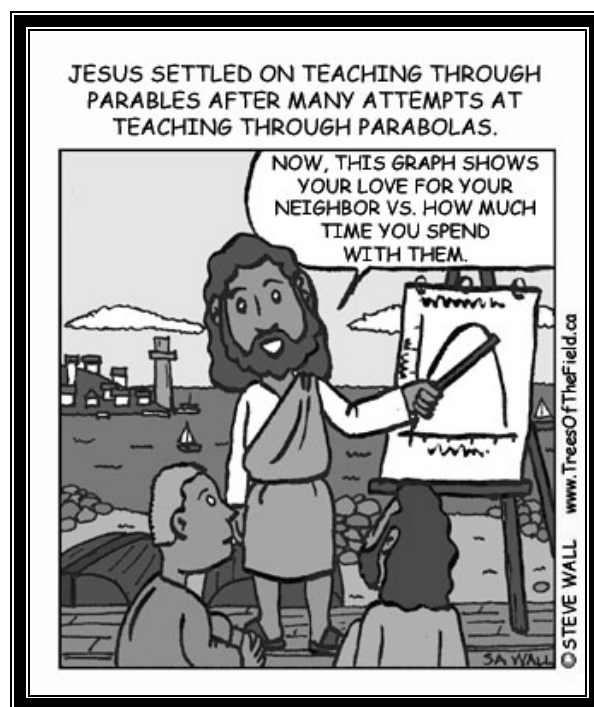


## UNIT 1

### Quadratic Relations in Vertex Form



# Modelling Quadratic Relations

Complete the chart below.

| EQUATION   | TABLE OF VALUES |                         |     |                      |                     | GRAPH |
|--|-----------------|-------------------------|-----|----------------------|---------------------|-------|
| The distance travelled by a boy on a bike is modelled by the equation $y = 3x$ .         | $x$             | $3x$                    | $y$ | 1 <sup>st</sup>      | 2 <sup>nd</sup>     |       |
|  | 0               | $3(0)$                  | 0   | $\frac{3-0}{3} = 3$  | $\frac{3-3}{3} = 0$ |       |
|  | 1               | $3(1)$                  | 3   | $\frac{6-3}{3} = 3$  | $\frac{3-3}{3} = 0$ |       |
|  | 2               | $3(2)$                  | 6   | $\frac{9-6}{3} = 3$  | $\frac{3-3}{3} = 0$ |       |
|  | 3               | $3(3)$                  | 9   | $\frac{12-9}{3} = 3$ |                     |       |
|  | 4               | $3(4)$                  | 12  |                      |                     |       |
| The height of a falling marble is recorded and modelled by the equation $y = -5x + 2$ .  | $x$             | $-5x + 2$               | $y$ | 1 <sup>st</sup>      | 2 <sup>nd</sup>     |       |
|  | 0               | $-5(0) + 2$             | 2   |                      |                     |       |
|  | 1               | $-5(1) + 2$             |     |                      |                     |       |
|  | 2               | $-5(2) + 2$             |     |                      |                     |       |
|  | 3               | $-5(3) + 2$             |     |                      |                     |       |
|  | 4               | $-5(4) + 2$             |     |                      |                     |       |
| The height of a ball thrown by a child is modelled by the equation $y = -2x^2 + 8$ .     | $x$             | $-2x^2 + 8$             | $y$ | 1 <sup>st</sup>      | 2 <sup>nd</sup>     |       |
|  | 0               | $-2(0)^2 + 8$           | 8   |                      |                     |       |
|  | 1               | $-2(1)^2 + 8$           |     |                      |                     |       |
|  | 2               | $-2(2)^2 + 8$           |     |                      |                     |       |
|  | 3               | $-2(3)^2 + 8$           |     |                      |                     |       |
|  | 4               | $-2(4)^2 + 8$           |     |                      |                     |       |
| The depth of a submarine is tracked and modelled by the equation $y = x^2 - 30x + 125$ . | $x$             | $x^2 - 30x + 125$       | $y$ | 1 <sup>st</sup>      | 2 <sup>nd</sup>     |       |
|  | 5               | $(5)^2 - 30(5) + 125$   | 0   |                      |                     |       |
|  | 10              | $(10)^2 - 30(10) + 125$ |     |                      |                     |       |
|  | 15              | $(15)^2 - 30(15) + 125$ |     |                      |                     |       |
|  | 20              | $(20)^2 - 30(20) + 125$ |     |                      |                     |       |
|  | 25              | $(25)^2 - 30(25) + 125$ |     |                      |                     |       |

**MBF 3C1**

Name: \_\_\_\_\_

 A **MATHEMATICAL MODEL** is a mathematical description of a real situation.

 Use the real-life models you completed to **SUMMARIZE** the differences between *linear* and *quadratic* relations.

| Type of Mathematical Model                | Linear Relations | Quadratic Relations |
|---|------------------|---------------------|
| EQUATION<br>OR<br>DEGREE                  |                  |                     |
| TABLE OF VALUES<br>OR<br>DIFFERENCE TABLE |                  |                     |
| GRAPH<br>OR<br>DIAGRAM                    |                  |                     |

**Example 1**

 Examine each equation. Determine the **degree** and the **type of relation** it represents (linear, quadratic or neither).

|                |                     |                     |
|----------------|---------------------|---------------------|
| $y = -5x + 18$ | $y = 6x^3 + 2x - 1$ | $y = 2x^2 + 7x - 1$ |
| Degree: _____  | Degree: _____       | Degree: _____       |
| Type: _____    | Type: _____         | Type: _____         |

**Example 2**

 Complete each table. Determine the **type of relation** it represents. Give a **reason** for your answer.

| $x$ | $-2x+1$    | $y$ | 1 <sup>st</sup> | 2 <sup>nd</sup> |
|-----|------------|-----|-----------------|-----------------|
| -4  | $-2(-4)+1$ |     |                 |                 |
| -2  | $-2(-2)+1$ |     |                 |                 |
| 0   | $-2(0)+1$  |     |                 |                 |
| 2   | $-2(2)+1$  |     |                 |                 |
| 4   | $-2(4)+1$  |     |                 |                 |

Type: \_\_\_\_\_

Reason: \_\_\_\_\_

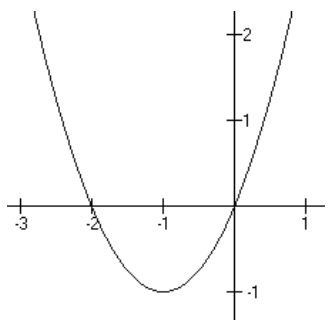
| $x$ | $2x^2-3$    | $y$ | 1 <sup>st</sup> | 2 <sup>nd</sup> |
|-----|-------------|-----|-----------------|-----------------|
| -2  | $2(-2)^2-3$ |     |                 |                 |
| -1  | $2(-1)^2-3$ |     |                 |                 |
| 0   | $2(0)^2-3$  |     |                 |                 |
| 1   | $2(1)^2-3$  |     |                 |                 |
| 2   | $2(2)^2-3$  |     |                 |                 |

Type: \_\_\_\_\_

Reason: \_\_\_\_\_

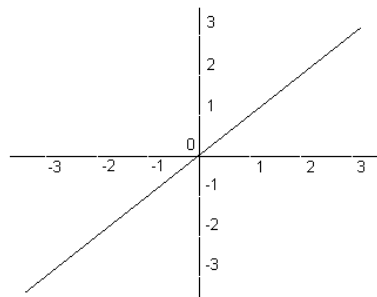
**Example 3**

Examine each graph. Determine the **type of relation** it represents. Give a **reason** for your answer.



Type: \_\_\_\_\_

Reason: \_\_\_\_\_



Type: \_\_\_\_\_

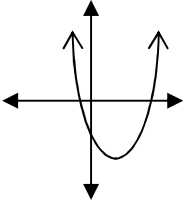
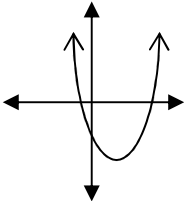
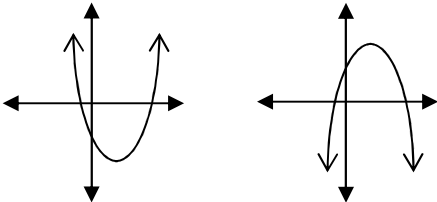
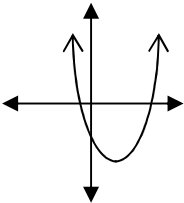
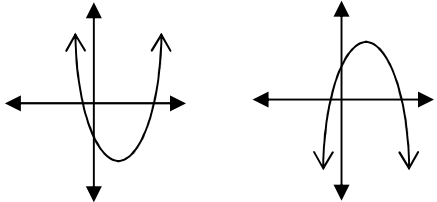
Reason: \_\_\_\_\_

# Changing Quadratic Relations: The value of 'a'

## Quadratic Vocabulary:

A quadratic relation is modeled by a smooth symmetrical curve, known as a \_\_\_\_\_.

