Unit6JOURNAL

August 29, 2016 2:39 PM

Unit6JOURNAL see below

UNIT 6 - Trigonometry JOURNAL

Big idea/Learning Goals

For this unit you must make sure your calculator is in DEGREE mode so that the answers will always come up correctly. In this unit you will learn how to use SINe, COSine, and TANgent buttons on the calculator to solve for sides or angles of right triangles. Not all triangles you see in real life may be right angled. But SOH CAH TOA and Pythag are only used on right triangles. Then you till learn about: Sine and Cosine Laws. (There is no Tangent Law, since having the two that we develop enables us to solve ANY type of triangle). This unit is an introduction to what you will learn in grade 11. There are a lot of real life applications for trigonometry – these you will see in the word problems of this unit.

✓DEG DV ✓DEG X GRAD XR X RAD

Activity before Break Dec 22/23

Tentative TEST date: TEST on Unit6 Fri Jan 13

Date	Topics	Finished the journal? Made corrections?	How many questions did you finish from HW? Total /124	Questions to ask the teacher:
2days	**Congruent triangles** DAY 1 HW Handout – find online on mrsk.ca website under this unit and this topic DAY 2 HW Handout – find online on mrsk.ca website under this unit and this topic		/12 /11	
2days	**Similar Triangles** DAY 3 HW text pg333 #4,5,6,7,8,14,15 DAY 4 HW text pg347 #1,5,7,9,11,12,16,19		/15 /15	
2days	**SOH CAH TOA** DAY 5 HW text pg362 #1ef,2ef,3gh,4gh,5cd,6cd,9,12 DAY 6 HW text pg374 #12,15,16,17,25,29		/17 /12	
	Solve word problems DAY 7 HW text pg382 #11,13,14,15,18,20,26		/14	
2days	**STRAND assignment**			
	Sine Law DAY 1 HW text pg402 #2,4,6,9,10,13,15		/9	
	Cosine Law DAY 2 HW text pg409 #3,5,8 Pg418 #2,6,9		/12	
	Word Problems DAY 3 HW text pg427 #3,4,7,10,12		/7	

Reflect – previous TEST mark __________, Overall mark now__________.

Calculate your potential final mark, show your calculations here:

potential final mark = (overall mark now)(0.60) + (future unit marks)(0.10) + (final exam marks)(0.30)

=(   )(0.60) + (   )(0.10) + (   )(0.30)

=

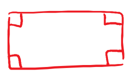
Looking back, what can you improve upon? ____

DAY 1 & 2 – Congruent Triangles

→ Same Shape + Size
maybe rotated, shifted, reflected.

1. **Proving something is FALSE**
Conjectures can be proved **false** with a
hypothesis ← single ____counter example____
Example Conjecture: In a quadrilateral, if all angles are congruent, then all sides are congruent. same

False, counter example
rectangle

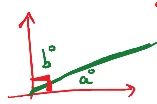


- all angles 90°
- but sides are not the same.

2. **Proving something is TRUE**
Conjectures can be proved **true** by using a logical argument, based on known facts. When a conjecture has been proved true, it is called a ____theorem____.
ex. "All angles in △ add to 180°"

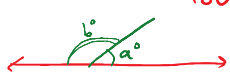
A proof is a logical argument. In math, something is considered true if it has been proved. It is not enough for something to *seem* true. In writing a proof, you can only use facts that have previously been proved, or facts that are assumed true without proof. In this class, we will assume the following facts are true without proving them:

Complementary Angles
two angles add to 90°



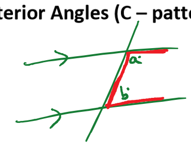
$a° + b° = 90°$

Supplementary Angles
two angles add to 180°



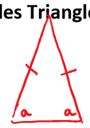
$a° + b° = 180°$

Sum of Interior Angles (C – pattern)



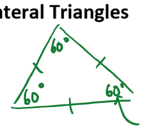
- the angles on the inside of "C" are the supplementary.
$a + b = 180°$

Isoceles Triangles



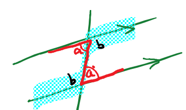
- two sides and two angles are the same.

Equilateral Triangles



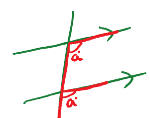
- three sides and three angles are the same
since all angles add to 180°

Alternate Angles (Z pattern)



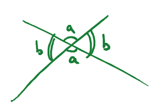
- angles on the inside of the "Z" are the same

Corresponding Angles (F pattern)



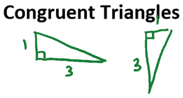
- angles on the inside of the "F" are the same

Opposite Angles (X pattern)



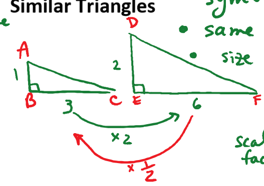
- opposite angles of two intersecting lines are the same.

Congruent Triangles



- symbol for congruent ≅
- same shape (angles)
- same size (lengths)

Similar Triangles



- symbol for similar ~
- same shape (angles)
- size is scaled up/down by the same scale factor.

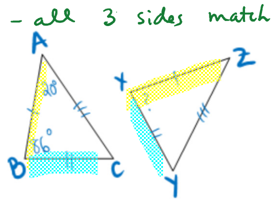
scale factor = Ratio of sides

$\frac{EF}{BC} = \frac{DE}{AB}$

Prove Congruent then find the value of "?"

3. SSS

- all 3 sides match



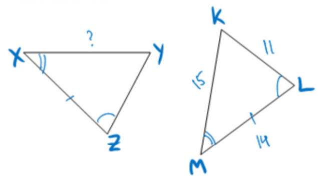
Statements	Reason	
AB = XZ	given by ⊢	S
BC = XY	given by ⫲	S
AC = YZ	given by ⫵	S

$\therefore \triangle ABC \cong \triangle ZXY$ by S.S.S.

corresponding order

Now the value of "?" = 86°

4. ASA or AAS

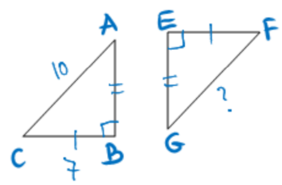


Statement	Reason	
∠M = ∠X		Angle
ML = XZ		Side
L = Z		Angle

$\therefore \triangle XYZ \cong \triangle MKL$ by A.S.A.

