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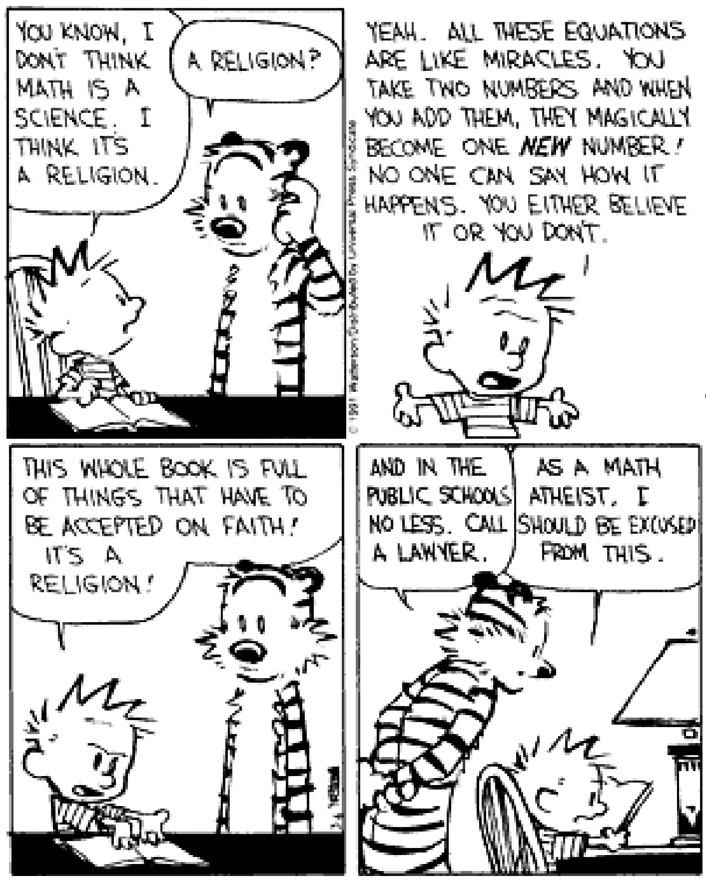
UNIT 2

Quadratic Relations in Factored Form



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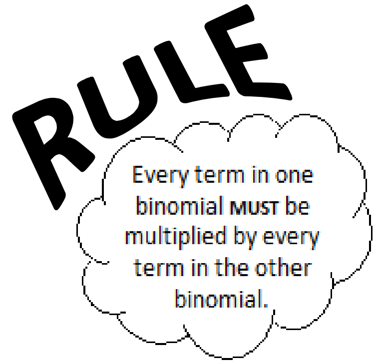
Expanding Binomials

A binomial is a polynomial with <u>two</u> terms.

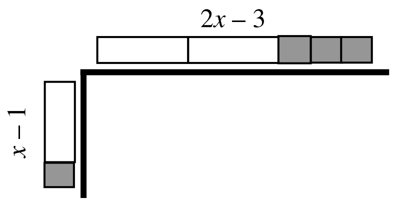


Binomials can be simplified using the following methods:

Expand $(2x-3)(x-1)$.

1. Algebra Tiles/Area Model

i. draw a multiplication frame
ii. use algebra tiles to model each binomial
iii. fill the rectangle using algebra tiles
iv. state the answer



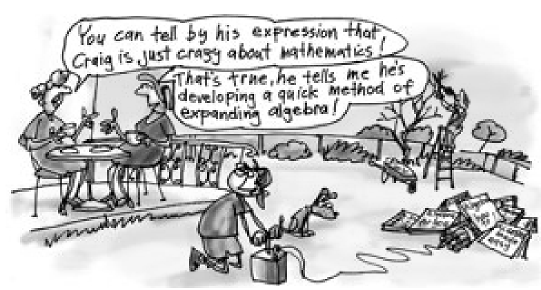
2. Chart

i. draw a chart
ii. place each term from one binomial at the top of each column and each term from the other binomial at the beginning of each row
iii. multiply columns and rows
iv. state the answer