The **key features** of a parabola are:

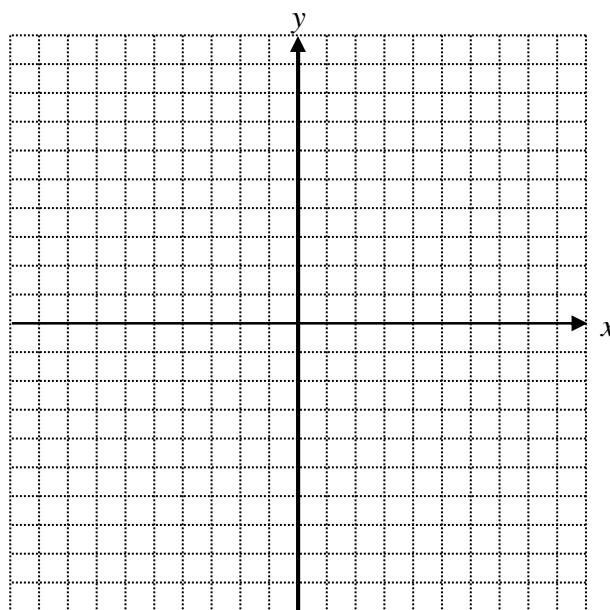
|                  |   |  |
|------------------|---|--|
| zeros            |    |  |
| y-intercept      |    |  |
| vertex           |  |  |
| axis of symmetry |  |  |
| optimal value    |  |  |

**The Basic Parabola:**

1. Complete the table of values for the relation  $y = x^2$ , including finite differences.

| $x$ | $x^2$ | $y$ | 1 <sup>st</sup> | 2 <sup>nd</sup> |
|-----|-------|-----|-----------------|-----------------|
| -4  |       |     |                 |                 |
| -3  |       |     |                 |                 |
| -2  |       |     |                 |                 |
| -1  |       |     |                 |                 |
| 0   |       |     |                 |                 |
| 1   |       |     |                 |                 |
| 2   |       |     |                 |                 |
| 3   |       |     |                 |                 |
| 4   |       |     |                 |                 |

2. Plot the data on the coordinate grid.  
Draw a curve of best fit.



**Summarize the properties of the basic quadratic relation  $y = x^2$ .**

- The vertex is \_\_\_\_\_ and is also known as the \_\_\_\_\_.
- The optimum value is \_\_\_\_\_ and it is a \_\_\_\_\_ because \_\_\_\_\_.
- The axis of symmetry is \_\_\_\_\_. The graph is symmetrical about \_\_\_\_\_.
- The zeros of the relation are \_\_\_\_\_.
- To graph the basic parabola without creating a table of values first:
  - Start at the vertex:  $(0,0)$ .
  - Go right 1 and up 1, plot a point.  
Go right 1 and up 3, plot a point.  
Go right 1 and up 5, plot a point.  
*These points are located on the right side of the y-axis.*
  - Find the points on the left side using symmetry.  
*OR repeat the pattern going left rather than right.*
  - Draw a curve of best fit.

**Investigate  $y = ax^2$** 

| Function         | Value of $a$ in $y = ax^2$ | Direction of Opening | Vertex | Axis of Symmetry | Same shape as $y = x^2$ ? |
|------------------|----------------------------|----------------------|--------|------------------|---------------------------|
| a. $y = x^2$     | 1                          | up                   | (0, 0) | $x = 0$          |                           |
| b. $y = 2x^2$    |                            |                      |        |                  |                           |
| c. $y = 0.5x^2$  |                            |                      |        |                  |                           |
| d. $y = -2x^2$   |                            |                      |        |                  |                           |
| e. $y = -0.5x^2$ |                            |                      |        |                  |                           |

**How does the value of  $a$  affect the basic parabola?**

The **sign** of  $a$  indicates \_\_\_\_\_:

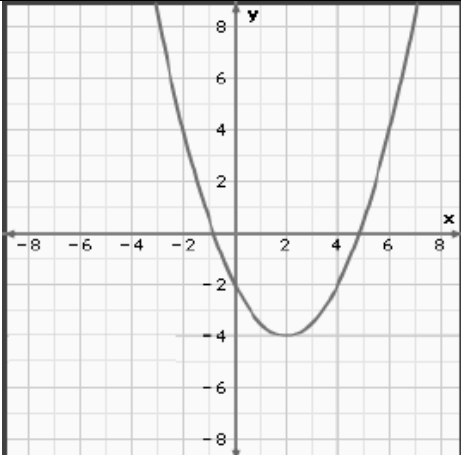
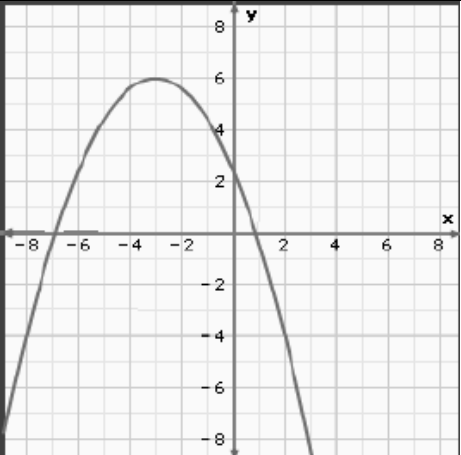
- when  $a$  is positive, the parabola \_\_\_\_\_ and has a \_\_\_\_\_
- when  $a$  is negative, the parabola \_\_\_\_\_ and has a \_\_\_\_\_; this is known as a \_\_\_\_\_

The **value** of  $a$  describes \_\_\_\_\_:

- when  $a$  is between 0 and 1, the parabola is \_\_\_\_\_ than  $y = x^2$ ; known as a \_\_\_\_\_
- when  $a$  is greater than 1, the parabola is \_\_\_\_\_ than  $y = x^2$ , known as a \_\_\_\_\_

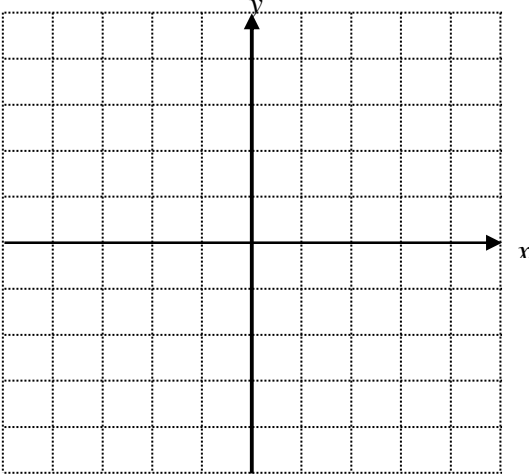
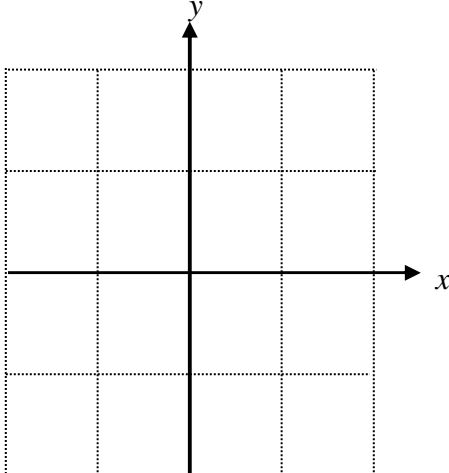
**Example 1**

State the key features of each graph. (Round answers to the nearest 0.5.)

|                  |   |  |
|------------------|---|--|
|                  |  |  |
| zeros            |   |  |
| y-intercept      |   |  |
| vertex           |   |  |
| axis of symmetry |   |  |
| optimal value    |   |  |

**Example 2**

For each of the following, (i) state the transformations, and (ii) graph the parabola.

|                     | $y = -2x^2$  | $y = \frac{1}{4}x^2$  |
|---------------------|--|---|
| (i) TRANSFORMATIONS |  |   |
| (ii) GRAPH          |  |  |



# Changing Quadratic Relations: The values of 'h' and 'k'

## Investigate $y = x^2 + k$

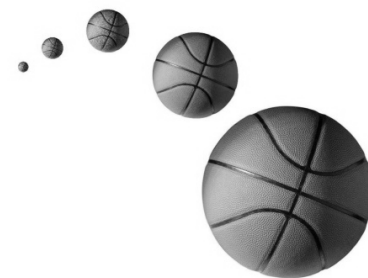
| Function         | Value of $k$ in $y = x^2 + k$ | Direction of Opening | Vertex | Axis of Symmetry | Same shape as $y = x^2$ ? |
|------------------|-------------------------------|----------------------|--------|------------------|---------------------------|
| a. $y = x^2$     | 0                             | up                   | (0, 0) | $x = 0$          |                           |
| b. $y = x^2 + 2$ |                               |                      |        |                  |                           |
| c. $y = x^2 + 4$ |                               |                      |        |                  |                           |
| d. $y = x^2 - 1$ |                               |                      |        |                  |                           |
| e. $y = x^2 - 3$ |                               |                      |        |                  |                           |

### How does the value of $k$ affect the basic parabola?

- when  $k$  is greater than 0, the parabola shifts \_\_\_\_\_
- when  $k$  is less than 0, the parabola shifts \_\_\_\_\_

The value of  $k$  describes the \_\_\_\_\_ of the parabola.