Now the value of "?" = 15

5. SAS



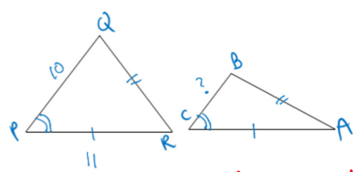
Statement	Reason	
CB = EF		Side
B = E	90°	Angle
AB = GE		Side

$\therefore \triangle ABC \cong \triangle GEF$ by SAS

Now the value of "?" = 10

Talk about why SSA or AAA is not enough to prove congruency:

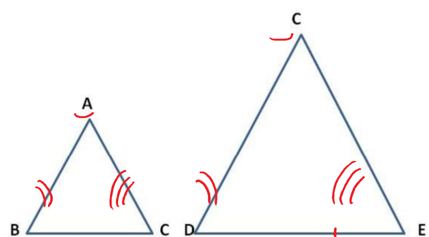
6. Not SSA



if SSA values match you can not say Δ's are congruent !

"?" unknown

7. Not AAA



just having angles match does not say anything about sides.

∴ can't use AAA for congruency.

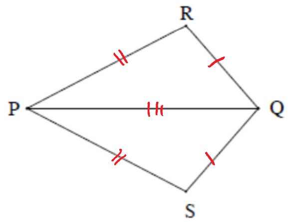
(can use for similarity)

Δ - triangle
∠ - angles

Two Formats for Proof

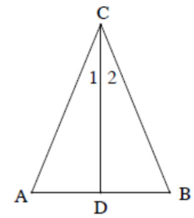
paragraph form – used by mathematicians *two-column form – students prefer this one*

8.



Given: RQ=SQ and RP=SP.
Prove: ∠R = ∠S.
The given information is: __RQ = SQ__ and __RP = SP__. PQ=PQ because __shared side__. So ΔPRQ ≅ ΔPSQ because of the __SSS__ property for congruent triangles. Therefore, __∠R = ∠S__ because corresponding parts of congruent triangles are congruent.

9.



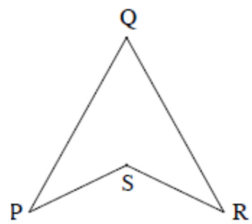
Given: AC=BC and ∠1 = ∠2.
Prove: ∠A = ∠B.

Proof.

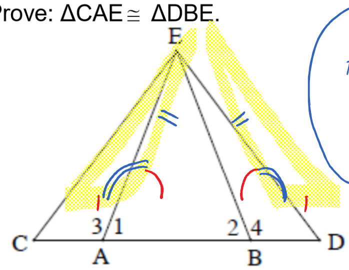
Statements	Reasons
1. AC = BC	1.
2.	2. Given
3. CD = CD	3.
4.	4. by SAS property
5. ∠A = ∠B	5.

NOTES:
- each statement must have a valid reason, don't jump steps
- "go around" the shape to see if you have $\underbrace{\text{SSS / SAS / AAS / ASA}}_{\text{only for Δ's statement}}$
- use congruency to answer question

10. Given: PQ = RQ and PS = RS.
Prove: ∠P = ∠R.



11. Given: AC = BD and ∠1 = ∠2.
Prove: ΔCAE ≅ ΔDBE.



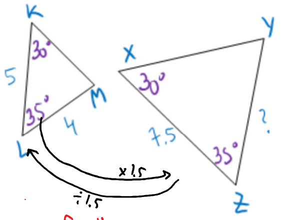
	Statement	Reason
A	∠3 = ∠4	Since they are supplementary to ∠1 and ∠2 and we're given ∠1=∠2
S	CA = BD	given
S	AE = BE	isosceles Δ with ∠1=∠2

∴ ΔCAE ≅ ΔDBE by $\underbrace{\text{S.A.S.}}_{\text{go around the shape.}}$

DAY 3 & 4 – Similar Triangles

Prove Similar then find the value of "?"

1. AAA or just AA ← enough to show just 2 angles, since all angles in Δ add to 180°.

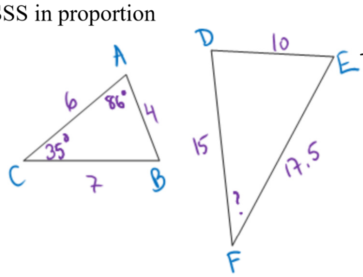


Scale factor $\frac{\text{Big } \Delta}{\text{Small } \Delta}$ or $\frac{\text{Small}}{\text{Big}}$

statements	reason	
∠K = ∠X	given = 30°	A
∠L = ∠Z	given = 35°	A
∴ ΔKLM ~ ΔXZY	by AA	

∴ "?" equals to 6

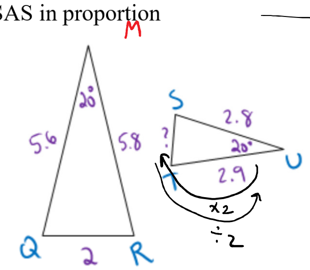
2. SSS in proportion



statement	reason
rough: $\frac{15}{6} \stackrel{?}{=} \frac{10}{4} \stackrel{?}{=} \frac{17.5}{7}$	
2.5 2.5 2.5	
$\frac{DF}{AC} = \frac{DE}{AB} = \frac{FE}{BC}$	same scale factor
∴ ΔABC ~ ΔDEF	by SSS~ (in proportion)

∴ ? is equal to 35°

3. SAS in proportion



rough work

$\frac{5.6}{2.8} \stackrel{?}{=} \frac{5.8}{2.9}$

2   2

statement	reason
$\frac{QM}{SU} = \frac{RM}{TU}$	same scale factor
∠M = ∠U	given = 20°
$\boxed{\therefore \Delta MQR \sim \Delta UST}$	by SAS~

∴ ? equals to 1

$\frac{QM}{SU} = \frac{RM}{TU} = \frac{QR}{ST}$

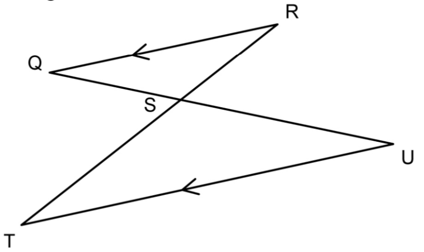
$\frac{5.6}{2.8} = \frac{5.8}{2.9} = \frac{2}{x}$

$\frac{5.8x}{5.8} = \frac{5.8}{5.8}$

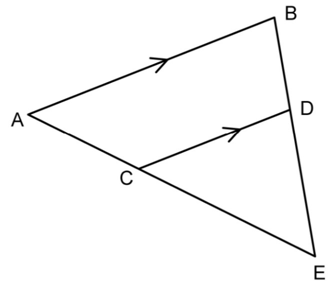
$x = 1$

Prove triangles are similar, then record the ratio of sides statement.

