Unit5JOURNAL

August 26, 2016 5:09 PM

Unit5JOURNAL

See below

 10D Date:___________ Name: ______________________

UNIT 5 - Quadratic Equations JOURNAL

Big idea/Learning Goals

In the last unit you practiced how to work with quadratic expressions by expanding and factoring. In this unit you will learn more complicated examples of where quadratics are used in real life. Here are some examples: If you were to open up a business selling an item you produce, how do you maximize profit and minimize cost? If you are an engineer, how do you find dimensions of a shape that will minimize cost of material yet maximize space inside; or how to model a flight path of a launched rocket.

Date	Topics	Finished the journal?	How many questions did you finish from HW?	Tentative TEST date: Mon, Dec.12 Questions to ask the teacher:
	Complete the square DAY 1 HW text pg270 #11,17		/11	
	Solve Quadratics & Quad Formula DAY 2 HW Handout – find online on mrsk.ca website under this unit and this topic		/12	
	# of Zeros & Graphs DAY 3 HW Handout – find online on mrsk.ca website under this unit and this topic		/13	
3days	**Solve Word Problems WITH Equations Given** DAY4HW text pg271#12,15,18 pg290#8,12,16 DAY 5 HW text pg301 #10,11,12,13,14 DAY 6 HW Handout – find online on mrsk.ca website under this unit and this topic		/14 /9 /24	
3days	**Solve Word Problems WITHOUT Equations Given** DAY 7 HW text pg272#16,23,24 pg280#8,11,14,17 DAY 8 HW text pg311 #2,11,12,13,16,17,23 DAY 9 HW Handout – find online on mrsk.ca website under this unit and this topic		/10 /17 /14	

Reflect – previous TEST mark __________, Overall mark now__________.

Calculate your potential final mark, show your calculations here:

potential final mark = (overall mark now)(0.40) + (future unit marks)(0.30) + (final exam marks)(0.30)

=(   )(0.40) + (   )(0.30) + (   )(0.30)

=

Looking back, what can you improve upon? ____

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Steps + notes in JOURNAL ∅

DAY 1 – Completing the Square

Explain and show how to FIND THE VERTEX from:

Standard form

$y=ax^2+bx+c$

Factor

OR

Complete sq.

Factored form

$y=a(x-r)(x-t)$

to find vertex

1. Find zeros.
2. a.o.s
3. opt. val

} vertex(a.o.s , opt. val.)

Vertex form

$y=a(x-h)^2+k$

vertex (h, k)

switch sign.

Find vertex.

1. Standard form

$y=-2x^2+18x-1$

$y=-2\left(\frac{-2x^2}{-2}+\frac{18x}{-2}\right)-1$

$y=-2\left(x^2-9x+\frac{81}{4}-\frac{81}{4}\right)-1$

mult by "a"

rough: $\left(\frac{-9}{2}\right)^2$

perfect sq. trinomial

$y=-2\left(x^2-9x+\frac{81}{4}\right)-\frac{81}{\cancel{4}_2}\left(\frac{-\cancel{2}}{1}\right)-1$

Sum Prod.

x x

$-9/2$ $-9/2$

$+\frac{81}{2}-\frac{1\cdot 2}{1\cdot 2}$

$y=-2(x-9/2)(x-9/2)+\frac{79}{2}$

half.

$y=-2(x-9/2)^2+\frac{79}{2}$

∴ vertex $\left(\frac{9}{2}, \frac{79}{2}\right)$

2. Factored form

$y=(4x+5)(2x-1)$

$y=4\left(x+\frac{5}{4}\right)2\left(x-\frac{1}{2}\right)$

$y=8\left(x+\frac{5}{4}\right)\left(x-\frac{1}{2}\right)$

1. Find zeros

$-5/4$ and $\frac{1}{2}$

2. $\text{a.o.s}=\frac{\text{add zeros}}{2}$

$=\frac{-\frac{5}{4}+\frac{1\cdot 2}{2\cdot 2}}{2}$

$=\frac{\left(-\frac{3}{4}\right)}{\left(\frac{2}{1}\right)}$

$=-\frac{3}{4}\div\left(\frac{2}{1}\right)$

$=-\frac{3}{4}\times\frac{1}{2}$

$x=-\frac{3}{8}$

3. $\text{opt.val}=8\left(-\frac{3}{8}+\frac{5\cdot 2}{4\cdot 2}\right)\left(-\frac{3}{8}-\frac{1\cdot 4}{2\cdot 4}\right)$

$=\frac{\cancel{8}}{1}\left(\frac{7}{\cancel{8}}\right)\left(\frac{-7}{8}\right)$

$y=-\frac{49}{8}$

∴ vertex $\left(-\frac{3}{8}, -\frac{49}{8}\right)$

3. Vertex form

$y=3(x-6)^2-8$

vertex $(6, -8)$

4. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square. Then, sketch labelling the vertex and two other points on the graph.