It is known as the \_\_\_\_\_ or \_\_\_\_\_



### Example 1

For each of the following, (i) state the transformations, and (ii) graph the parabola.

|                     | $y = x^2 + 3$ | $y = x^2 - 2$ |
|---------------------|---------------|---------------|
| (i) TRANSFORMATIONS |               |               |
| (ii) GRAPH          |               |               |

**Investigate  $y = (x - h)^2$** \* To determine the value of  $h$ , remove it from the brackets by setting the expression equal to zero and solving. \*

| Function           | Value of $h$ in $y = (x - h)^2$ | Direction of Opening | Vertex | Axis of Symmetry | Same shape as $y = x^2$ ? |
|--------------------|---------------------------------|----------------------|--------|------------------|---------------------------|
| a. $y = x^2$       | 0                               | up                   | (0, 0) | $x = 0$          |                           |
| b. $y = (x - 2)^2$ | $x - 2 = 0$<br>$x = 2$          |                      |        |                  |                           |
| c. $y = (x - 4)^2$ |                                 |                      |        |                  |                           |
| d. $y = (x + 1)^2$ |                                 |                      |        |                  |                           |
| e. $y = (x + 3)^2$ |                                 |                      |        |                  |                           |

**How does the value of  $h$  affect the basic parabola?**

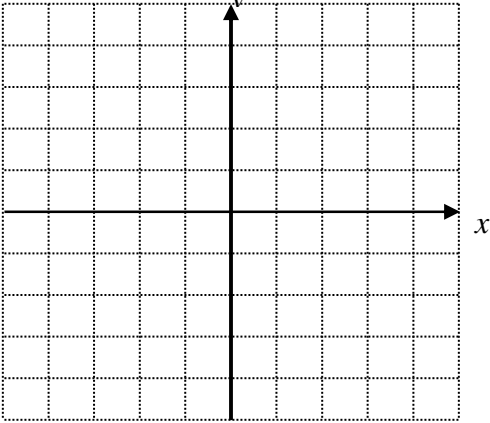
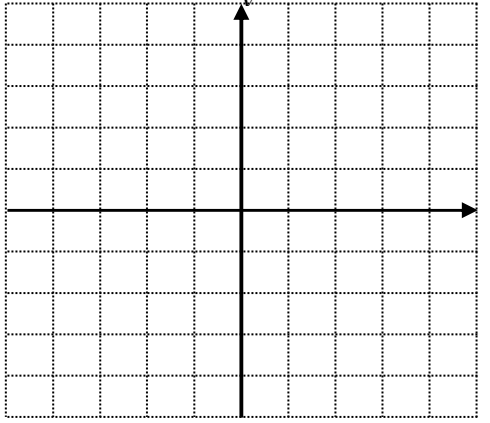
- when  $h$  is greater than 0, the parabola shifts \_\_\_\_\_
- when  $h$  is less than 0, the parabola shifts \_\_\_\_\_

The value of  $h$  describes the \_\_\_\_\_ of the parabola.

It provides the value for the \_\_\_\_\_ and is the \_\_\_\_\_.

**Example 2**

For each of the following, (i) state the transformations, and (ii) graph the parabola.

| $y = (x + 2)^2 - 3$ |   | $y = (x - 3)^2 + 1$  |  |
|---------------------|---|--|--|
| (i) TRANSFORMATIONS |   |  |  |
| (ii) GRAPH          |  |  |  |

# Vertex Form of Quadratic Relations: $y = a(x - h)^2 + k$

## Summary

The transformed parabola  $y = a(x - h)^2 + k$  is known as **vertex form**.

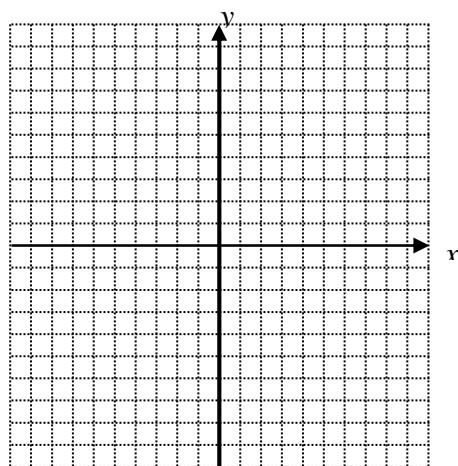
- $a$  represents the \_\_\_\_\_ and \_\_\_\_\_
- $k$  represents the \_\_\_\_\_
- $h$  represents the \_\_\_\_\_
- the coordinates of the vertex of the parabola are \_\_\_\_\_



## Sketching Vertex Form: $y = -4(x - 3)^2 + 5$

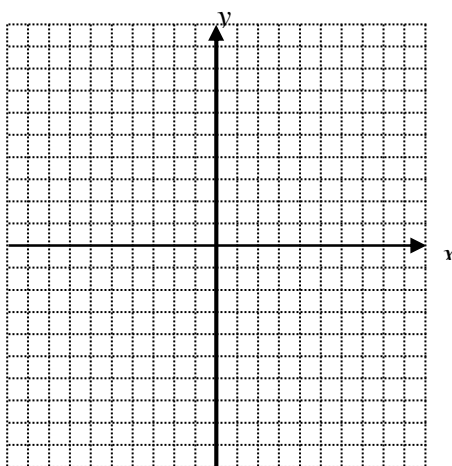
1. Graph the basic parabola.
2. Plot the vertex  $(h, k)$  by determining the horizontal and vertical translations.
3. Find additional points on the parabola by multiplying the value of  $a$  by the step pattern of the basic parabola.

### step 1

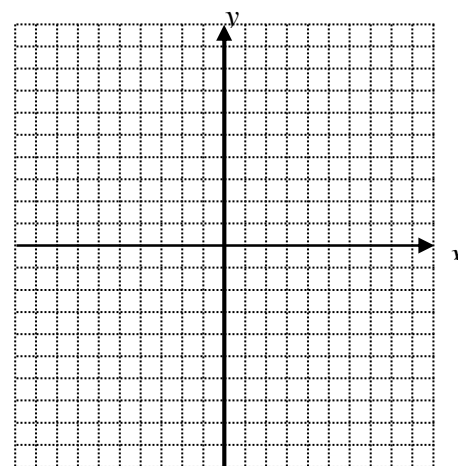


Copyright 1997 Randy Glasbergen. www.glasbergen.com

### step 2



### step 3

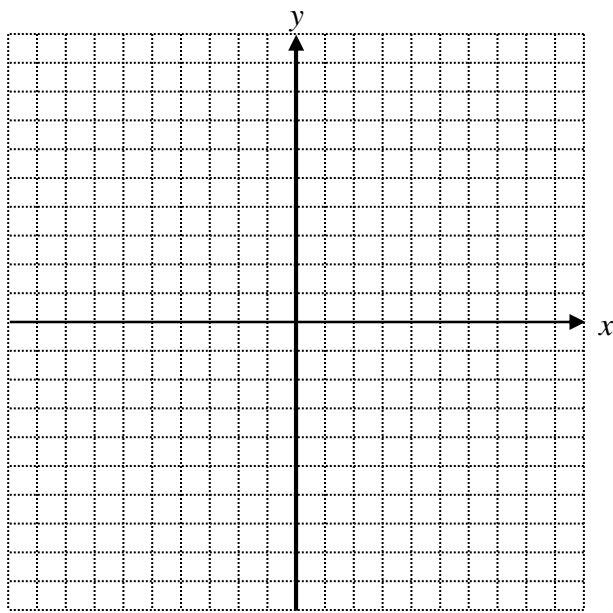
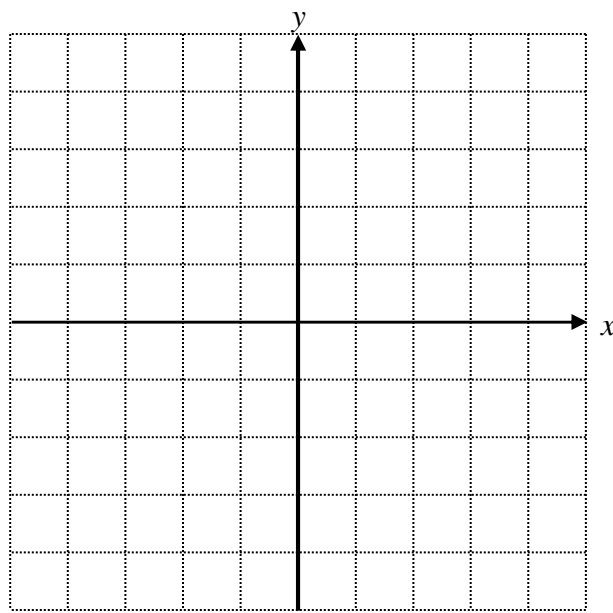


"Algebra class will be important to you later in life because there's going to be a test six weeks from now."



**Example 1**

For each of the following quadratic relations in vertex form, (i) state the transformations, and (ii) graph the parabola.

| $y = -3(x - 2)^2 - 1$ |  | $y = \frac{1}{2}(x + 3)^2 + 1$  |  |
|-----------------------|--|---|--|
| (i) TRANSFORMATIONS   |  |   |  |
|                       |  |   |  |
| (ii) GRAPH            |  |  |  |
|                       |  |   |  |



"If you think I do a good job teaching quadratic equations, like me on Facebook."



## Creating an Equation in Vertex Form

The vertex of a parabola is  $(3, -7)$ . It also passes through the point  $(1, -5)$ . State the equation of the quadratic.