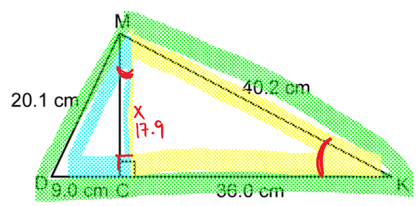
4.



5.



6.



rough work.

$\frac{40.2}{20.1} \overset{?}{=} \frac{36}{17.9} \overset{?}{=} \frac{17.9}{9}$

2 ~2 ~2

answers a bit off because of rounding

statement	reason
$\frac{MK}{DM} = \frac{CK}{MC} = \frac{MC}{DC}$	same scale factor
$\triangle MCD \sim \triangle KCM$ ($\cancel{MCK}$)	by SSS~

∴ proportion statement is

$\boxed{\frac{MK}{DM} = \_\_\_ = \_\_\_}$

$a^2 + b^2 = c^2$

$x^2 + 36^2 = 40.2^2$

$x^2 + 1296 = 1616.04$

-1296.00

$\sqrt{x^2} = \sqrt{320.04}$

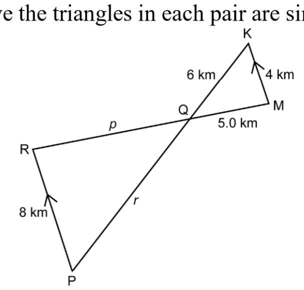
$x = 17.9$

Note is the Big △ a right △?

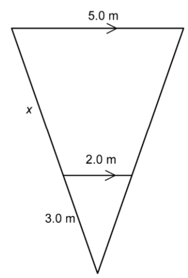
$a^2 + b^2$	$\neq c^2$
$20.1^2 + 40.2^2$	45^2
$404.01 + 1616.04$	2025
2020.05	∴ not a right △

Prove the triangles in each pair are similar. Then find the unknown side lengths

7.



8.



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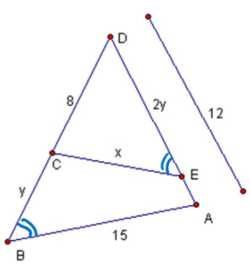


Proof.

$\angle B = \angle CED$ given

$\angle D = \angle D$ shared

$\therefore \triangle ABD \sim \triangle CED$ by AA

$\frac{AB}{CE} = \frac{BD}{ED} = \frac{AD}{CD}$

$\frac{15}{x} = \frac{8+y}{2y} = \frac{12}{8}$ scale factor

$\frac{15}{x} = \frac{12}{8}$

$120 = 12x$

$10 = x$

$\frac{(8+y)}{2y} = \frac{12}{8}$

$8(8+y) = 24y$

$64 + 8y = 24y$ $-8y$

$64 = 16y$

$4 = y$

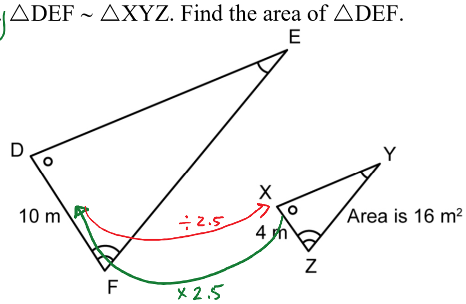
NOTES:

1. prove Δ's are similar using AA, SSS~, SAS~
2. Set up side proportion statement
3. Cross mult. to solve for unknown side.

OK use scale factor $\frac{\text{Big}}{\text{small}}$

10. A right triangle has side lengths 5 cm, 12 cm, and 13 cm.
    - **a)** A similar triangle has a hypotenuse 52 cm long. What is the scale factor?
    - **b)** What are the lengths of the legs of the triangle in part a)?
    - **c)** Find the area of each triangle.
    - **d)** How are these areas related?

11. $\triangle DEF \sim \triangle XYZ$. Find the area of $\triangle DEF$.



$\frac{\text{Big}}{\text{small}} = \text{scale factor} = 2.5$

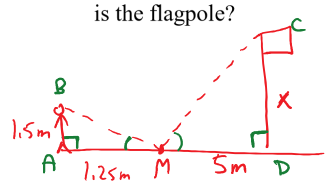
$A = \frac{b^{\times 2.5} h^{\times 2.5}}{2}$

∴ areas must be scaled up twice

$\therefore 16(2.5)^2$
$= 16(6.25)$
$= 100$

∴ $\triangle DEF$'s area is 100m^2

12. Bill placed a mirror on the ground 5 m from the base of a flagpole. He stepped back until he could see the top of the flagpole reflected in the mirror. Bill is 1.5 m tall and saw the reflection when he was 1.25 m from the mirror. How high is the flagpole?



Proof

$\angle AMB = \angle DMC$ "refraction" reflected angle

$\angle A = \angle D$ vertical at 90°

∴ Δ's are similar by AA

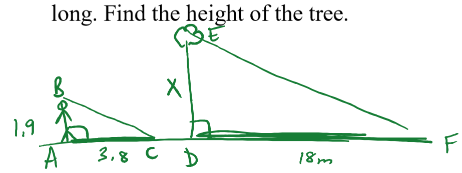
$\frac{\text{Big}}{\text{Small}} = \frac{x}{1.5} = \frac{5}{1.25}$

$1.25x = 7.5$

$x = 6$

∴ flagpole is 6m tall.

13. A person 1.9 m tall casts a shadow 3.8 m long. At the same time a tree casts a shadow 18 m long. Find the height of the tree.



Proof

$\angle A = \angle D$ assumed at 90°

$\angle B = \angle E$ angle of the sun

∴ Δ's are similar by AA

$\frac{\text{Big}}{\text{Small}} \rightarrow \frac{18}{3.8} = \frac{x}{1.9}$

$\frac{18(1.9)}{3.8} = x$

$9 = x$

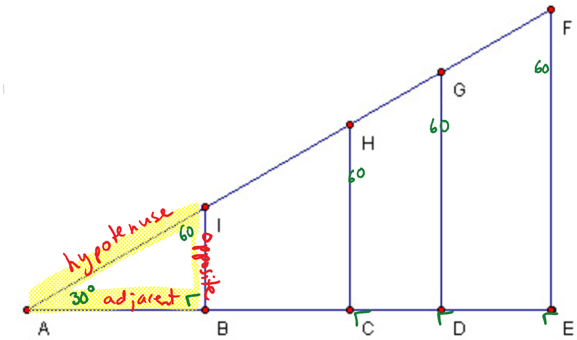
∴ tree is 9m tall.