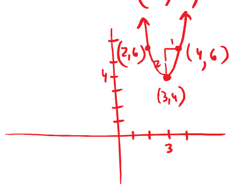
   a) $y = 2x^2 - 12x + 22$

   $y = 2(x^2 - 6x + 9 - 9) + 22$

   rough: $\left(\frac{-6}{2}\right)^2$ $\quad -9(2) + 22$ $\quad -18 + 22$

   $y = 2(x-3)^2 + 4$

   ∴ vertex $(3,4)$ step $(1,3,5\ldots)a$ $\quad 2,6,10,\ldots$



   other pts. $(5,12)$ $(6,22)$

   b) $y = -x^2 + 2x + 4$

5. Find the maximum or minimum value for each parabola. opt. val.

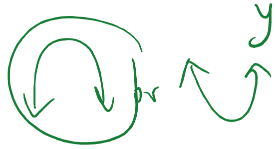
   a) $y = -\frac{1}{2}x^2 - 4x - 7$

   $y = -\frac{1}{2}(\underbrace{x^2 + 8x}_{\text{sum}} + \overset{\text{Prod}}{16} - 16) - 7$

   rough: $\left(\frac{8}{2}\right)^2$ $\quad -16\left(-\frac{1}{2}\right) - 7$ $\quad 8 - 7$

   $y = -\frac{1}{2}(x+4)^2 + 1$

   ∴ opt. val $y = 1$, MAX



   vertex $(-4,1)$

   b) $y = 1.5x^2 + 6x - 5$

DAY 2 – Solving Quadratics & Quadratic Formula

Solve.

1. $(5x+6)(4x+3)=0$

$20\left(x+\frac{6}{5}\right)\left(x+\frac{3}{4}\right)=0$

$20 \neq 0$

$x+\frac{6}{5}=0$ $\quad x=-\frac{6}{5}$ and

$x+\frac{3}{4}=0$ $\quad x=-\frac{3}{4}$

2. $(3x-1)(10x-3)=0$

3. $-x^2+5x+6=0$

4. $4a^2+12a=-9$

$4a^2+12a+9=0$

$(2a+3)(2a+3)=0$

$\sqrt{(2a+3)^2}=\pm\sqrt{0}$

$2a+3=0$

$2a=-3$

$a=-\frac{3}{2}$

5. $6x=3x^2$ if ÷ by 3x you'll lose a solution

$0=3x\left(\frac{3x^2}{3x}-\frac{6x}{3x}\right)$

$0=(3x)(x-2)$

prop. of zero

$\frac{0}{3}=\frac{3x}{3}$ or $x-2=0$

$0=x$ $\qquad x=2$

NOTES:

- If x appears once
  → undo operations in SAMDEB order
- If x appears more than once, and you can't combine unlike terms
  → Ø on one side
  → Factor * see next page if can't.
  → use prop. of Ø to separately solve each factor.

6. $3x^2+2x=0$

7. $9c^2=49$

8. $(6)\frac{x^2}{6}+(6)2x+\frac{10}{3}(6)=0(6)$

9. $(12)\frac{x^2}{4}-(12)\frac{x}{3}=(12)\frac{1}{3}$

* to get rid of fractions
→ mult each fraction by LCD
→ do to both sides.

$3x^2-4x=4$

$3x^2-4x-4=0$

$3(3x+2)(x-2)=0$ one neg.

$3\left(\frac{3x}{3}+\frac{2}{3}\right)(x-2)=0$

$3\left(x+\frac{2}{3}\right)(x-2)=0$

prop. of zero

$3x+2=0$ $\qquad x-2=0$

$3x=-2$ $\qquad x=2$

$x=-\frac{2}{3}$

10. $(2)\frac{3y^2+7}{2}=5(2)$

$3y^2+7=10-7$

$3y^2=3$

$\sqrt{y^2}=\pm\sqrt{1}$

$y=\pm 1$

$y=1$ $\qquad y=-1$

11. $0 = x^2 + x - 7$

$a=1 \quad b=1 \quad c=-7$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 28}}{2}$$

$$x = \frac{-1 \pm \sqrt{29}}{2}$$

if this was 1, 4, 9, 16, 25, ... the quest can be factored.