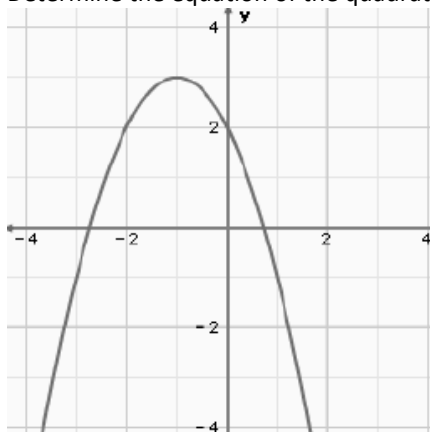
|   |                      |
|---|----------------------|
| 1. State the generalization for a quadratic in vertex form.   | $y = a(x - h)^2 + k$ |
| 2. Substitute the vertex into the generalization for $(h, k)$ .                                       |                      |
| 3. Substitute the other point that the parabola passes through into the generalization for $(x, y)$ . |                      |
| 4. Solve for the variable $a$ .   |                      |
| 5. Sub the values for $a$ , $h$ , and $k$ into the generalization for vertex form.                    |                      |

### Example 2

Determine the equation of the quadratic relation that passes through the point  $(3, 1)$  and has a vertex at  $(5, -7)$ .

### Example 3

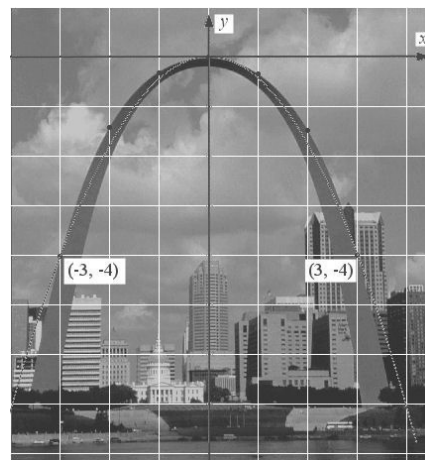
Determine the equation of the quadratic relation in the graph provided.



# Understanding Problems Involving Quadratic Relations

## Example 1

A grid has been superimposed on The Gateway Arch in St. Louis. Find the equation that models The Arch.

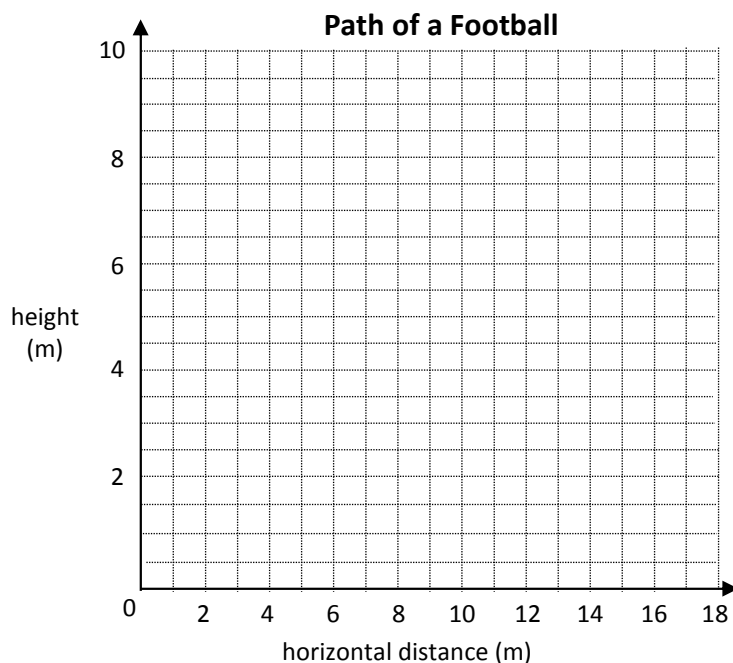


## Example 2

A football was kicked. Its path can be modelled by the relation  $h = -0.1(d - 8.7)^2 + 7.6$  where  $h$  is the football's height above the ground and  $d$  is the horizontal distance from where the football was kicked, both in meters.

- a. What is the vertex of the parabola?                      b. What is the football's initial height?

- c. Graph of the football's path.



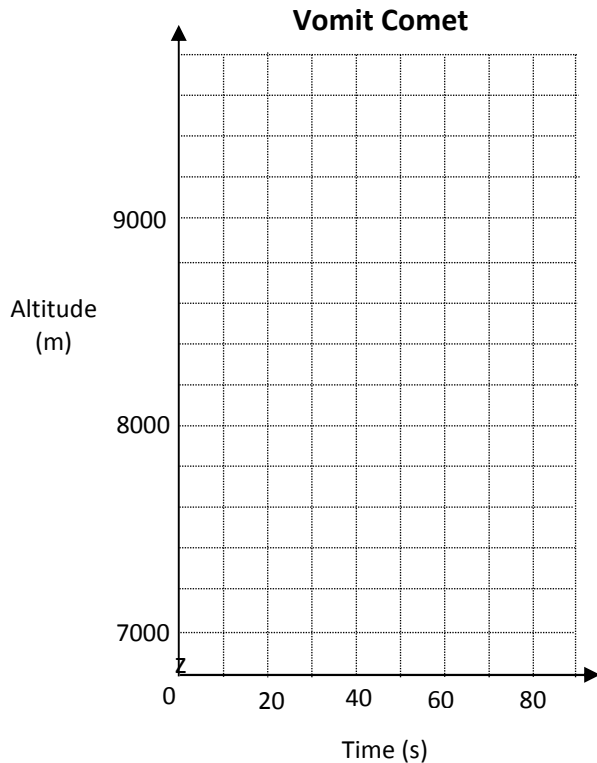
- d. What does the vertex represent in terms of the football's path?



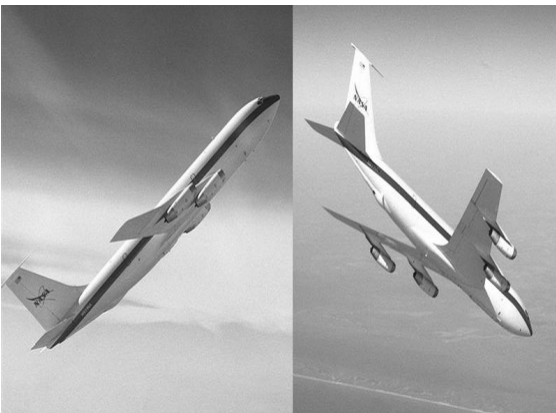
**Example 3**

"The Vomit Comet" is the nickname of a jet used to simulate zero gravity (0-g) for astronauts. To simulate 0-g, the jet flies in a parabolic arc, starting at an altitude of about 7300 m. After climbing for about 30 s, the jet reaches its maximum altitude at about 9800 m, where the weightlessness effect occurs. The jet descends back to an altitude of 7300 m after about 60 s and then repeats the process.

- Sketch the parabola.
- Write a quadratic relation to model the path of the jet. The parabola also passes through the point (31, 9799.999).

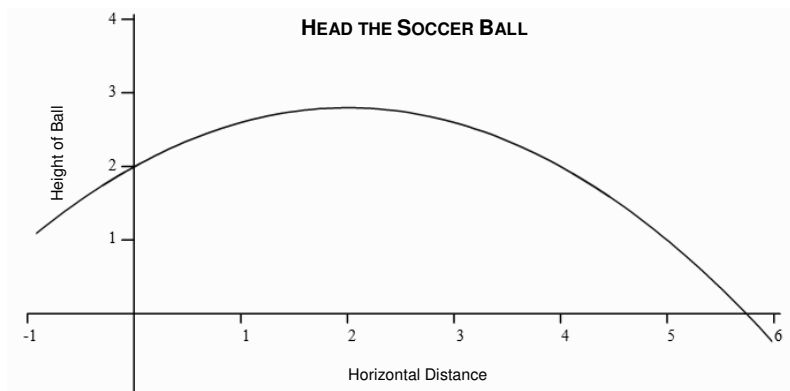


- The effects of simulated 0-g start being felt at about 20 s. What is the jet's altitude at this time?



# EXTRA: Graphing Quadratic Equations

1. Teresa heads the ball in soccer. The path of the ball is described by the equation  $h = -0.2d^2 + 0.8d + 2$ , where  $h$  metres is the height of the ball and  $d$  metres is its distance measured horizontally from where Teresa hits it.



- a. When the equation is graphed, why are only positive values of  $h$  and  $d$  used?

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- b. What is the height of the ball when Teresa hits it? \_\_\_\_\_

- c. What is the maximum height of the ball? \_\_\_\_\_

- d. How far has the ball travelled horizontally when it reaches its maximum height? \_\_\_\_\_

- e. How far has the ball travelled from its original position when it hits the ground? \_\_\_\_\_

2. A company manufactures and sells computer games. The daily profit,  $P$  dollars, is given by the equation  $P = -10x^2 + 750x - 9000$ , where  $x$  dollars is the price of each game.

- a. Complete the table.

| $x$ | $-10x^2 + 750x - 9000$ | $P$ | 1 <sup>st</sup> Differences | 2 <sup>nd</sup> Differences |
|-----|------------------------|-----|-----------------------------|-----------------------------|
| 0   |                        |     |                             |                             |
| 15  |                        |     |                             |                             |
| 30  |                        |     |                             |                             |
| 45  |                        |     |                             |                             |
| 60  |                        |     |                             |                             |
| 75  |                        |     |                             |                             |



Is the relationship quadratic? Provide a reason for your answer.

