100}$	**b)** $\frac{1}{16}$	**c)** $\frac{1}{8}$	**d)** 100
e) 16	**f)** 8	**g)** $\frac{1}{36}$	**h)** $\frac{1}{32}$
i) $\frac{1}{81}$	**j)** 64	**k)** 64	**l)** 10
3. a) 2	**b)** 3	**c)** $\frac{3}{2}$	**d)** $\frac{64}{27}$
e) $\frac{81}{25}$	**f)** $\frac{8}{27}$	**g)** 32	**h)** $\frac{81}{16}$
i) $\frac{64}{49}$			

MBF 3C1 Name: _____

EXTRA: Exponent Laws

1. Simplify each of the following. Write all answers with positive exponents.

a. $(5mn)(2mn^2p^3)$

b. $\dfrac{-18p^8q^9}{3p^2q^2}$

c. $\dfrac{(8x^4y^5)(-3x^3y)}{2x^2y^4}$

d. $(v^3wx^2)(-2vy)^2$

e. $(4abc)^2(2a^2bc)(ab^3c^3)^{-1}$

f. $\left(\dfrac{x^2y^5}{x^2y^7}\right)^0\left(\dfrac{x^5y^3}{x^{-4}y^2}\right)^{-2}$

MBF 3C1 Name: _____

EXTRA: Exponential Problems

1. Durham Region's Population, P, is projected to grow until 2031 based on the relation $P = 610\,000(1.029)^n$, where n is the number of years after 1990.
a) What does 610 000 represent?
b) What is the projected population in 2010?
c) What is the projected population in 2025?

2. A stamp has a current value of $1.50. The value is going to increase by 7% every year.
a) What is the equation that represents the value of the stamp?
b) What is the value of the stamp 5 years from now?
c) What is the value of the stamp in 10 years?

3. You purchased a 2010 Camaro for $35 000. The value of the Camaro decreases by 12% every year.
a) What is the equation that represents the value of the Camaro?
b) What is the value of the Camaro in 3 years?
c) What is the value of the Camaro in 7 years?

4. You purchased a $500 Canada Savings Bond. It's value is going to increase by 3% every year.
a) What is the equation that represents the value of the Bond?
b) What is the value of the bond in 10 years?
c) What is the value of the bond in 20 years?

5. The Beluga Whale is one of Canada's Endangered Species. It's current population is 1000 animals and it's projected to decrease by 5% every year.
a) What is the equation that represents the population of the Beluga Whale?
b) What will the population be in 10 years?
c) What was the population 5 years ago?