8

DAY 5 – Introduction to Solving Right Triangles

In early times, similar triangles were used to solve problems about measurement.
One individual, Hipparchus, 140 B.C.E., found that right angle triangles had a special property.
Let's investigate what this special property is.

This diagram shows some similar **right** triangles with a common angle A. Measure the side lengths of each triangle and record your findings in the table below.



round to one decimal place

Triangles	Side Opposite to $\angle A$ (cm)	Side Adjacent to (beside) $\angle A$ (cm)	Hypotenuse (cm)	Calculate these Trigonometric Ratios		
				$\frac{opposite}{hypotenuse}$	$\frac{adjacent}{hypotenuse}$	$\frac{opposite}{adjacent}$
ΔABI	1.9	3.4	3.9	0.5	0.9	0.6
ΔACH	3.5	6	7	0.5	0.9	0.6
ΔADG	4.4	7.7	8.9	0.5	0.9	0.6
ΔAEF	5.5	9.6	11.2	0.5	0.9	0.6

1. Explain why the ratios of the triangles are the same.

   Since the Δ's are simila → ∠A shared → all have 90°

2. Measure the angle A using the protractor: $< A =$ 30°

   D
   DEG DRG

   ~~RAD~~ ~~R~~ G~~RAD~~

3. MAKE SURE your calculator is in DEGREE mode!
   Use the calculator to calculate the following trigonometric functions

$\sin A =$ 0.5 $\cos A =$ 0.9 $\tan A =$ 0.6

sin 30°   cos 30°   tan 30°

$\frac{opposite}{hypotenuse}$ is called Sine or SIN for short

$\frac{adjacent}{hypotenuse}$ is called Cosine or Cos for short

$\frac{opposite}{adjacent}$ is called tangent or tan for short

A way to remember Trig Ratios

SOH CAH TOA

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

X variable

"theta" θ angle variable (Greek letter)

4. Try $\sin 90° =$ ___1___ $\cos 90° =$ ___0___ $\tan 90° =$ ___error___

5. What can you conclude about which angles you can use (highlight) when labeling opposite/adjacent/hypotenuse for SOH CAH TOA on the right triangle? only use acute angles for focus angles to work with.

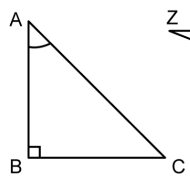
NOTES: How and when to round decimals

- Round only at the end. (keep at least 4 digits in middle of your steps)
- At the end → follow instructions
  → round to whole # for angles
  → round to one decimal place for sides.

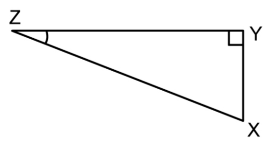
hyp → across from 90° opp → far from focus adj → beside focus angle

6. Practice labelling triangles: Label the hypotenuse, the opposite, and the adjacent sides relative to each marked angle.

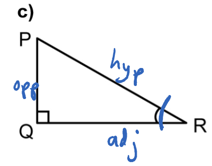
a)



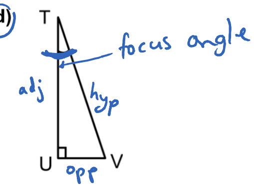
b)



c)



d)



7. Practice using the calculator: Make sure your calculator is in DEGREE (DEG) mode

Given the angle find the ratio

a) sin 45°

b) cos 98°

c) tan 4° = 0.0699

d) $\cos 76° = \frac{3}{x}$

$(\cos 76)x = 3$

$\frac{\cancel{(\cos 76)}x}{\cancel{\cos 76}} = \frac{3}{\cos 76}$

$x = \frac{3}{0.2419}$

$x = 12.4$

Given the angle fnd the angle (use the SHIFT or 2ND buttons)

e) sin A = 0.557

f) cos C = 0.705

g) $\tan^{-1}(\tan B) = \tan^{-1}(2.984)$

$B = \tan^{-1}(2.984) = 71°$

h) $\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{5}{6}\right)$

$x = \sin^{-1}\left(\frac{5}{6}\right)$

$x = \sin^{-1}(0.8333)$

$x = 56°$

I

NOTES:

Solving for sides

1. Identify focus angle, and label sides as hyp/opp/adj
2. Cross off the side that doesn't have a # or variable on it. Decide to use SOH CAH TOA
3. Cross multiply, round at the end to one decimal.

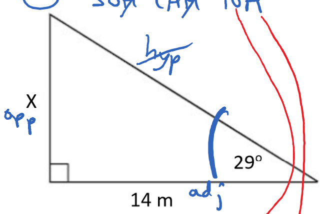
Solving for angles

1. } same as before.
2. }
3. Use inverse buttons $\sin^{-1}$ / $\cos^{-1}$ / $\tan^{-1}$ to get angle isolated, round to whole #.

8. Practice solving triangles: Determine the measure of the missing angles or sides

a.

S O ~~H~~ C A ~~H~~ T O A



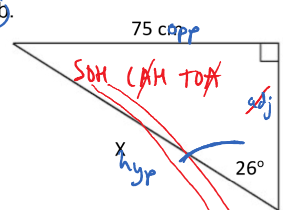
$$\frac{\tan 29^\circ}{1} = \frac{x}{14}$$

$$(\tan 29^\circ)(14) = x$$

$$(0.5543)(14) = x$$

$$7.8\text{ m} = x$$

b.



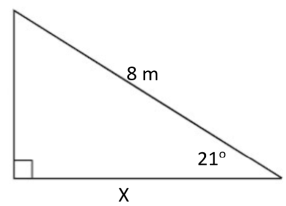
$$\frac{\sin 26^\circ}{1} = \frac{75}{x}$$

$$\frac{(\cancel{\sin 26^\circ})x}{\cancel{\sin 26}} = \frac{75}{\sin 26}$$

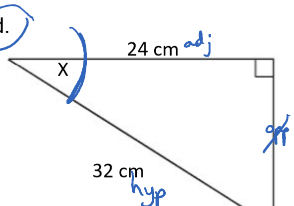
$$x = \frac{75}{0.4384}$$

$$x = 171.1\text{ cm}$$

c.



d.



S ~~O~~ H C A H T ~~O~~ A

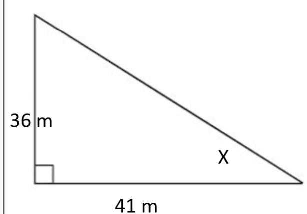
$$\cos X = \frac{24}{32}$$

$$X = \cos^{-1}\left(\frac{24}{32}\right)$$

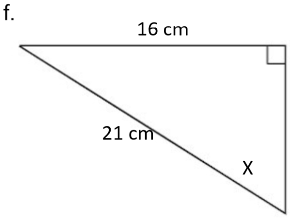
$$X = \cos^{-1}(0.75)$$

$$X \doteq 41^\circ$$

e.