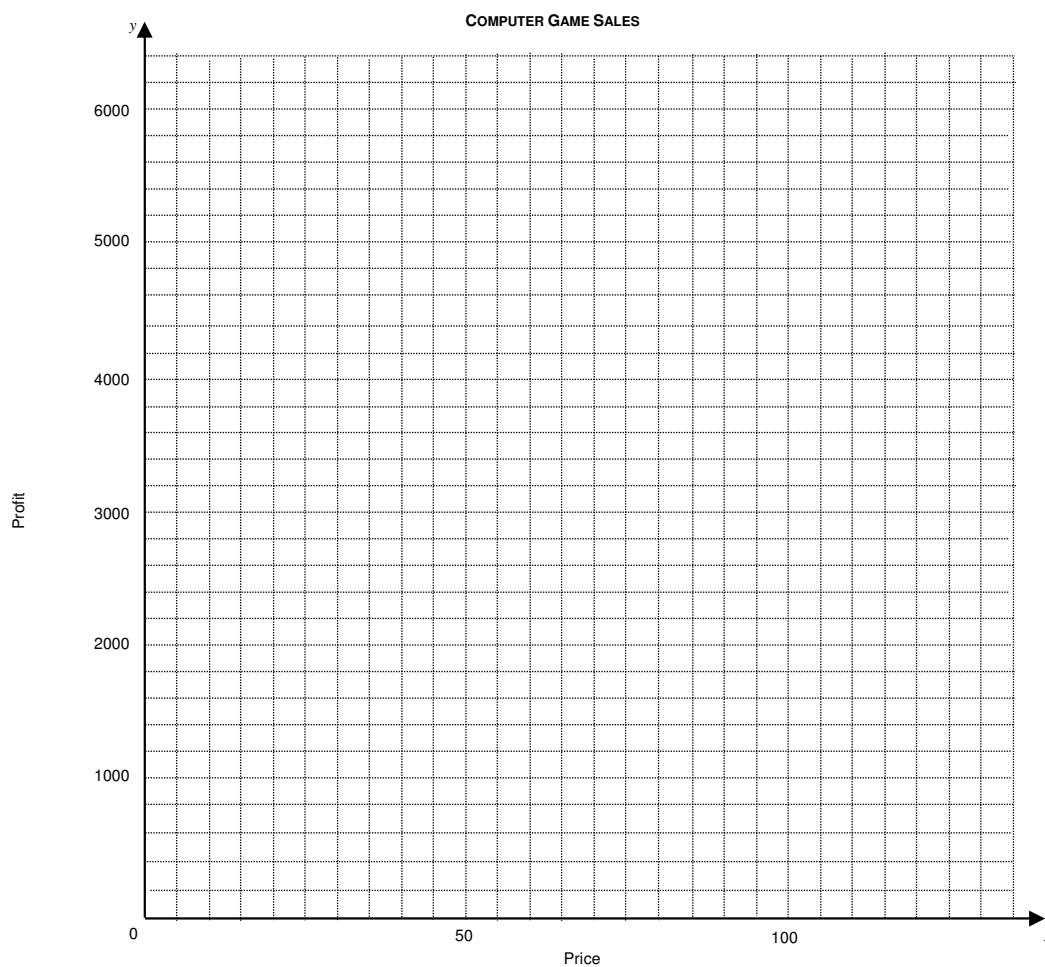
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- b. Graph the relationship on the grid provided.  
[To locate the vertex, find the point where  $x = 37.5$ .]



- c. Are negative values of  $x$  meaningful in this situation? Explain.

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- d. For what range of prices does the company make a profit? Explain.

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- e. What price gives the maximum profit? \_\_\_\_\_

- f. What is this profit? \_\_\_\_\_

3. A quarterback throws a football. The height,  $h$  metres, of the ball is given by the equation  $h = -5t^2 + 20t + 2$ , where  $t$  is the time in seconds after the ball is thrown.



- a. Is the relationship described quadratic? Provide a reason for your answer.

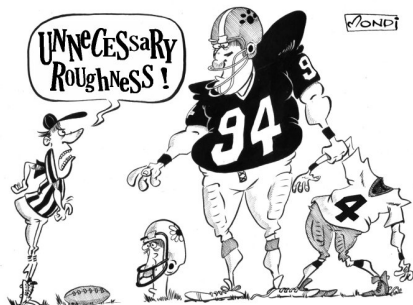
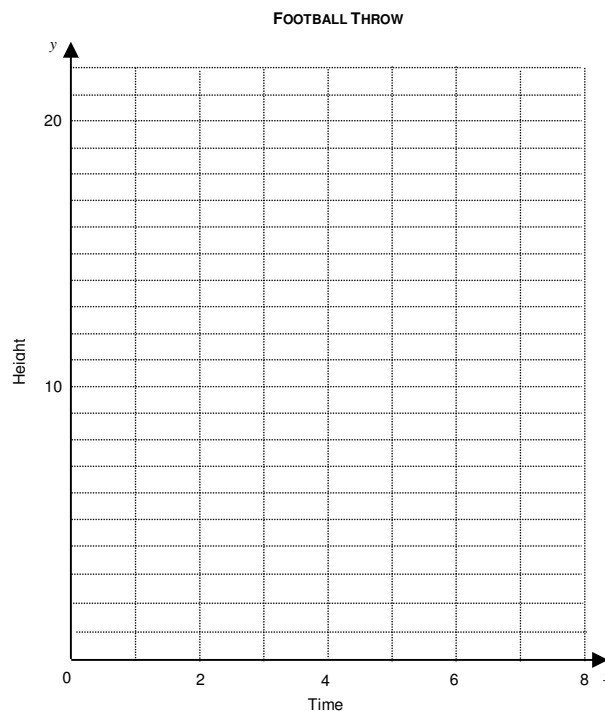
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- b. Complete the table.

| $t$ | $-5t^2 + 20t + 2$ | $h$ |
|-----|-------------------|-----|
| 0   |                   |     |
| 1   |                   |     |
| 2   |                   |     |
| 3   |                   |     |
| 4   |                   |     |
| 5   |                   |     |



- c. Graph the relationship on the grid provided.

- d. Why are only positive values of  $h$  and  $t$  used? \_\_\_\_\_

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- e. What is the height of the ball 1 second after it is thrown? \_\_\_\_\_

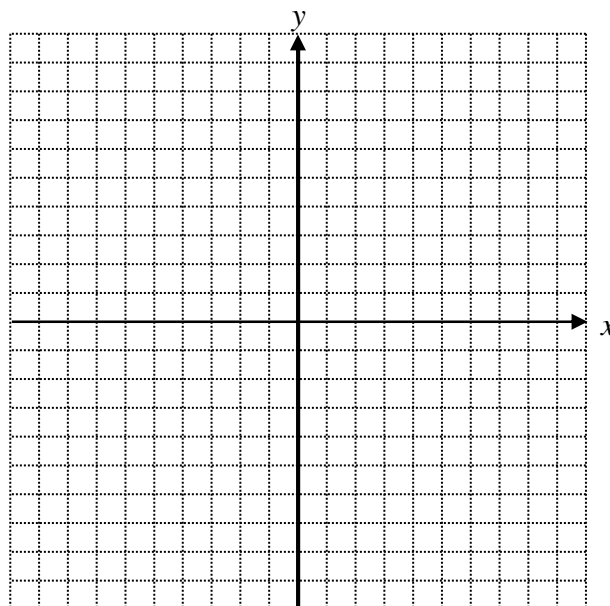
- f. What is the maximum height of the ball? \_\_\_\_\_

- g. How long does it take for the ball to reach the maximum height? \_\_\_\_\_

- h. For how long is the ball more than 10 metres above the ground? \_\_\_\_\_

4. a. For the relation  $y = x^3$ , complete the table and graph the relation.

| $x$ | $x^3$    | $y$ | 1 <sup>st</sup> | 2 <sup>nd</sup> |
|-----|----------|-----|-----------------|-----------------|
| -3  | $(-3)^3$ |     |                 |                 |
| -2  | $(-2)^3$ |     |                 |                 |
| -1  | $(-1)^3$ |     |                 |                 |
| 0   | $(0)^3$  |     |                 |                 |
| 1   | $(1)^3$  |     |                 |                 |
| 2   | $(2)^3$  |     |                 |                 |
| 3   | $(3)^3$  |     |                 |                 |



Is this relation quadratic? Explain your answer.

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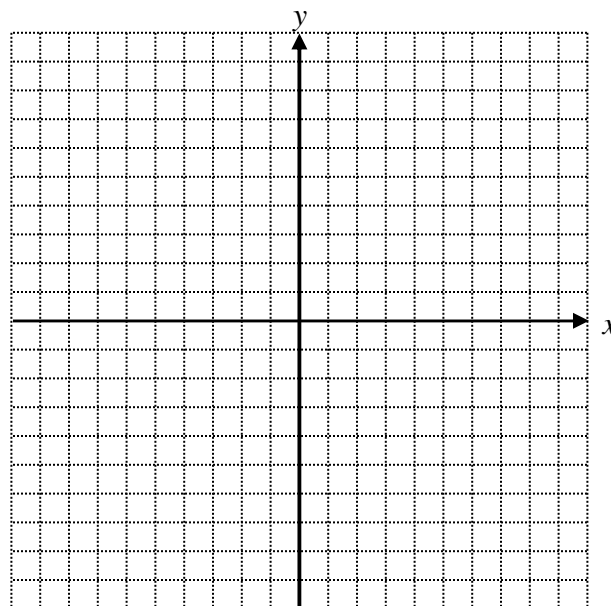
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- b. For the relation  $y = x^4$ , complete the table and graph the relation.

| $x$ | $x^4$    | $y$ | 1 <sup>st</sup> | 2 <sup>nd</sup> |
|-----|----------|-----|-----------------|-----------------|
| -3  | $(-3)^4$ |     |                 |                 |
| -2  | $(-2)^4$ |     |                 |                 |
| -1  | $(-1)^4$ |     |                 |                 |
| 0   | $(0)^4$  |     |                 |                 |
| 1   | $(1)^4$  |     |                 |                 |
| 2   | $(2)^4$  |     |                 |                 |
| 3   | $(3)^4$  |     |                 |                 |



Is this relation quadratic? Explain your answer.