$x = \frac{-1+\sqrt{29}}{2} \quad x = \frac{-1-\sqrt{29}}{2}$ ← exact

$x \doteq 2.19 \quad x \doteq -3.19$ ← approx (sketch word prob)

12. $x^2 - x = 5$ LONG WAY

NOTES:

- If x appears more than once and you can't factor
  - → complete square
  - → SAMDEB

  } Long ☹

OR

Quadratic Formula

need $\varnothing$ on one side:

Ques: $ax^2 + bx + c = 0$

Sol: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ } shorter ☺

13. $2c^2 + 7c = 4$ LONG WAY

$$2\left(c^2 + \frac{7}{2}c + \frac{49}{16} - \frac{49}{16}\right) = 4$$

rough: $\left(\frac{\frac{7}{2}}{\frac{2}{1}}\right)^2 = \left(\frac{7}{2}\cdot\frac{1}{2}\right)^2 = \left(\frac{7}{4}\right)^2$ $\quad -\frac{49}{16}\left(\frac{2}{1}\right) = -\frac{49}{8}$

$$2\left(c + \frac{7}{4}\right)^2 - \frac{49}{8} = \frac{8}{8}\cdot\frac{4}{1} + \frac{49}{8}$$

now variable appears once.

$$\cancel{2}\left(c + \frac{7}{4}\right)^2 = \frac{81}{8}\cdot\frac{1}{2}$$

$$\sqrt{\left(c + \frac{7}{4}\right)^2} = \pm\sqrt{\frac{81}{16}}$$

$$c + \frac{7}{4} = \pm\frac{9}{4} - \frac{7}{4}$$

$c = \frac{2}{4} \quad c = \frac{-16}{4}$

$c = \frac{1}{2} \quad c = -4$

14. $x(2x - 3) = 7$

15. $3y^2 - (5y + 1)(2y - 3) = 3$

$$3y^2 + (-5y - 1)(2y - 3) = 3$$

$$3y^2 - 10y^2 + 15y - 2y + 3 = 3$$
$-3 \quad -3$

$$-7y^2 + 13y = 0$$

$a = -7 \quad b = 13 \quad c = 0$

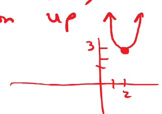
$$y = \frac{\quad \pm \sqrt{\quad}}{\quad}$$

DAY 3 – # of Zeros

For each quadratic relation, state the coordinates of the vertex, the direction of opening, and the number of x-intercepts.

1. $y = (x - 2)^2 + 3$

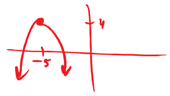
vertex $(2, 3)$
direction up
sketch



$\therefore$ # of zeros = NONE

2. $y = -2(x + 5)^2 + 4$

vertex = $(-5, 4)$
dir down



$\therefore$ # of zeros = TWO

3. $y = -(x + 1)^2$

vertex $(-1, 0)$
dir. down

# of zeros = ONE

NOTES: To find # of zeros from vertex form: $y = a(x-h)^2 + k$

- if $k = 0$ then parabola has ONE x-int/zeros
- if "a" and "k" are opposite sign then → TWO x-int
- if "a" and "k" are same sign then → NONE

Find the x-intercepts, to the nearest hundredth; the vertex; and the equation in factored form.

4. $y = 3x^2 + 6x + 4$

x-int sub $y = 0$

$0 = 3x^2 + 6x + 4$

$a = 3 \quad b = 6 \quad c = 4$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(4)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{-12}}{6}$

x = error to do $\sqrt{neg}$!

$\therefore$ no x-int.

vertex convert to vertex form by completing sq.

factored form can't be done. no zeros.

5. $0 = -2x^2 + 4x + 7$

x-int

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-2)(7)}}{2(-2)}$

$x = \frac{-4 \pm \sqrt{72}}{-4}$

$x = \frac{-4}{-4} + \frac{\sqrt{72}}{-4} \qquad x = \frac{-4 - \sqrt{72}}{-4}$

$x \doteq -1.12 \qquad x \doteq 3.12$

vertex OR find (a.o.s, opt. val)

factored form $y = a(x-r)(x-t)$

$y \doteq -2(x - -1.12)(x - 3.12)$

6. $0 = -x^2 + 8x - 16$

x-int

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-1)(-16)}}{2(-1)}$

$x = \frac{-8 \pm \sqrt{0}}{-2}$

$x = \frac{-8}{-2}$

$x = 4$ ← ONLY!