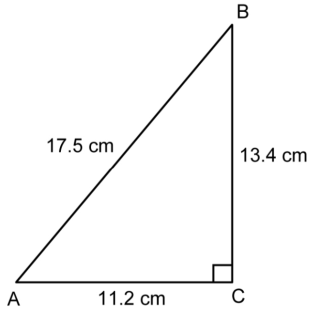
f.



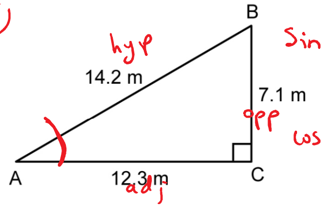
DAY 6 – More Trig Ratios

1. Find the three primary trigonometric ratios for ∠A, to four decimal places.

   a)



   b)



$\sin A = \frac{7.1}{14.2}$
$= 0.5$

$\cos A = \frac{12.3}{14.2}$
$\doteq 0.8662$

$\tan A = \frac{7.1}{12.3}$
$\doteq 0.5772$

2. Find the measures of both angles A and B in part a) above. Discuss several methods of doing so.

3. Evaluate with a calculator. Round your answers to four decimal places.
   a) sin 72°

   b) cos 36°

   c) tan 57.4°

4. Find the measure of each angle, to the nearest degree.
   a) sin θ = 0.5189

   b) cos B = $\frac{9}{10}$

   c) tan θ = $\left(\frac{9}{14}\right)$

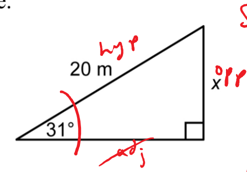
$\tan^{-1}$ $\tan^{-1}$

$\theta = \tan^{-1}\left(\frac{9}{14}\right)$
$\theta = 33°$

5. Find the value of *x*, to the nearest tenth of a metre.

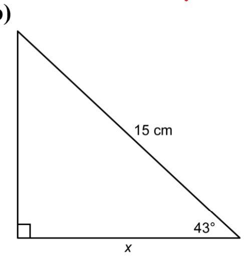
SOH CAH ~~TOA~~

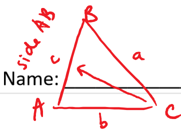
   a)



$\frac{\sin 31°}{1} = \frac{x}{20}$
$(\sin 31°)(20) = x$
$(0.5150)(20) \doteq x$
$10.3\,m \doteq x$

   b)



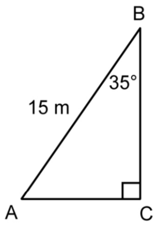


Capitals for angles
small case for sides

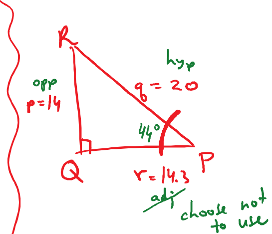
find all sides, all angles

6. Solve each triangle. Round side lengths to the nearest tenth of a metre.

a)



b) In △PQR, ∠Q = 90°, p = 14 m and q = 20 m.



find side r

$a^2+b^2=c^2$

$14^2+r^2=20^2$

$196+r^2=400$ -196

$\sqrt{r^2}=\sqrt{204}$

$r \doteq 14.3$ m

find ∠P

SOH ~~CAH~~ ~~TOA~~

$\sin P = \frac{14}{20}$

$P = \sin^{-1}\left(\frac{14}{20}\right)$

$P \doteq 44°$

find ∠R

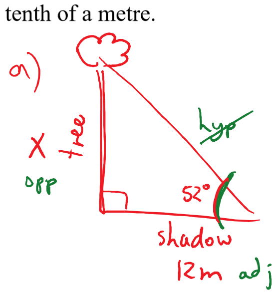
$\angle R = 180° - 44° - 90°$

$\angle R = 46°$

$\therefore$ ∠R = 46°, ∠P = 44°, ∠Q = 90°; r = 14.3 m, p = 14 m, q = 20 m

7. In order to measure the height of a tree, Dan calculated that its shadow is 12 m long and that the line joining the top of the tree to the tip of the shadow forms an angle of 52° with the flat ground.
   a) Draw a diagram to illustrate this problem.
   b) Find the height of the tree, to the nearest tenth of a metre.

a)



b) ~~SOH~~ ~~CAH~~ TOA

$\frac{\tan 52°}{1} = \frac{x}{12}$

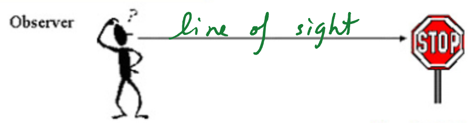
$12(\tan 52°) = x$

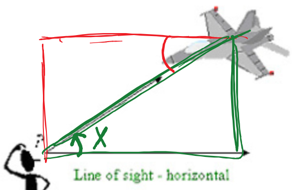
$15.4 = x$

$\therefore$ the height of the tree is 15.4 m

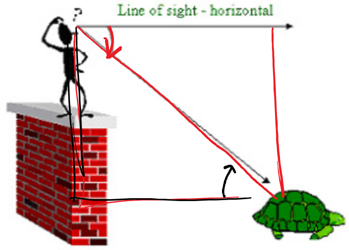
Angles – inside △
sides – outside of △

Angles of elevation and depression





When the line of sight moves from the horizontal in an upwards direction (you lift your eyes) this is referred to as an angle of elevation



When the line of sight moves from the horizontal in an downwards direction (you lower your eyes) this is referred to as an angle of depression

NOTES:

* always label acute angles (elevation/depression) from horizontal line
NOT vertical!

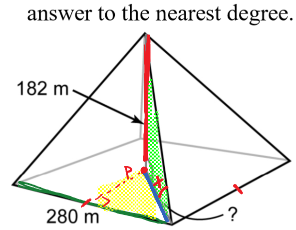
* by Z pattern
angle of elevation = angle of depression.

- use pythagorean if no angles involved
- use SOH CAH TOA if acute angle is involved
- use similar Δ's if can prove they are similar (you must have 2 Δ's!)