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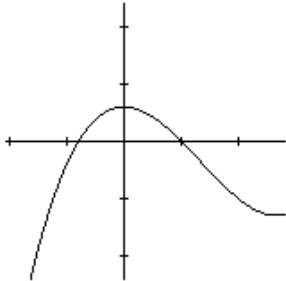
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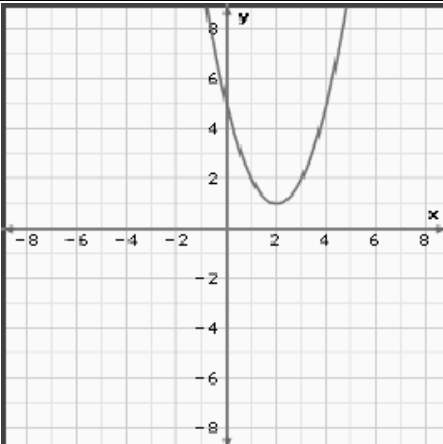
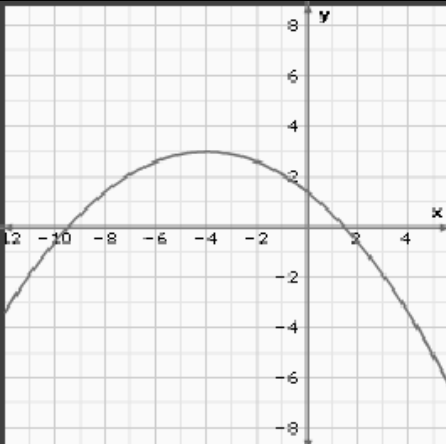
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# EXTRA: Quadratics in Vertex Form

1. State whether each mathematical models is linear, quadratic or neither (circle the appropriate answer). Give a reason for each answer.

| <p>a.</p> $y = 9x^3 + 6x - 7$ <p style="text-align: center;"><i>linear or quadratic or neither?</i></p> <p>Reason: _____</p> | <p>b.</p> <table border="1" style="margin: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> <th>1<sup>st</sup></th> <th>2<sup>nd</sup></th> </tr> </thead> <tbody> <tr><td>-4</td><td>12</td><td></td><td></td></tr> <tr><td>-2</td><td>0</td><td></td><td></td></tr> <tr><td>0</td><td>-4</td><td></td><td></td></tr> <tr><td>2</td><td>0</td><td></td><td></td></tr> <tr><td>4</td><td>12</td><td></td><td></td></tr> </tbody> </table> <p style="text-align: center;"><i>linear or quadratic or neither?</i></p> <p>Reason: _____</p> | $x$             | $y$             | 1 <sup>st</sup> | 2 <sup>nd</sup> | -4 | 12 |  |  | -2 | 0 |  |  | 0 | -4 |  |  | 2 | 0 |  |  | 4 | 12 |  |  | <p>c.</p>  <p style="text-align: center;"><i>linear or quadratic or neither?</i></p> <p>Reason: _____</p> |
|--|---|-----------------|-----------------|-----------------|-----------------|----|----|--|--|----|---|--|--|---|----|--|--|---|---|--|--|---|----|--|--|--|
| $x$  | $y$   | 1 <sup>st</sup> | 2 <sup>nd</sup> |                 |                 |    |    |  |  |    |   |  |  |   |    |  |  |   |   |  |  |   |    |  |  |  |
| -4   | 12  |                 |                 |                 |                 |    |    |  |  |    |   |  |  |   |    |  |  |   |   |  |  |   |    |  |  |  |
| -2   | 0   |                 |                 |                 |                 |    |    |  |  |    |   |  |  |   |    |  |  |   |   |  |  |   |    |  |  |  |
| 0  | -4  |                 |                 |                 |                 |    |    |  |  |    |   |  |  |   |    |  |  |   |   |  |  |   |    |  |  |  |
| 2  | 0   |                 |                 |                 |                 |    |    |  |  |    |   |  |  |   |    |  |  |   |   |  |  |   |    |  |  |  |
| 4  | 12  |                 |                 |                 |                 |    |    |  |  |    |   |  |  |   |    |  |  |   |   |  |  |   |    |  |  |  |

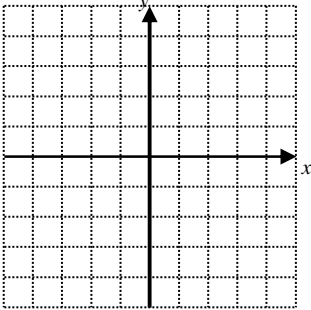
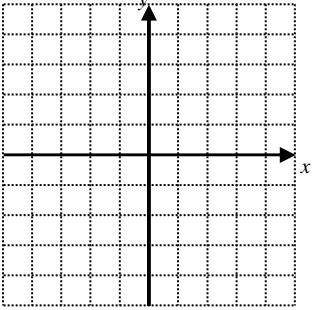
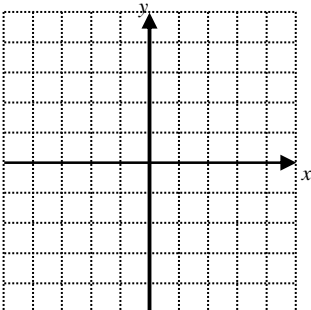
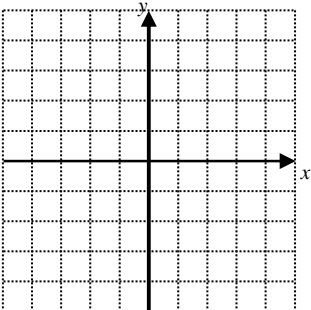
2. Locate the key features for each graph.

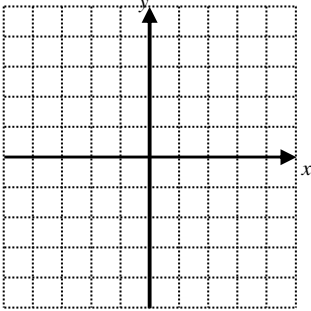
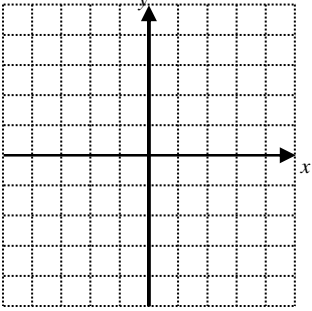
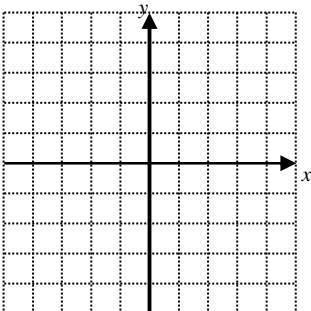
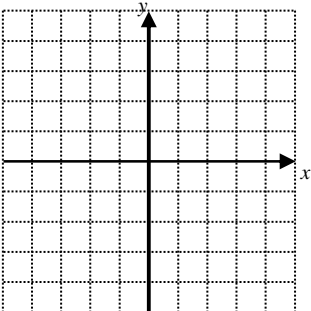
|                  |  |  |
|------------------|--|--|
|                  |  |  |
| zeros            |  |  |
| $y$ -intercept   |  |  |
| vertex           |  |  |
| axis of symmetry |  |  |
| optimal value    |  |  |

3. State the equation for each set of transformations described.

- a. A parabola is reflected and translated 5 units up. \_\_\_\_\_
- b. A parabola is stretched by 4 and translated 8 units left. \_\_\_\_\_
- c. A parabola is reflected and compressed by  $\frac{1}{5}$ . \_\_\_\_\_
- d. A parabola is stretched by 5, translated down 1 unit and right 8 units. \_\_\_\_\_

4. State the transformations and sketch each of the following quadratic relations.

| a. $y = -4x^2$  |   | b. $y = 0.5x^2$  |   | c. $y = (x - 3)^2$  |  | d. $y = x^2 + 4$ |  |
|-----------------|---|--|---|---|--|------------------|--|
| TRANSFORMATIONS |   |  |   |   |  |                  |  |
| GRAPH           |  |  |  |  |  |                  |  |

| e. $y = 2x^2 - 4$ |   | f. $y = -\frac{1}{3}x^2 + 3$   |   | g. $y = \frac{1}{4}(x - 2)^2 + 1$   |  | h. $y = 2(x + 1)^2 - 2$ |  |
|-------------------|---|--|---|---|--|-------------------------|--|
| TRANSFORMATIONS   |   |  |   |   |  |                         |  |
| GRAPH             |  |  |  |  |  |                         |  |

## EXTRA: Designing Gaming Software

Gaming software designers use mathematics and physics to make figures and vehicles move realistically. Often, the motion can be modelled by a quadratic relation.