	$2x$	-3
x		
-1		

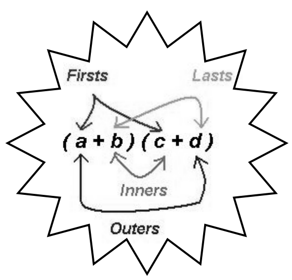
3. Expand Using The Distributive Property

$(2x-3)(x-1)=$



4. Multiplication Pattern: FOIL

$(2x-3)(x-1)=$



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Example 1

Expand each of the following.

a. $(5x-2)(x-3)$ b. $(2x+1)(3x-2)$ c. $(6x+1)^2$ d. $(x-1)^2$

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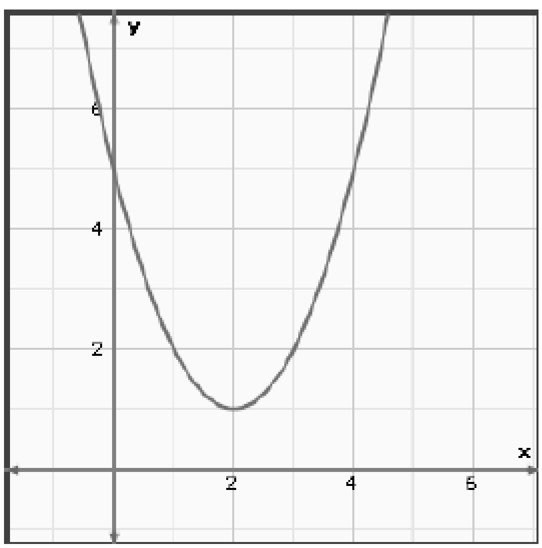
Converting Vertex Form to Standard Form

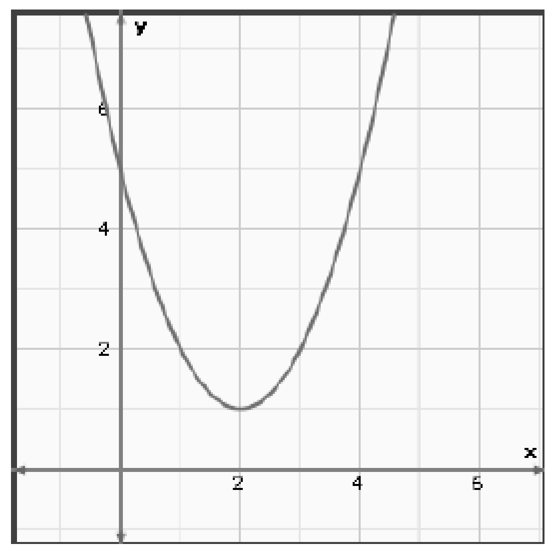
Quadratics can be written in different forms. Each form provides specific information about the quadratic.

Compare the graphs of the same quadratic when it is in vertex form and standard form:

$y=(x-2)^2+1$



$y=x^2-4x+5$

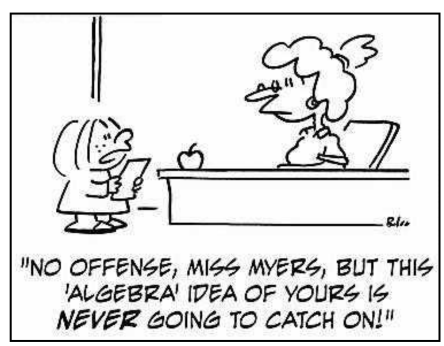


What information (about the key features of the parabola) can you take from each form?

- An equation in **standard form**, ________________, tells the location of the ________________.
- An equation in **vertex form**, ________________, tells the location of the ________________, ________________________________ and ________________________________.

Vertex Form can be converted to **Standard Form** by ________________________________.