vertex since only ONE x-int

x-int = vertex $= (4, 0)$

factored form = vertex form

$y = -1(x-4)(x-4)$

$y = -1(x-4)^2$

NOTES: To find # of zeros from standard form $y = ax^2 + bx + c$

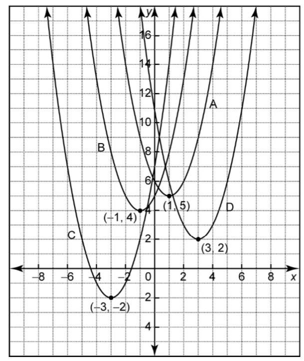
Discriminant = under root of quadratic formula = $b^2 - 4ac$

- if $b^2 - 4ac = 0$ → ONE x-int
- if $b^2 - 4ac < 0$ (neg) → NONE
- if $b^2 - 4ac > 0$ (pos) → TWO x-int

6

7. A parabola has a vertex of (1, 8) and one x-intercept is 3. → (3, 8)
   - **a)** Find the equation of the parabola in the form $y = a(x - h)^2 + k$.
   - **b)** Find the other x-intercept. → symmetry ok sub $y=0$
   - **c)** Find the y-intercept. sub $x=0$

8. Match each graph with the appropriate equation.
   - **a)** $y = (x - 3)^2 + 2$ graph —
   - **b)** $y = (x + 1)^2 + 4$
   - **c)** $y = (x - 1)^2 + 5$
   - **d)** $y = (x + 3)^2 - 2$



9. Write a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers and the roots are $\frac{1}{5}$ and $-\frac{2}{3}$.

many solutions

no fractions, no decimals

zeros, x-int, solutions

start with $y = a(x-r)(x-t)$

$y = a\left(x - \frac{1}{5}\right)\left(x - -\frac{2}{3}\right)$

choose a to make fractions disappear

$a = 15$

$y = (5)(3)\left(x - \frac{1}{5}\right)\left(x + \frac{2}{3}\right)$

$y = (5x - 1)(3x + 2)$ FOIL

$\boxed{0 = 15x^2 + 7x - 2}$

10. Write a quadratic equation in factored form, using integers, the roots of the equation are $\frac{1}{2}$ and -5.

11. Find the value of the constant so that there is only one zero $y = -4x^2 + bx - 10$

discrim = 0

$b^2 - 4ac = 0$

$b^2 - 4(-4)(-10) = 0$

$b^2 - 160 = 0$

$b^2 = 160$

$b = \pm\sqrt{160}$

7

if $b = \sqrt{160}$ or $b = -\sqrt{160}$ then parabola will have one x-int.

DAY 4 & 5 & 6 – Solve Word Problems WITH Equations Given

NOTES:

If you see "initial": find y-int
- for standard → look at "c"
- for others → sub x=0

If you see "maximum" or "minimum": find vertex (a.o.s; opt.val)
x ; y
- for vertex form → easily seen.
- for standard form → complete square
- for factored form → zeros then a.o.s then opt. val.

OTHERWISE sub in given values and try to find zeros/x-int
- factored form → easily seen.
- vertex form → isolate x by SAMDEB
- standard form → Factor if it works
($b^2-4ac = 1, 4, 9, 16, \ldots$)
→ Quadratic Formula

1. The flight of a baseball is modelled by $y = -4.9x^2 + 9.8x + 14.7$ where x is the time, in sec, and y is the height, in m, above the ground.

a) What is the initial height of the ball?
b) What is the height of the ball 0.5 seconds after it was hit?
c) How long does it take for the ball to reach the ground?
d) Find the maximum height.

2. A regular polygon with n sides has $\frac{n(n-3)}{2} = d$ diagonals. Find the number of sides of a regular polygon that has 44 diagonals.

let d be # of diagonals

sub. $d = 44$

$\cancel{(2)}\frac{n(n-3)}{\cancel{2}} = 44(2)$

$n(n-3) = 88$

$n^2 - 3n - 88 = 0$

Sum Prod

$(n-11)(n+8) = 0$

$n = 11$ ~~$n = -8$~~ can't have neg. sides.

$b^2 - 4ac$ under root of Quad formula

$(-3)^2 - 4(1)(-88)$ not a decimal

$\sqrt{361} = 19$

∴ factor is faster than Quad formula

∴ polygon with 44 diagonals has 11 sides.

4. Sipapu Natural Bridge is in Utah. Find the horizontal distance, x, in metres, across this natural arch at the base by solving the equation $-0.04x^2 - 1.56x + 3.28 = 0$.

typo

a b c

$x = \frac{1.56 \pm \sqrt{(-1.56)^2 - 4(-0.04)(3.28)}}{2(-0.04)}$

$x = \frac{1.56 \pm \sqrt{2.9584}}{-0.08}$

$x = \frac{1.56 \pm 1.72}{-0.08}$

$x = -41$

$x = 2$

across

-41 2

origin is your point of reference.