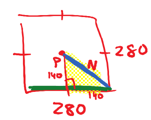
DAY 7 – Problem Solve with Trig

1. Aimee and Russell are facing each other on opposite sides of an 8-m telephone pole. From Aimee's point of view, the top of the telephone pole is at an angle of elevation of 52°. From Russell's point of view, the top of the telephone pole is at an angle of elevation of 38°. How far apart are Aimee and Russell?

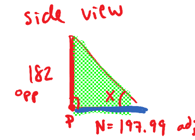
2. A square-based pyramid has a height of 182 m and a base length of 280 m. Find the angle, to the nearest degree, that one of the edges of the pyramid makes with the base. Round your answer to the nearest degree.



top view



side view



find side N

$a^2 + b^2 = c^2$

$140^2 + 140^2 = N^2$

$19600 + 19600 = N^2$

$\sqrt{39200} = \sqrt{N^2}$

$197.99 \doteq N$

$\tan X = \frac{182}{197.99}$

$X = \tan^{-1}\left(\frac{182}{197.99}\right)$

$X \doteq 43°$

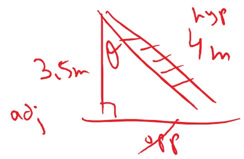
∴ pyramid makes 43° with the base.

3. A monument casts a shadow 13 m long. The sun's rays form an angle of 63° with the ground. Calculate the height of the monument to on decimal place.

4. A ladder leans against a wall forming a 25° angle with the wall. If the ladder reaches 2.8 m up the wall, how long is the ladder?

5. Brook is flying a kite while standing 24 m from the base of a tree at the park. Her kite is directly above the 3-m tree and the 25-m string is fully extended. Approximately how far above the tree is her kite flying?

6. A carpenter leans a 4 m ladder against a wall. It reaches 3.5 m up the wall. What is the angle the ladder makes with the wall?



$\cos\theta = \frac{3.5}{4}$

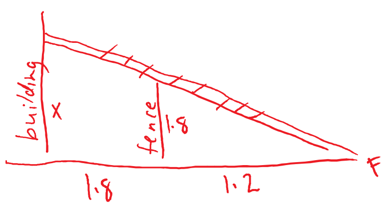
$\theta = \cos^{-1}\left(\frac{3.5}{4}\right)$

$\theta = 29°$

SOH CAH TOA

∴ ladder makes 29° with the wall

7. The foot of a ladder is 1.2 m from a fence that is 1.8 m high. The ladder touches the fence and rests against a building that is 1.8 m behind the fence. Draw a diagram, and determine the height on the building reached by the top of the ladder.



Δ's are similar since building and fence are at 90°, and both share ∠F.

Big Δ / Small Δ $\quad \frac{x}{1.8} = \frac{1.8+1.2}{1.2}$

$x = \frac{(1.8)(3)}{1.2}$

$x = 4.5$ ∴ ladder reaches 4.5 m high

8. A tower casts a shadow 7 m long. A vertical stick casts a shadow 0.6 m long. If the stick is 1.2 m high, how high is the tower?

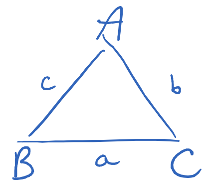
NOTES:

~~**Sine Law**~~ ~~**Cosine Law**~~

Naming Δ's

- use capitals for angles
- use small case letters for sides

ex.



side $BC = a$

SINE LAW → must be given 3 #'s, 2 of which are opposite angle and side

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

* use sides on top if solving for side

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

* use angles on top if solving for the angle.

COSINE LAW → must be given 3 #'s → all sides SSS

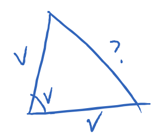
OR → 2 sides and contained angle SAS

SAS

Solving for side

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

sides angle



$$c^2 - a^2 - b^2 = -2ab(\cos C)$$

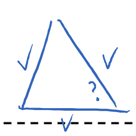
$$\frac{c^2 - a^2 - b^2}{-2ab} = \cos C$$

$$\frac{-c^2 + a^2 + b^2}{2ab} = \cos C$$

SSS

Solving for angle

$$\boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$$



Practice writing with other letters

ΔXYZ

$$y^2 = x^2 + z^2 - 2xz\cos Y$$

ΔDEF

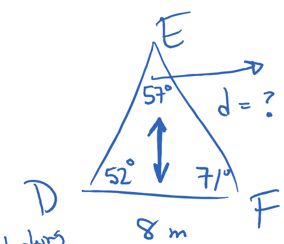
$$\cos F = \frac{d^2 + e^2 - f^2}{2de}$$

DAY 8 – Sine Law

1. Draw a diagram and label the given information. Then, find the measure of the indicated side in each triangle, to the nearest tenth of a unit.

   a) In acute △ABC, ∠A = 72°, ∠B = 68°, and *a* = 12 cm. Find side *b*.

   b) In acute △DEF, ∠D = 52°, ∠F = 71°, and *e* = 8.0 m. Find side *d*.



$180° - 52° - 71° = 57°$

looking for d

given pair of opposites

not asked for f

$$\frac{d}{\sin D} = \frac{e}{\sin E} = \cancel{\frac{f}{\sin F}}$$

$$\frac{d}{\sin 52°} = \frac{8}{\sin 57°}$$

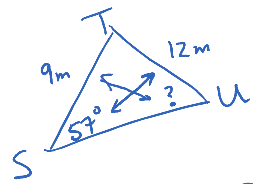
$$d = \frac{8(\sin 52°)}{\sin 57°} = \frac{8(0.7880)}{(0.8387)}$$

$$d \doteq 7.5\text{ m}$$

2. Draw a diagram and label the given information. Then, find the measure of the indicated angle in each triangle, to the nearest degree.

   a) In acute △PQR, ∠P = 64°, *p* = 5.7 cm, and *r* = 4.1 cm. Find ∠R.

   b) In acute △STU, ∠S = 57°, *s* = 12 m, and *u* = 9 m. Find ∠U.



$$\frac{\sin U}{u} = \frac{\sin S}{s}$$

$$\frac{\sin U}{9} = \frac{\sin 57°}{12}$$

$$\sin U = \frac{9(\sin 57°)}{12}$$

$$U = \sin^{-1}\left[\frac{9(\sin 57°)}{12}\right] = \sin^{-1}\left[\frac{9(0.8387)}{12}\right]$$

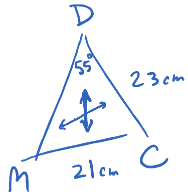
$$= \sin^{-1}(0.625)$$

$$\angle U \doteq 39°$$

17

3. Draw a diagram and label the given information. Then, solve the triangle.
In $\triangle DMC$, $\angle D = 55°$, $d = 21$ cm, and $m = 23$ cm.