$y=(x-2)^2+1$



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Example 1

Given the equation for a quadratic in vertex form, $y=-3(x+1)^2-4$

i. state the key features you can determine from vertex form
ii. expand the equation to put it into standard form
iii. state the key feature(s) you can determine from standard form

key features from vertex form	
expand	$y=-3(x+1)^2-4$
key features from standard form	

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Common Factoring and Difference of Squares

The opposite of expanding is ______________.

A ______________________________ is an expression that can be divided into each term in a polynomial.

For example, $3x$ is a common factor for the polynomial $9x^3 - 15x^2 + 3x$, because $3x$ divides evenly into each of the three terms.

TO COMMON FACTOR:

1. Determine what the greatest common factor is for all the terms in the polynomial.	$9x^3 - 15x^2 + 3x$ *Choose the greatest number and the most variables that will divide evenly into each term:*
2. Divide each term in the polynomial by the common factor.	
3. Write the answer in proper form: GCF (“left overs”).	

Example 1

a. $8x^4 - 4x^3 + 20x^2$

b. $25x^2 - 100$

c. $-54x^4 + 135x$

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A number is a ______________________________ if when square rooted the result is a whole number. For example, 9, 16, 25, 81 and 100 are all perfect squares because when square rooted the answers are whole numbers → $\sqrt{9}=3$, $\sqrt{16}=4$, $\sqrt{25}=5$, $\sqrt{81}=9$ and $\sqrt{100}=10$.

If a variable has an **even** exponent then it is considered a perfect square.

To determine the square root of a variable, keep the base and divide the exponent by 2.

For example, x^8 is a perfect square, its square root is x^4.

A **difference of squares** is a **binomial** with **two perfect square terms being subtracted**.

TO FACTOR USING DIFFERENCE OF SQUARES:

1. Check to ensure that difference of squares factoring is possible.	$9x^2-16$ Two terms both terms perfect squares, even powers subtraction
2. Create two sets of brackets. 3. Square root the first term of the polynomial and place one in each bracket in first position 4. Square root the second term of the polynomial and place one in each bracket in second position. 5. Separate the terms in one bracket with + and in the other with −.	

Example 2

a. x^2-25

b. $16x^2-1y^2$

c. $81x^2-4y^4$

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Factoring by Sum & Product

Factor by Sum & Product when given a trinomial in the form ax^2+bx+c, where $a=1$.

TO FACTOR USING SUM & PRODUCT:

1. Find two factors of c which also add to be b. (start with combinations for the product, *c* If product is positive use two positives or two negatives If product is negative use one of each)	$x^2+4x-12$
2. Create two sets of brackets. 3. Square root the first term of the polynomial and place one in each bracket in the first position. 4. Put one of the factors found in step 1 in each of the brackets in the second position.	

Helpful Hints:

- if b is + and c is + then both factors are +
- if b is − and c is + then both factors are −
- if b is + and c is − then the larger factor is + and the smaller factor is −
- if b is − and c is − then the larger factor is − and the smaller factor is +

Example 1

a. $x^2+8x+12$

b. $x^2-7x-18$

c. x^2-5x+6

d. $x^2+12x+11$

e. $x^2-2x-15$

f. $x^2-13x+22$

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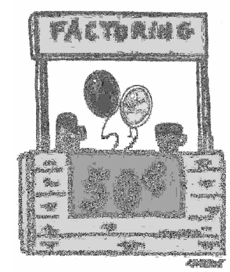
Factoring: Multiple Methods



When factoring, always

i. common factor if possible
ii. factor using appropriate method (difference of squares or sum & product)
iii. check your answer to see any factors can be factored further

Example 1

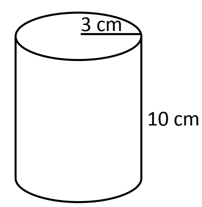
a. $3x^2+21x+36$ b. $2x^2-50$ c. $4x^2-8x-60$

Example 2

The surface area of a cylinder is given by the formula $SA=2\pi r^2+2\pi rh$.

a. Factor the expression for the surface area.

b. A cylinder has radius 3 cm and height 10 cm. Use both the original expression and the factored expression to find the surface area of this cylinder to the nearest square centimetre.



c. Why would the factored form of the formula be more useful than the original form?