- For part of a new extreme sports video game, you have to model the path of a snowboarder jumping off a ledge. The mathematical model developed from a video clip is  $h = -0.05d^2 + 11.25$ , where  $h$  is the snowboarder's height above the base of the cliff and  $d$  is the snowboarder's horizontal distance from the base of the cliff, both in metres.

- Create a table of values for the relation. Choose consecutive  $d$ -values.
- Graph the relation.
- At what horizontal distance from the cliff will the snowboarder land?



- You are creating a computer model of a skateboarder jumping off a ramp for the next part of your game. The path of the skateboarder is modelled by the relation  $h = -0.85t^2 + 2$ , where  $h$  is the skateboarder's height above the ground, in metres, and  $t$  is the time in seconds.

- How far from the ground will the skateboarder be after 0.5 s?
- For how long will the skateboarder be in the air?



- In another part of the video game, you are modelling a mountain biker performing a stunt. The mountain biker is speeding along the edge of a mesa toward a cliff, marked  $P$ . The mountain biker must judge when to jump off the mesa to land safely on the ramp. The mountain biker's path through the air can be modelled using the relation  $y = -\frac{5}{v^2}(x-h)^2$ , where  $v$  is the mountain biker's speed, in metres per second,  $y$  is the mountain biker's height above the top of the mesa, in metres, and  $x$  is the mountain biker's horizontal distance from the cliff,  $P$ , in metres.

- The value of  $h$  is the distance from the cliff at which the mountain biker must jump to land safely on the ramp, for a given speed. Suppose the mountain biker's speed is 5 m/s. Find the value of  $h$ .
- If the mountain biker's speed increases, will the value of  $h$  from part (a) increase, decrease, or remain the same? Explain your reasoning.
- Check your answer to part (b) by calculating a value of  $h$  for a speed of 10 m/s.



4. The next part of your video game models a biker jumping off a ramp at a motocross event. The biker's path can be modelled by the relation  $h = -0.03(d - 9.5)^2 + 5$ , where  $h$  is the biker's height above the ground and  $d$  is the biker's horizontal distance from the end of the ramp, both in metres.



- What is the biker's initial height above the ground (when  $d = 0$ )?
- What is the vertex of the parabola? What information do the coordinates of the vertex give about the biker's position?
- Sketch the graph of the biker's path.
- Use a graphing calculator or graphing software to determine where the biker will land.

5. When studying ballistics, Galileo Galilei found that by changing the angle between a cannon and the ground, a cannonball could be fired different distances using the same amount of explosive.

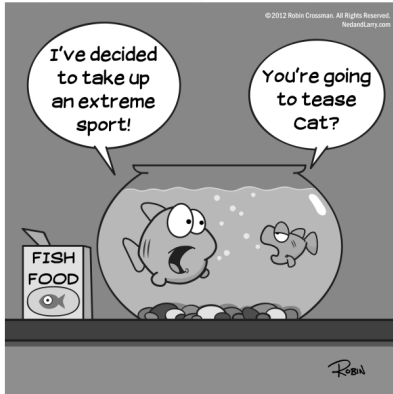
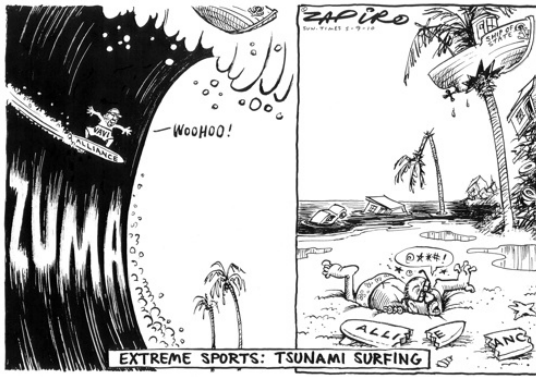
The same method is used by video game designers when determining the angle of a ramp in a ski jump game. For each relation in the table,  $h$  is the ski jumper's height above the ground and  $d$  is the ski jumper's horizontal distance from the ramp, both in metres.

| Ramp Angle | Quadratic Relation          |
|------------|-----------------------------|
| 25°        | $h = -0.018d^2 + 0.47d + 2$ |
| 35°        | $h = -0.022d^2 + 0.70d + 2$ |
| 45°        | $h = -0.029d^2 + 1.00d + 2$ |
| 55°        | $h = -0.044d^2 + 1.43d + 2$ |
| 65°        | $h = -0.081d^2 + 2.15d + 2$ |
| 75°        | $h = -0.216d^2 + 3.73d + 2$ |



- Use a graphing calculator or graphing software to graph each relation in the table on the same set of axes.
  - Which angle gives the greatest horizontal distance?
  - Which angle gives the greatest height?
  - What is the meaning of the constant 2 in each relation?
6. There are many more extreme sports that involve flying through the air. Think of a different sport and develop a relation that can model the sport's motion to complete your video game.





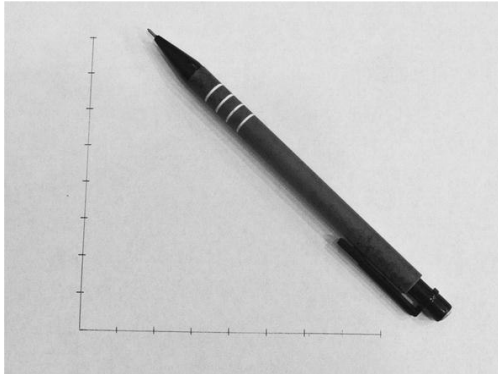
Newest X Games Event-The 5000 Meters With Scissors



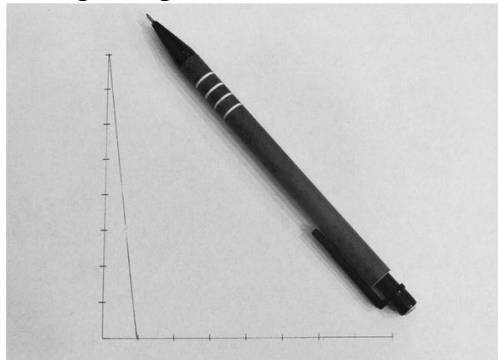
# EXTRA: Parabola Art Instructions

Create your own parabola using straight lines and a right angle. Follow the instructions below.

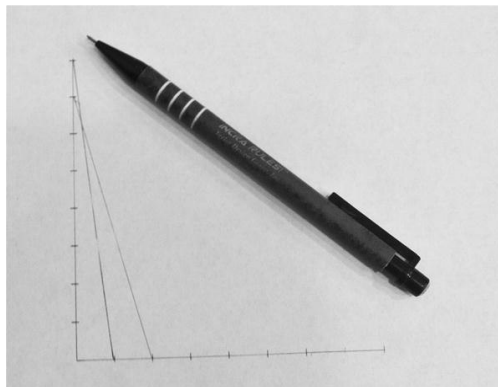
1. Draw a right angle and mark two lines of equal length at equal intervals. It should look like you are making a coordinate plane to graph an equation.



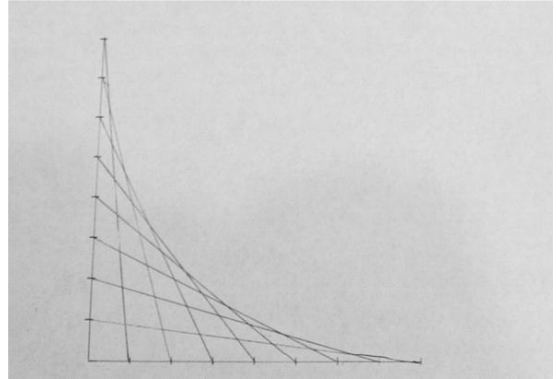
2. Draw a line: start at the farthest mark from the right angle on one line, to the closest mark to the right angle on the other line.



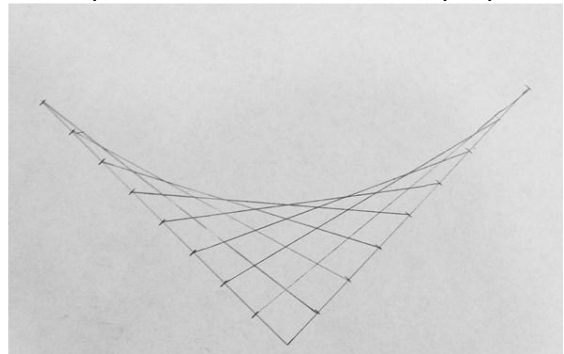
3. Now connect the 2nd farthest mark to the 2nd closest mark.



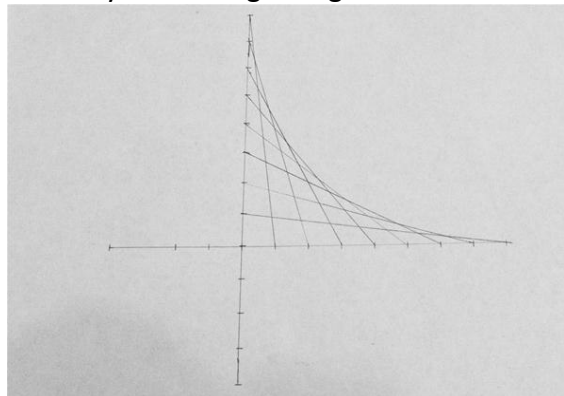
4. Continue connecting lines between the points as you step down one line and step up the other.