means find all sides all angles



find <M

$$\frac{\sin M}{23} = \frac{\sin 55°}{21}$$

$$\sin M = \frac{23(\sin 55°)}{21}$$

$$M = \sin^{-1}\left[\frac{23(\sin 55°)}{21}\right]$$

$$M \doteq 64°$$

find <C

$$\angle C = 180° - 64° - 55°$$

$$\angle C = 61°$$

find side c

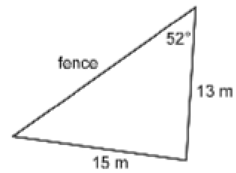
$$\frac{c}{\sin 61°} = \frac{21}{\sin 55°}$$

$$c = \frac{21(\sin 61°)}{\sin 55°}$$

$$c \doteq 22.4$$

$\therefore$ $\angle D = 55°$ $d = 21$ cm
$\angle M = 64°$ $m = 23$ cm
$\angle C = 61°$ $c = 22.4$ cm

4. Angela is building a garden in the shape of a triangle, as shown. She would like to put a fence on one side of the garden.



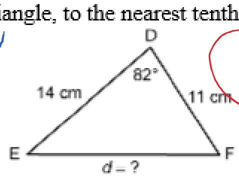
a) Find the angle formed by the fence and the side that is 15 m in length.
b) Find the length of the fence

DAY 9 – Cosine Law SAS version

1. Find the length of the indicated side in each triangle, to the nearest tenth of a unit.

notes on how I'll use my calc:

a)



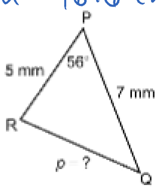
$d^2 = e^2 + f^2 - 2ef\cos D$

$d^2 = 11^2 + 14^2 - 2(11)(14)\cos 82^\circ$

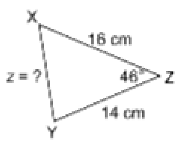
$\sqrt{d^2} = \sqrt{274.1346\ldots}$

$d = 16.6$ cm

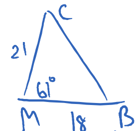
b)



c)



2. Sketch the triangle and label the given information. Then, solve the triangle.
   In $\triangle MCB$, $\angle M = 61^\circ$, $c = 18$ cm, and $b = 21$ cm.



find side m

$m^2 = 18^2 + 21^2 - 2(18)(21)\cos 61^\circ$

$m^2 = 398.4839$

$m = 20.0$

find ∠C

$\frac{\sin C}{18} = \frac{\sin 61^\circ}{20}$

$\sin C = \frac{18(\sin 61^\circ)}{20}$

$C = \sin^{-1}(0.7872)$

$= 52^\circ$

find ∠B

$\angle B = 180^\circ - 52^\circ - 61^\circ$

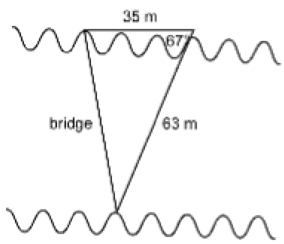
$= 67^\circ$

$\therefore \angle M = 61^\circ$ $m = 20$ cm

$\angle C = 52^\circ$ $c = 18$ cm

$\angle B = 67^\circ$ $b = 21$ cm

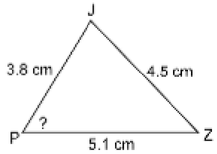
3. Find the length of the bridge, to the nearest metre.



Cosine Law SSS version

4. Solve for the indicated angle, to the nearest degree.

a)



$\cos P = \frac{j^2+z^2-p^2}{2jz}$

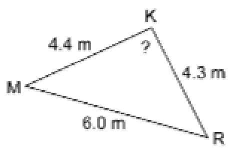
$\cos P = \frac{5.1^2+3.8^2-4.5^2}{2(5.1)(3.8)}$

$\cos P = \frac{20.2}{38.76}$

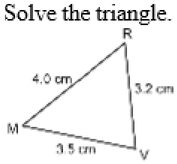
$P = \cos^{-1}(0.52115\ldots)$

$P = 59°$

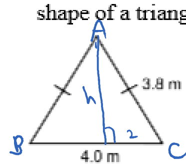
b)



5. Solve the triangle.



6. Laurissa is designing a reflecting pool, in the shape of a triangle, for her backyard.



a) Find the interior angles of the reflecting pool
b) Find the surface area of the water in the reflecting pool

ⓐ $\cos A = \frac{3.8^2+3.8^2-4.0^2}{2(3.8)(3.8)}$

$A = \cos^{-1}\left(\frac{12.88}{28.88}\right)$

$\angle A = 64°$

$\boxed{\angle B = \angle C}$ since Δ is isosceles

$\angle B = \frac{180° - 64°}{2}$

$\angle C = \angle B = 58°$

$\therefore \angle A = 64°$
$\angle B = 58°$
$\angle C = 58°$

ⓑ $A_\Delta = \frac{bh}{2}$

$= \frac{(4.0)(h)}{2}$

$= \frac{(4.0)(3.2)}{2}$

$= 6.4$

$\therefore$ Area is $6.4\ m^2$

find height

can use pythag

$2^2 + h^2 = 3.5^2$

OR

SOH CAH TOA

$\sin 58° = \frac{h}{3.8}$

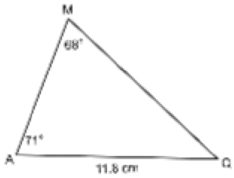
$3.8 \sin 58° = h$

$3.2 = h$

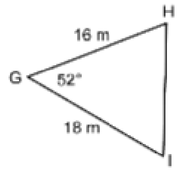
DAY 10 - Problem Solve with Trigonometry

Determine whether the primary trigonometric ratios, the sine law, or the cosine law should be used to solve each triangle.

1.



2.



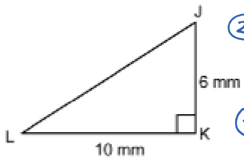
① Cosine for side g

② sine for ∠H $\frac{\sin H}{h} = \frac{\sin G}{g}$

OR cosine Law $\cos H = \frac{i^2 + g^2 - h^2}{2ig}$

③ $180° - \angle G - \angle H$ for ∠I

3.

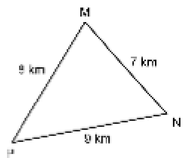


① cosine Law for side k or pythag

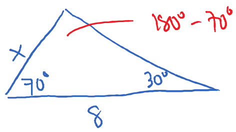
② For ∠L can use SOH CAH TOA or sine or Cosine

③ $\angle J = 180° - \angle L - \angle K$

4.