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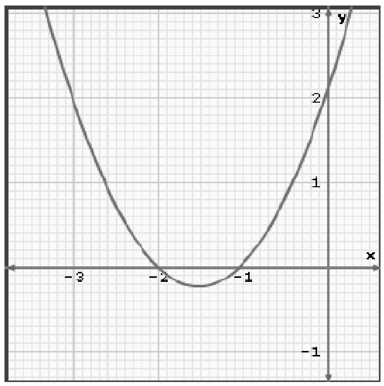
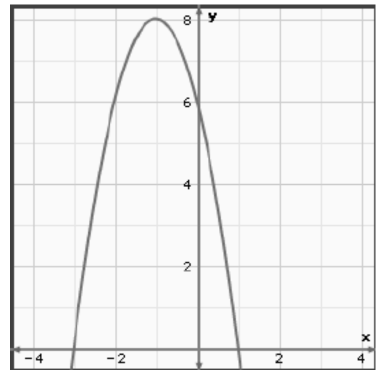
Quadratics in Factored Form

State the key features of each graph.

	$y=(x+2)(x+1)$	$y=-2(x+3)(x-1)$
		
zeros		
y-intercept		
vertex		
axis of symmetry		
optimal value		

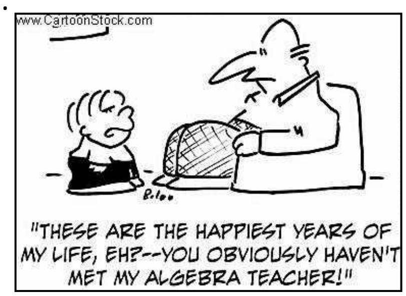
Compare the equation of each graph to it's key features. What information (about the key features of the parabola) can you take from this form of equation?

- An equation in **factored form**, ______________________, tells the location of the ________________.

  (The zeros must be removed from the brackets: $x-r=0$ *and* $x-t=0$*)*

Factored Form can be converted to **Standard Form** by ________________.

$y=3(x-2)(x+1)$



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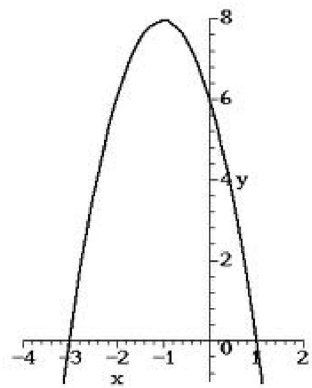
Example 1

a. Factor $y = -3x^2 + 24x - 48$ to find the zeros of the relation.

b. Factor $y = -24x^2 + 6$ to find the zeros of the relation.

c. Factor $y = -4x^2 + 24x$ to find the zeros of the relation.



Example 2

Find the equation of the quadratic in factored form using the picture provided above.

1. State the generalization for a quadratic in factored form.	$y = a(x - r)(x - t)$
2. Substitute the zeros into the generalization for *r* and *t*	
3. Substitute the other point that the parabola passes through into the generalization for (*x*, *y*).	
4. Solve for the variable *a*.	
5. Sub the values for *a*, *r*, and *t* into the generalization for factored form.	

Problems Involving Quadratic Relations in Factored Form

Example 1

The profit of a chocolate bar can be modeled by the equation $P = -0.3(n-7)^2 + 10.8$, where P is the profit in Thousands of dollars and n is the number of chocolates sold in thousands.

a) What is the coordinates of the vertex? Explain what it represents

b) What is the y-intercept? Explain what it represents

c) What are the zeros? Explain what they represent.