∴ 43 meters across

3. The path of a soccer ball can be defined by the relation $h = -0.025d^2 + d$, where h represents the height, in metres, and d represents the horizontal distance, in metres, that the ball travels before it hits the ground.

a) Find the d-intercepts. → x-int "zeros

b) Sketch a graph of the relation.

c) For what values of d is the relation invalid? Explain.

d) What is the maximum height? → vertex y-value

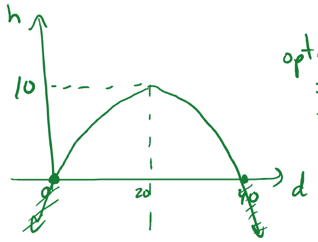
e) How far will the ball have travelled horizontally at its maximum height? → vertex x-value

(a) $h = -0.025d(d - 40)$

$\frac{-0.025d}{-0.025} = \frac{0}{-0.025}$ $d - 40 = 0$

$d = 0$ $d = 40$ ∴ d-int are 0 AND 40

(b)



opt. val = sub 20 in

$= -0.025(20)(20-40)$

$= -0.025(20)(-20)$

$= 10$

(c) Valid values of d are between ∅ and 40

$0 \le d \le 40$

(d) ∴ Max height is 10 m

(e) It travelled 20 m horizontally for max height.

5. The path of a skydiver can be modelled by the relation $h = -40t^2 + 6000$, where h represents the height of the skydiver in metres, and t represents time in seconds.

a) From what height does the skydiver jump out of the plane?

b) How long does the skydiver take to reach the ground?

6. A textbook falls from the top shelf of a shaky bookcase. The path of the book can be modelled by the relation $h = -9t^2 + 90$, where h represents the height of the book above the floor, in centimetres, and t represents time in seconds.

a) What is the height of the top shelf?

b) How long does it take the book to reach the floor?

7. A supporting arch of a bridge can be represented by the quadratic function $y = -0.0625x^2 + 9$ where x is the horizontal distance (in metres) and y is the height of the arch (in metres).

$-0.0625(x \pm 0)^2 + 9$

a) What is the vertex of this parabola?

b) What is the maximum height of the arch? → vertex y value

c) If the x-intercepts represent the beginning and the end of the arch, how wide is the base of the arch

a) vertex $= (0, 9)$

b) ∴ max height is 9m

c) to find x-int, sub $y = 0$

$0 = -0.0625x^2 + 9$ → SAMD&B choose this.

ok → Factor, find zeros

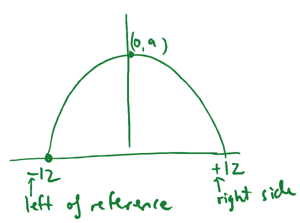
Ok → Quad formula, find zeros.

$-9 = -0.0625x^2$

$\div -0.0625 \quad \div -0.0625$

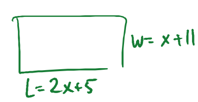
$\pm\sqrt{144} = \sqrt{x^2}$

$\pm 12 = x$



∴ the base is 24m wide.

8. A rectangle has dimensions $x + 11$ and $2x + 5$, both measured in centimetres. Determine the value of x so that the area is 117 cm².



$A = LW$

$117 = (2x+5)(x+11)$

not ∅ but looking for zeros

$0 = 2x^2 + 22x + 5x + 55 - 117$

$0 = 2x^2 + 27x - 62$ → will factor since $b^2 - 4ac = 1225$, $\sqrt{\ } = 35$

$a = 2 \quad b = 27 \quad c = -62$

$x = \frac{-27 \pm \sqrt{(27)^2 - 4(2)(-62)}}{2(2)}$ but since 62 has many factors can still use Quad. Formula.

$x = \frac{-27 \pm \sqrt{1225}}{4}$

$x = \frac{-27 \pm 35}{4}$

$x = \frac{-27+35}{4}$ $\qquad$ $x = \frac{-27-35}{4}$

$x = \frac{8}{4}$ $\qquad$ $x = \frac{-62}{4}$

$x = 2$ → $L = 2(2)+5 = 9$, $W = 2+11 = 13$ valid :)

$x = \frac{-31}{2}$ → $L = 2(-15.5)+5 = \cancel{-26}$ not valid.

∴ only $x = 2$cm gives area of 117cm²

10

9. Michael owns a trampoline. He wants to see how high he can jump. The path of one jump can be modelled by the relation $h = -4t^2 + 80t + 12$, where h represents Michael's height above the ground in centimetres and t represents time in seconds.

a) What is the height of the trampoline?

b) What is the maximum height Michael reaches?

c) How long does it take Michael to reach this height?

d) What is the height at 2 seconds?

e) How long would it take for Michael to reach a height of 348 cm?