5. Here the image has been rotated 45 degrees so the parabola is oriented in the proper way.



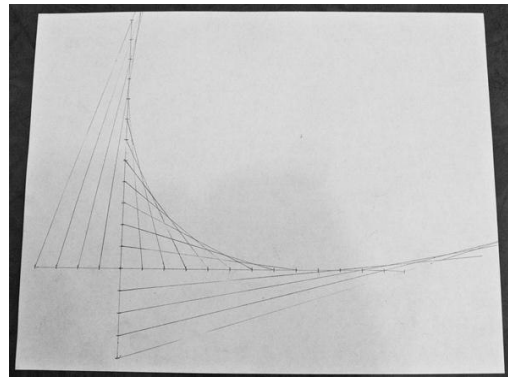
6. For the parabola to continue, extend the lines both beyond the right angle.



## MBF 3C1

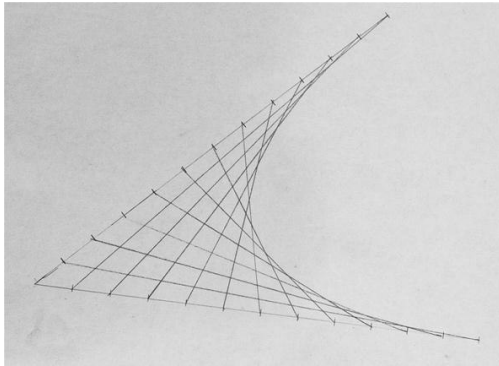
7. If you want the parabola to continue getting steeper, you can extend the marks past the right angle and connect them up as shown below. Remember to count the right angle as a mark.

Name: \_\_\_\_\_

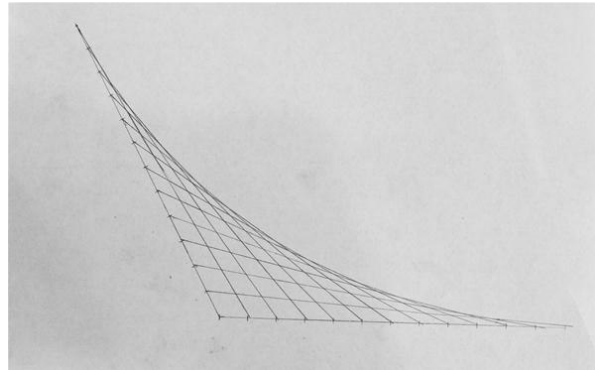


## Making Narrower or Wider Parabolas by Using Non-Right Angles

If you want a narrower parabola, you can follow the process above, but use an angle less than 90 degrees. The angle below is 45 degrees.

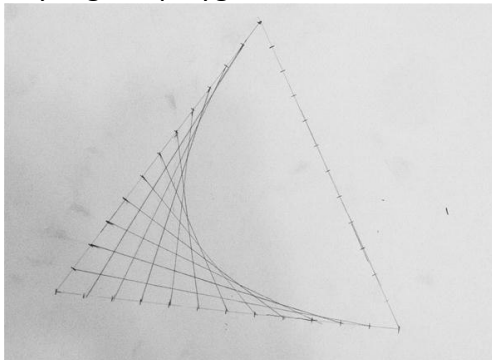


If you want a wider parabola, you can follow the process above, but use an angle greater than 90 degrees. The angle below is 120 degrees.

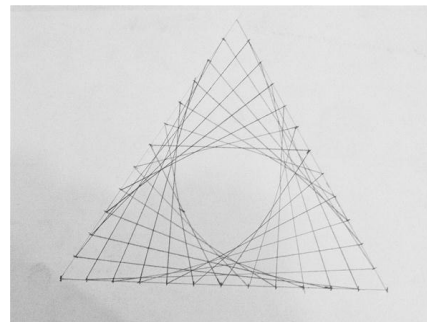


## Creating Parabolic Sections Inside Polygons

Since the process of creating parabolic sections can be done with any angle, you can use angles that make polygons and use the process on all of the sides. Using the process on regular polygons where all of the angles and sides are the same results in pleasing figures. A regular triangle is pictured below, but the process would work for any regular polygon.

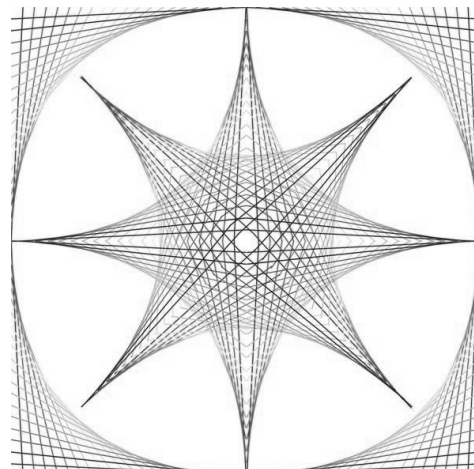
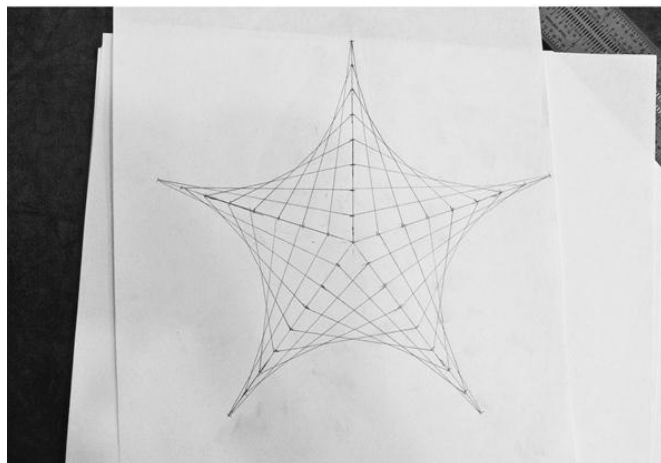


After all three sides have been connected.

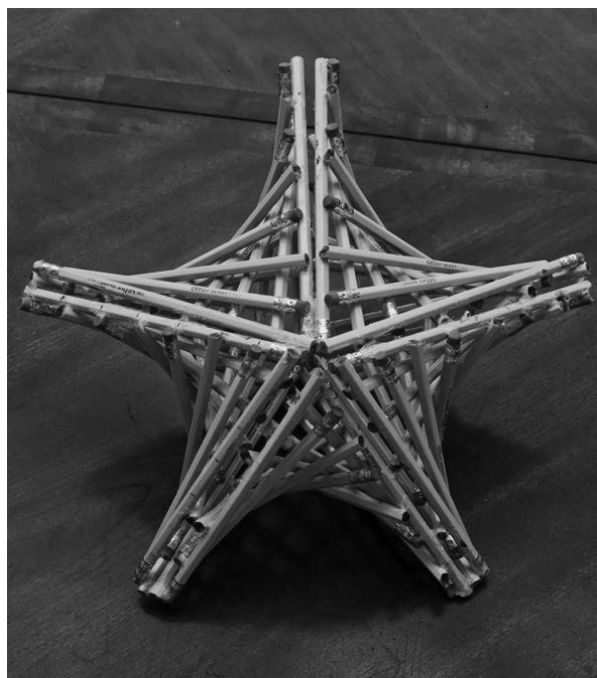


## Creating Star Figures from Parabolic Sections

Use the lines that connect up the center of a regular polygon to each of the sides, you get star figures made out of your parabolas. The image below used a pentagon, which has 5 central angles of 72 degrees.



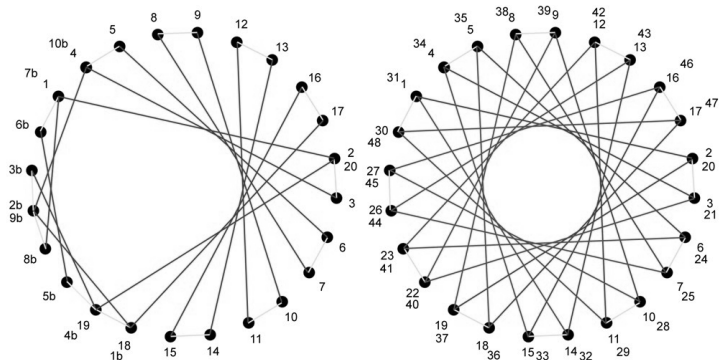
Below is a three dimensional version made out of pencils.



## EXTRA: Parabola Art Assignment

1. Follow the instructions provided to make a parabola using straight lines on the page provided.
2. Once your parabola is complete, determine it's equation:
  - a. Plot and number a coordinate grid.
  - b. Locate the vertex of the parabola. \_\_\_\_\_
  - c. Find the value of 'a' using the step pattern. \_\_\_\_\_
  - d. Sub the values for 'a', 'h' and 'k' into vertex form [ $y = a(x - h)^2 + k$ ].
3. Try to make your own design. Create either a second parabola that has been stretched or compressed or use a regular shape (triangle, square, pentagon, etc.). Use colour to make your design more interesting.

**MBF 3C1**



Name: \_\_\_\_\_