UNDERSTANDING PROBLEMS RELATED TO FACTORED FORM

- draw sketches to help visualize the situation
- consider how key features relate to the context of the problem:
  - initial point = _________________
  - break-even points/distance/time/etc. = _______
  - max/min profit/distance/height/etc. = ________ _________________
  - point at which max/min occurs = ___________________

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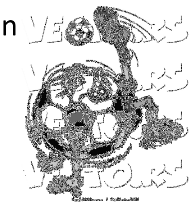
Example 2

The path of a soccer ball can be modelled by the relation $h = -0.1d^2 + 0.5d + 0.6$, where *h* is the ball's height and *d* is the horizontal distance from the kicker. Both measured in meters.



a. Find the zeros of the relation.

b. What do the zeros mean in the context of the question?

Example 3

The arch of a small suspension bridge over a gorge can be modelled by the equation $y = -2x^2 + 4x + 6$ where *x* is the distance in meters from the edge of the gorge and *y* is the height above the ground, also in meters.

a. What are the zeros? What do they represent?

b. How long is the bridge? Justify your answer.

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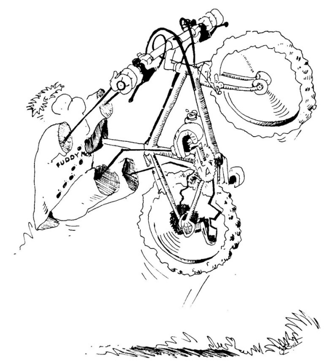
Example 4

A rider on a mountain bike jumps off a ledge. Her path is modelled by the relation $h = -0.3d^2 + 1.2d + 1.5$, where *h* is her height above the ground and *d* is her horizontal distance from the ledge, both in metres.

a. What is the height of the ledge?

b. How far was the rider from the ledge when she landed?



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Finding the Vertex of a Quadratic

To find the **vertex** of a quadratic when it is in the **factored form** **y=a(*x* - *r*)(*x* - *t*)** you must:

To find the **vertex** of a quadratic when it is in the **standard form** **y=*ax*^{2} + *bx* + *c*** you must:

To find the **vertex** of a quadratic when it is in the **vertex form** **y=*a(x* - *h)*^{2} + k** you must:

Factored Form	**Zeros or x-int**	**x** (axis of symm)	**y** (optimal val or max/min value)	**Vertex and vertex Form**	**Convert to standard form**	**y-int**
$y=-3(x-2)(x-8)$						

Standard Form	**y-int**	**Zeros or x-int**	**x** (axis of symm)	**y** (optimal val or max/min value)	**Vertex and vertex form**
$y=x^2-3x-18$					

Vertex Form	**Vertex**	**Is the vertex a max or a min?**	**Convert to standard form**
$y=-3(x-2)^2+5$			

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Example

The flight of a baseball is modelled by $y = -4.9x^2 + 9.8x + 14.7$ where x is the time, in sec, and y is the height, in m, above the ground.

a. What is the height of the ball 0.5 seconds after it was hit?

b. What is the height of the ball when it was hit?

c. How long does it take for the ball to reach the ground?

d. Find the maximum height.

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Factoring Review

1. Common Factor
   a. $2x + 4$
   b. $-3x - 18$
   c. $18x^2 - 6x + 6$
   d. $60x^2 - 25x + 15$
   e. $10x^2 - 4x$
   f. $-16xy - 4x$
   g. $-2x + 6y$
   h. $-mx + my$
   i. $24x - 30$
   j. $-7x - 21$
   k. $24x^3 - 15x^2 - 20x$
   l. $21x^4 - 28x^3 - 35x^2$
   m. $3x + 3y$
   n. $-5a - 5b$
   o. $4m + 6n$
   p. $5 - 10b$
   q. $4r^2 - 8r$
   r. $-15x^2 - 25x$
   s. $-ax + ay - 2a$
   t. $6a + 9b - 3c$
   u. $3a^2 + 6a$
   v. $36a^4 - 9b^4$
   w. $x^2y^3z - 2xy^2$
   x. $x^2 - x$