10. A family restaurant has daily expenses that can be modelled by the quadratic relation $C = 4t^2 - 28t + 40$, where C represents the total cost in dollars, and t represents the time in hours the restaurant is open.

Balance in account

a) What is the minimum cost of running the restaurant each day?

vertex y-value

b) What is the number of hours the restaurant is open for this minimum cost? vertex x-value

c) What is the cost per day when the restaurant is not open for business? $x = t = 0$

d) How many hours was the restaurant open if the total cost per day was $160?

e) What is the cost per day if the restaurant is open for 8 hours?

a) Find vertex by completing sq.

$C = 4\left(\frac{4x^2}{4} - \frac{28x}{4}\right) + 40$

$C = 4\left(x^2 - 7x + \frac{49}{4} - \frac{49}{4}\right) + 40$

rough: $\left(\frac{-7}{2}\right)^2$

$-\frac{49}{4}\left(\frac{4}{1}\right) + 40$

$C = 4\left(x - \frac{7}{2}\right)^2 - 9$ vertex $(3.5, -9)$ x y

∴ Minimum cost is -9

money in pocket $+9

b) ∴ for min cost restaurant is to be open for 3.5 hours

c) sub $x = 0$ → not open, no time.

$C = 4(0)^2 - 28(0) + 40$

$C = 40$ ∴ Cost is $40 of loss.

"initial" Cost.

d) sub $C = 160$

$160 = 4x^2 - 28x + 40 - 160$

$0 = 4x^2 - 28x - 120$

$x = \frac{+28 \pm \sqrt{(-28)^2 - 4(4)(-120)}}{2(4)}$

$x = \frac{28 \pm 52}{8}$ → $x = 10$, ~~$x = -3$~~ time can't be neg.

∴ at 10 hours cost $160.

e) $C = 4(8)^2 - 28(8) + 40$

$C = 72$ ∴ Cost is $72 at 8 hours.

11. A model rocket is launched from the deck and the path followed by the rocket can be modelled by the relation $h = -5t^2 + 100t + 15$, where h, in metres, is the height that the model rocket reaches after t seconds. $t=0$ for y-int

a) What is the initial height of the rocket?
b) What is the height of the model rocket after 2 s?
c) What is the maximum height reached by the model rocket? vertex → y-value.
d) When was the rocket **at** a height of 200m?
e) How long was the model rocket **above** 200 m?
f) When did the rocket land on the ground?

a) initial height was 15 m

b) sub $t=2$

$h = -5(2)^2 + 100(2) + 15$

$h = -5(4) + 200 + 15$

$h = 195$

∴ at 2 sec rocket was 195 m height.

c) find vertex:

$h = -5(t^2 - 20t + 100 - 100) + 15$

$-5(t^2 - 20t + 100) - 100(-5) + 15$

$h = -5(t-10)^2 + 515$

vertex $(10, 515)$

∴ max height is 515 m

d) sub $h = 200$

$200 = -5t^2 + 100t + 15 - 200$

$0 =$

then Quad. Formula

OR

$200 = -5(t-10)^2 + 515$

-515 SAMDEB

$\frac{-315}{-5} = \frac{-5}{-5}(t-10)^2$

$\pm\sqrt{63} = \sqrt{(t-10)^2}$

$\pm\sqrt{63} + 10 = t$

$t = \sqrt{63} + 10$

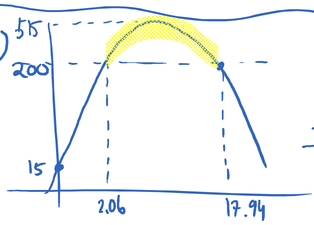
$t = 17.94$

$t = -\sqrt{63} + 10$

$t = 2.06$

∴ rocket reaches 200 m at 2.06 sec (on the way up) and at 17.94 sec (on way down)

e)



$17.94 - 2.06$

∴ rocket was above 200 m for 15.88 sec

f) sub $h = 0$

$0 = -5(t-10)^2 + 515$

$\frac{-515}{-5} = \frac{-5}{-5}(t-10)^2$

$103 = (t-10)^2$

$\pm\sqrt{103} = \sqrt{(t-10)^2}$

$\pm\sqrt{103} + 10 = t$

$+\sqrt{103} + 10$ $\quad$ $-\sqrt{103} + 10$

$t \doteq 20.15$ $\quad$ $\cancel{t \doteq -0.15}$

time can't be neg.