5. A shed is 8 ft wide. One rafter makes an angle of 30° with the horizontal on one side of the roof. A rafter on the other side makes an angle of 70° with the horizontal. Calculate the length of the shorter rafter to the nearest foot.



$180° - 70° - 30° = 100$

$\frac{x}{\sin 30°} = \frac{8}{\sin 100°}$

$x = \frac{8 \sin 30°}{\sin 100°}$

$x \doteq 4$

so shorter rafter is 4 ft

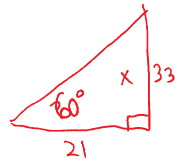
6. A 10 m ladder leans against a wall. The top of the ladder is 9 m above the ground. Safety standards call for the angle between the base of the ladder and the ground to be between 70° and 80°. Is the ladder safe to climb?

7. An intersection between two country roads makes an angle of 68°. Along one road, 5 km from the intersection, is a dairy farm. Along the other road, 7 km from the intersection, is a poultry farm. How far apart are the two farms? Round the answer to the nearest tenth of a kilometre.

8. A triangle is built using three poles with lengths 17 m, 15 m and 9 m. What is the measure of the largest angle in the triangle?

9. Three islands - Fogo, Twillingate and Moreton's Harbour - form a triangular pattern in the ocean. Fogo and Twillingate are 15 nautical miles apart. The angle between Twillingate and Moreton's Harbour from Fogo is 45°. The angle between Moreton's Harbour and Fogo from Twillingate is 65°. How far is Moreton's Harbour from the other two islands to the nearest nautical mile?

10. A golfer is faced with a shot that has to pass over some trees. The trees are 33 ft tall. The golfer finds himself 21 ft behind these trees, which obstruct him from the green. he decides to go for the green by using a 60° lob wedge. This club will allow the ball to travel at an angle of elevation of 60°. Did he make the right choice? Explain.



Ball | tree

$\tan 60° = \frac{x}{21}$ 33

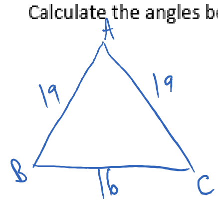
$21(\tan 60°) = x$

$36 = x$

∴ yes ball will clear the trees

11. Two tracking stations, 5 km apart, track a weather balloon floating between them. The tracking station to the west tracks the balloon at an angle of elevation of 52°, and the station to the east tracks the balloon at an angle of elevation of 60°. How far is the balloon from the closest tracking station?

12. Three cell phone towers form a triangle. The distance between the first tower and the second tower is 16 km. The distance between the second tower and the third tower is 19 km. The distance between the first tower and the third tower is 19 km. Calculate the angles between the cell phone towers.



$\cos A = \frac{19^2 + 19^2 - 16^2}{2(19)(19)}$

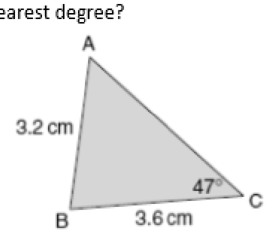
$A = \cos^{-1}\left(\frac{466}{722}\right)$

$A = 50°$

$\therefore \angle B = \angle C = \frac{180° - 50°}{2} = 65°$

13. From the top of a cliff that is 70 m in height, the angle of depression of a sailboat on a lake is 41°. What is the distance from the base of the cliff to the sailboat?

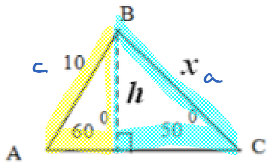
14. What are the measures of the other two angles in this triangle, to the nearest degree?



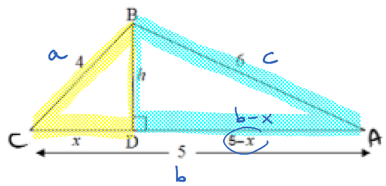
2

Proofs of the laws

Sine Law



Cosine Law



Proof of Cosine Law

$a^2 = x^2 + h^2$ $\qquad$ $c^2 = (b-x)^2 + h^2$

$a^2 - x^2 = h^2$ $\qquad$ $c^2 - (b-x)^2 = h^2$

same

$a^2 - x^2 = c^2 - (b^2 - 2bx + x^2)$

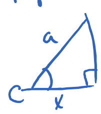
$a^2 - x^2 + (b^2 - 2bx + x^2) = c^2$

$a^2 - \cancel{x^2} + b^2 - 2bx + \cancel{x^2} = c^2$

$a^2 + b^2 - 2bx = c^2$

replace x using expression with Angles

CAH



$\boxed{\frac{\cos C}{1} = \frac{x}{a}}$

$a \cos C = x$

$\boxed{a^2 + b^2 - 2b\, a \cos C = c^2}$

Proof of Sine Law

Quote length h in two ways:

"SOH" $\sin 60^\circ = \frac{h}{10}$ → variables only $\frac{\sin A}{1} = \frac{h}{c}$ $\xrightarrow{\text{isolate } h}$ $c \sin A = h$ ①

"SOH" $\sin 50^\circ = \frac{h}{a}$ → $\sin C = \frac{h}{a}$ → $a \sin C = h$ ②

① = ② Both are "h"

$\frac{\cancel{c} \sin A}{a\cancel{c}} = \frac{\cancel{a} \sin C}{\cancel{a} c}$ } to get a law that's more easily memorized you divide by sides "ac"

$\boxed{\frac{\sin A}{a} = \frac{\sin C}{c}}$

23

Proof of Quadratic Formula

$$\boxed{ax^2+bx+c=0 \longrightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

① complete square to get x appearing once
② SAMDEB

$$a\left(\frac{ax^2}{a}+\frac{bx}{a}\right)+c=0$$

$$a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}\right)+c=0$$

$\cdot a$

rough $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2=\frac{b^2}{4a^2}$

$$a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)\underbrace{-\frac{b^2(\cancel{a})}{4a\cancel{a}}+\frac{c}{1}}_{\text{rough}}=0$$

$$\frac{-b^2}{4a}+\frac{c\cdot 4a}{1\cdot 4a}$$

$$\frac{-b^2+4ac}{4a}$$

$$a\left(x+\frac{b}{2a}\right)^2+\frac{-b^2+4ac}{4a}=0$$

~~SAMD~~EB

switch sign on top.

$$\frac{\cancel{a}}{\cancel{a}}\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a}\cdot\frac{1}{a}$$

$$\sqrt{\left(x+\frac{b}{2a}\right)^{\cancel{2}}}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b-4ac}}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$