2. Factor using the difference of squares, remember to check for a common factor first.
   a. $x^2 - 1$
   b. $81x^2 - 16$
   c. $-9m^2 + 49$
   d. $-16 + 9y^2$
   e. $-(1 - a^4)$
   f. $x^2 - 9$
   g. $m^4 - 16$
   h. $x^3 - xy^2$
   i. $-x^2 + 16$
   j. $2x^2 - 8$
   k. $-1 + 9x^2$
   l. $x^8 - 1$
   m. $\frac{x^2}{25} - 1$

3. Factor using sum and product, remember to check for a common factor first.
   a. $x^2 + 7x + 12$
   b. $x^2 - 1x - 12$
   c. $x^2 + 8x + 12$
   d. $2x^2 + 6x - 8$
   e. $-x^2 + 7x - 12$
   f. $3a^2 - 36a - 39$
   g. $x^2 - 5x - 24$
   h. $x^2 + 3x - 88$
   i. $m^2 - 9m - 112$
   j. $a^2 - 5a - 36$
   k. $4y^2 + 8y - 60$
   l. $x^2 - 13x + 30$
   m. $a^2 + 10a + 21$
   n. $y^2 - 5y - 14$
   o. $x^2 + 13x - 30$
   p. $4x^2 + 28x + 24$
   q. $2x^2 - 2x - 24$
   r. $-a^2 - 2ab - b^2$

4. Factor fully.
   a. $36 - 16x^2$
   b. $x^4 - 16$
   c. $x^4 - 13x^2 + 36$
   d. $3x^2 - 18x^2 - 216$
   e. $x^2 + 3x + 2$
   f. $a^4 - 5a^2 - 36$
   g. $3x^2 + 9x + 6$
   h. $8x^2 - 98$
   i. $x^2 - 10x + 25$
   j. $-x^2 - 7x - 12$
   k. $3x^2 + 9x - 12$
   l. $5x^2 - 10x + 15$
   m. $x^4 - 64$
   n. $m^4 - 9m^2 - 112$

5. Fully factor.
   a. $16x^2 - 25$
   b. $36y^2 - 121$
   c. $25a^2b^2 - 49$
   d. $81m^4 - 625y^6$
   e. $8e^2 - 50$
   f. $48x^2 - 27y^2$
   g. $20m^4n^4 - 180$
   h. $h^4 - 256$
   i. $4x^8 - 64y^8$
   j. $x^2 - 12x + 36$
   k. $x^2 + 18xy + 81y^2$
   l. $m^2b^2 - 10mb + 25$
   m. $3x^2 - 6x + 3$
   n. $4a^2 - 80a + 400$
   o. $x^3 - 14x^2 + 49x$

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STRAND 1 TASK

Quadratic Relations

1. Many suspension bridges hang from cables that are supported by two towers. The shape of the hanging cables is very close to a parabola. A typical suspension bridge has large cables that are supported by two towers that are 20 m high and 80 m apart. The bridge surface is suspended from the large cables by many smaller vertical cables. The shortest vertical cable is 4 m long.

   a. Sketch the bridge. Use the bridge surface as the *x*-axis, and the left tower as the y-axis.

   b. State the vertex (*h*, *k*). ____________

   c. Find the value of '*a*'.

   d. Write an equation for the shape of the bridge in vertex form.

   e. Convert the equation which models the bridge to standard form

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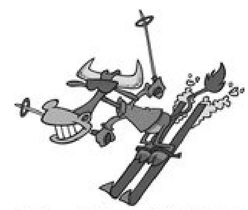
2. A student running a ski trip over March Break determined that his break-even point occurs if he can sell ski packages to 12 students. He also knows that when he sells 16 ski packages he will maximize his profit at $2000.
   a. Assume that the relation for his profit is quadratic. Sketch a graph that models his situation.



   b. State the zeros (r and *t*). ____________
   c. Find the value of '*a*'.
   d. Write an equation for his profit in factored form
   e. Convert the equation for his profit to standard form
   f. What is his profit if he sells 18 packages?
   g. What is the initial profit?