∴ Land on ground at 20.15 sec.

12

DAY 7 & 8 & 9 – Solve Word Problems WITHOUT Equations Given

Revenue Problem → $ coming in.

Cost − $ coming out

Profit = Revenue − Cost

Revenue = (price)(quantity)

1. Angie sold 1200 tickets for the holiday concert at $20 per ticket. Her committee is planning to increase the prices this year. Their research shows that for each $2 increase in the price of a ticket, 60 fewer tickets will be sold.
   - **a)** Determine the revenue relation that describes the ticket sales.
   - **b)** What should the selling price per ticket be to maximize revenue? → not x, not R; vertex
   - **c)** How many tickets will be sold at the maximum revenue? vertex
   - **d)** What is the maximum revenue?

a) let R be revenue
let x be # of times you increase price by $2.

$R = (20 + 2x)(1200 - 60x)$

original price; original quantity

Method 1: for vertex → x-int → a.of.s → opt. val

b) find x-int

$0 = (20 + 2x)(1200 - 60x)$

$20 + 2x = 0$ $\qquad$ $1200 - 60x = 0$

$2x = -20$ $\qquad$ $-60x = -1200$

x-int. $x = -10$ AND $x = 20$

find vertex

$a.o.s = \frac{-10 + 20}{2} = \frac{10}{2} = 5$

opt. val $= (20 + 2(5))(1200 - 60(5))$ (price, quantity)

$= (30)(900)$

$= 27\,000$

vertex $(5, 27000)$

x ↑ # of inc by $2; R ↑ Rev.

Method 2: for vertex → Foil → complete square

$R = 24000 - 1200x + 2400x - 120x^2$

$R = -120x^2 + 1200x + 24000$

$R = -120(x^2 - 10x + 25 - 25) + 24000$

$R = -120(x - 5)^2 + 27000$

∴ vertex

But asked for price

price $= 20 + 2x$

$= 20 + 2(5)$

$= 30$

∴ for max revenue price should be $30 per ticket.

c) quantity $= 1200 - 60x$

$= 1200 - 60(5)$

$= 900$

∴ 900 tickets will maximize revenue.

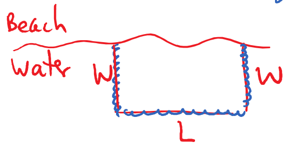
d) Max Revenue $= \$27000$

HW: do pg. 17 of journal

<u>**Fence/Rope off an Area Problem**</u> rectangular

2. For a park swimming area, 840 m of line is used to mark off the permissible area. One side not roped off is next to the beach. Find the dimensions of the swimming area that will make it a maximum.

let A be area



perimete → $840 = 2W + L$ isolate L then sub in.

$A = L \cdot W$

$840 - 2W = L$

Goal: $A = (840 - 2W)(W)$

now it's quadratic and only 2 variables

$A = 840W - 2W^2$

Since it's saying "max" → vertex.

$A = -2w^2 + 840w + 0$

$A = -2(w^2 - 420w + 44100 - 44100) + 0$

$A = -2(w - 210)^2 + 88200$

vertex$(210, 88200)$ W A

width area

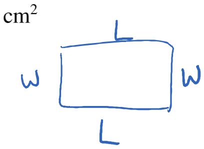
∴ length = 420m, width = 210 m } for max area.

$A = LW$ OR $L = 840 - 2W$

$\frac{88200}{210} = \boxed{\frac{A}{W} = L}$ $840 - 2(210)$

3. Suppose that half of a piece of 40 cm wire is bent to construct a rectangle. Use a quadratic model to determine the dimensions that will give an area of 24 cm^2

20cm, equation



① $20 = 2L + 2W$ → perimeter.

② $24 = LW$ → area

isolate L

$\frac{20}{2} - \frac{2W}{2} = \frac{2L}{2}$

$10 - W = L$

sub in ②

$24 = (10 - W)W$

$24 = 10W - W^2$ now a quadratic :)

no "initial/max/min" ∴ find zeros

$W^2 - 10W + 24 = 0$

-6, -4

$(W - 6)(W - 4) = 0$

W-int $W = 6$ AND $W = 4$

$L = 10 - W$
$L = 10 - 6$
$L = 4$

$L = 10 - W$
$L = 10 - 4$
$L = 6$

∴ for area 24 cm^2 dimensions must be 4cm and 6cm

Geometry Problem

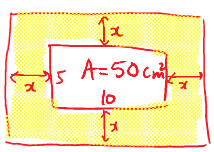
4. The hypotenuse of a right triangle measures 20 cm. The sum of the lengths of the legs is 28 cm. Find the length of each leg.

5. A rectangular skating rink measures 30 m by 20 m. It is doubled in area by extending each side of the rink by the same amount. Determine by how much each side was extended.

Frame-Border Problem

6. A picture that measures 10 cm by 5 cm is to be surrounded by a ~~mat~~ border. The mat is to be the same width on all sides of the picture. The area of the mat is to be twice the area of the picture. What is the width of the mat?

let A be area



$$A_{mat} = {}_{2x+5}\underbrace{\boxed{\quad\quad}}_{2x+10} - \boxed{50}$$

$$2(50) = (2x+5)(2x+10) - 50$$

$$100 = 4x^2 + 20x + 10x + 50 - 50$$

$$0 = \underset{a}{4x^2} + \underset{b}{30x} - \underset{c}{100}$$

$$x = \frac{-30 \pm \sqrt{30^2 - 4(4)(-100)}}{2(4)}$$

$$x = \frac{-30 \pm \sqrt{2500}}{8}$$

$$x = \frac{-30 \pm 50}{8}$$

$x = 2.5$ $\quad$ $\cancel{x = -10}$ can't be width

∴ the width is 2.5 cm

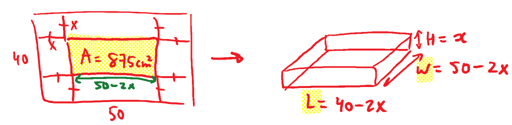
Volume Problem

7. A rectangular piece of tin 50 cm by 40 cm is made into a lidless box of base area 875 cm^2 by cutting squares of equal sizes from the corners and bending up the sides.

a) Find the side length of each removed square.

b) Find the volume of the box.

let A be area



$$A = LW$$

$$875 = (40-2x)(50-2x)$$

a) find x:

$$875 = 2000 - 80x - 100x + 4x^2$$

$$0 = 4x^2 - 180x + 1125$$

$$x = \frac{+180 \pm \sqrt{(-180)^2 - 4(4)(1125)}}{2(4)}$$

$$x = \frac{180 \pm \sqrt{14400}}{8}$$

∴ length of corner was 7.5 cm

$$x = \frac{180 \pm 120}{8}$$

$\cancel{x = 37.5}$ $\quad$ $x = 7.5$

if sub in to L or W get neg.

b) $V = LHW$

$V = (40-2(7.5))(7.5)(50-2(7.5))$

$V = (25)(7.5)(35)$

∴ $V = 6562.5$ cm^3 is the volume

Translate English to Math Problems

8. A triangle has an area of 308 cm^2. If the base is 2 cm more than three times the height of the triangle, find the base and height of the triangle

10. The sum of the squares of four consecutive integers is 630. Find the integers.

let x be 1st number
let $x+1$ be next
" $x+2$ " "
" $x+3$ " "

$(x+1)(x+1)$
$x^2+1x+1x+1$

$630 = x^2 + (x+1)^2 + (x+2)^2 + (x+3)^2$
$630 = x^2 + x^2+2x+1 +$

Falling object Problem

9. A model rocket is launched from the deck that is 15 meters high, with an initial speed of 100 m/sec.

a) What is the equation that would model this?
b) What is the height of the model rocket after 2 s?
c) What is the maximum height reached by the model rocket?
d) How long did the model rocket take to reach this height?

To know for any falling object affected by gravity.

$h = -4.9x^2 + Vx + C$

any height, time, initial speed (velocity), initial height

a) $h = -4.9x^2 + 100x + 15$

b) $h = -4.9(2)^2 + 100(2) + 15$ BEDMAS
$h = 195.4$ ∴ in 2 sec height is 195.4 m

c) $h = \left(-4.9x^2 + 100x\right) + 15$
$h \doteq -4.9\left(x^2 - 20.408x + 104.123 - 104.123\right) + 15$
$-4.9(-104.123) + 15$
$h \doteq -4.9\left(x - 10.2\right)^2 + 525.2$
vertex $(10.2, 525.2)$ — x: the time, h: height
∴ the max height was 525.2 m

d) it reaches max at 10.2 sec.

Revenue Problem

11. A harbour ferry service has about 240 000 riders per day for a fare of $2. The port authority wants to increase the fare to help with increasing operational costs. Research has shown that for every $0.10 increase in the fare the number of riders will drop by 10 000.
    - **a)** What is the revenue equation that will represent this?
    - **b)** How many times should the fare be increased to maximize the revenue? (show two methods)
    - **c)** What is the new fare that maximizes the revenue?
    - **d)** How many riders are needed for the maximum revenue?
    - **d)** What is the maximum revenue?