Unit3JOURNAL

August 26, 2016 5:40 PM

see below

Unit3JOURNAL

UNIT 3 – Quadratic Relations JOURNAL

Big idea/Learning Goals

Not everything in real life can be modeled by a linear relations which look like: $y=mx+b$. Non-linear relations can look like $y = x^2$ (quadratics – study this year) or $y = 2^x$ (exponentials – study next year). Since the last two types involve exponents we shall start this unit with laws of exponents. Exponents were invented as a shortcut of writing something that is repeated, or to avoid clumsy denominators that take up a lot of space. You will also learn how different versions of equations tell you different things, how to graph quadratics from different forms and how to interpret these graphs.

$(2)(2)(2)(2)(2) = 2^5$

$\frac{1}{3} = 3^{-1}$

Tentative TEST date: Mon. Nov. 14

Date	Topics	Finished the journal?	How many questions did you finish from HW?	Questions to ask the teacher:
2days	**Exponent Laws** DAY 1 HW text pg199 #1,2,3,8 DAY 2 HW text pg 199 #4,6,11		/24 /10	
	Non Linear relationships DAY 3 HW text pg166 #1,2,3,7		/11	
	Quadratic Relations DAY 4 HW text pg172 #1,2,3,9		/15	
FRI	**Transformation of Quadratics** DAY 5 HW text pg178 #4,6,7,8,11 DAY 6 HW Handout find online on mrsk.ca website under this unit and this topic		/19 /18	
MON	**Vertex Form** DAY 7 HW text pg185 #1efgh, 2efgh,5,6,9,10,14		/14	
3days TUE	**Factored & Standard Forms** DAY 8 HW text pg192 #4,5,8,10,13 DAY 9& 10 HW Handout find online on mrsk.ca website under this unit and this topic		/17 /42	

Reflect – previous TEST mark ___________, Overall mark now___________.

DO

Calculate your potential final mark, show your calculations here:

potential final mark = (overall mark now)(0.20) + (future unit marks)(0.50) + (final exam marks)(0.30) =( )(0.20) + ( )(0.50) + ( )(0.30) =

Looking back, what can you improve upon? ____

____

____

____

____

____

DAY 1 & 2 – Exponent Laws

1. Summarize the exponent laws you learned in grade 9 and provide examples.

LAW	GENERALIZATION	EXPLANATION	EXAMPLE
Multiplication	$x^a \cdot x^b = x^{a+b}$	- keep base - add exponents	$3^4 \cdot 3^5 = 3^9$ = Huge.
Division	$\frac{x^a}{x^b} = x^{a-b}$ record on top	- keep base - subtract exponent	$\frac{7^5}{7} = 7^4 = 2401$
Power of a Power	$(x^a)^b = x^{ab}$	- keep base - mult. exp.	$(3^2)^5 = 3^{10} = 59\,049$
Power of a Product		- keep all bases - multiply outer power with all inner powers	$(5x^6y^2)^3 = 5^3x^{18}y^6$ $= 125x^{18}y^6$
Power of a Quotient		same	$\left(\frac{2x^4}{3^2y^3}\right)^5 = \frac{2^5x^{20}}{3^{10}y^{15}} = \frac{32x^{20}}{3^{10}y^{15}}$
Zero Exponent	$\boxed{a^0 = 1}$	- always equals one.	$8^0 = 1$
Negative Exponent	$x^{-a} = \frac{1}{x^a}$	- switch the base with negative power to be a reciprocal, make a positive power.	$\frac{3x^{-2}}{(2y)^{-1}} = \frac{3^1(2y)^1}{x^2} = \frac{6y}{x^2}$
Power of Sum/Diff	$(x+y)^2$	can't distribute, must record twice.	$(2^3+x)^2 = (8+x)(8+x)$ $(8+x)^2 = 64+8x+8x+x^2$ $= 64+16x+x^2$

Practice

2. $\frac{8^{-2}}{3^1} = \frac{1}{3(8^2)} = \frac{1}{3(64)}$
$= \frac{1}{192}$

3. $\frac{3}{5^{-2}}$

4. $\frac{(4a)^{-1}}{5b^{-3}} = \frac{b^3}{5(4a)^1} = \frac{b^3}{20a}$

5. $3x^3 \cdot 4x^4$

6. $\frac{8x^6}{12x^4}$

7. $(3xy^3)^4 = 3^4x^4y^{12}$
$= 81x^4y^{12}$

square root $\sqrt{\ }$ cube root $\sqrt[3]{\ }$

Isolate in SAMDEB order

Solving if variable is in the base:

8. $\sqrt[3]{x^3} = \sqrt[3]{27}$

$x = 3$

9. $2x^2 - 4 = 12$ $+4$

$\frac{2x^2}{2} = \frac{16}{2}$

$\sqrt{x^2} = \pm\sqrt{8}$

$x = \pm\sqrt{8}$

10. $x^{-4} = \frac{1}{16}$

$\frac{1}{x^4} = \frac{1}{16}$

$\pm\sqrt[4]{16} = \sqrt[4]{x^4}$

$\pm 2 = x$

Solving if variable is in the exponent:

*do not ÷ by 3

11. $3^x = 243$ } equivalent

$3^x = 3^5$

compare

$\therefore x = 5$

12. $6^x + 5 = 221$ -5

$6^x = 216$

$6^x = 6^3$

compare

$\therefore x = 3$

13. $\left(\frac{1}{3}\right)^x = \frac{1}{81}$

$\frac{1^x}{3^x} = \frac{1}{81}$

$81 = 3^x$

$3^4 = 3^x$

$\therefore x = 4$

14. Determine the value of x that makes each statement true.

(changed questions repeated 10, 13)

a) $x^{-4} = \frac{1}{81}$

b) $\left(\frac{1}{2}\right)^x = \frac{1}{64}$

c) $\left(\frac{3}{4}\right)^x = \frac{64}{27}$

d) $5^x = \frac{1}{25}$

NOTES:

* If undoing even powers use $\pm\sqrt{\ }$, $\pm\sqrt[4]{\ }$, $\pm\sqrt[6]{\ }$ ...

* $1^x = 1$ any power.

* when solving for x and it is in the base → SAMDEB

* when solving for x and it is in the power → use power table and try to compare sides after making the bases match.

"Matching Bases Method"

1. Rewrite each power with a positive exponent.

a) 2^{-3} **b)** 4^{-1} **c)** 3^{-2}

d) $(-4)^{-2}$ $= \frac{1}{(-4)^2}$

e) -3^{-2} $= \frac{1}{-3^2}$ $= -\frac{1}{3^2}$

f) $(-14)^{-3}$ $= \frac{1}{(-14)^3}$

* base doesn't change but power becomes pos.

2. Evaluate.

a) 4^{-2} **b)** 3^0 **c)** 10^{-4} **d)** $(-3)^{-2}$

e) -8^{-2} $-1 \cdot 8^{-2}$ $= \frac{-1}{8^2}$ $= \frac{-1}{64}$

f) -7^0

g) $\left(\frac{1}{3}\right)^{-3}$ $= \left(\frac{3}{1}\right)^3 = \frac{1^{-3}}{3^{-3}}$ ✓ $= \frac{27}{1}$ $= 27$

h) $\left(-\frac{3}{7}\right)^{-2}$ $= \left(-\frac{7}{3}\right)^2$ $= \frac{49}{9}$ $\frac{1}{\left(\frac{-3}{7}\right)^2}$

3. Evaluate.

a) $3^4 + 3^{-1}$ **b)** $2^0 - 2^{-2}$

c) $(3 + 2)^0$ $= (5)^0$ $= 1$

d) $9 + 9^{-2} + 9^0$

no law for adding same bases ∴ BEDMAS

$= 9 + \frac{1}{9^2} + 1$

$= \frac{10}{1}\cdot\frac{81}{81} + \frac{1}{81}$ $\frac{810+1}{81}$

$= \frac{811}{81}$ reduce if you can

4. The half-life of radon-222 is 4 days. Determine the remaining mass of 300 mg of radon-222 after 20 days

Method 1 "Brute Force"

	Mass	Time
start	300 mg	0
	150	4 days
	75	8 days
	37.5	12 days
	18.75	16 days
	9.375	20 days

∴ after 20 days mass is 9.375 mg.

Method 2 Model with Equation

let M be remaining mass
let d be time in days

$$M = 300\left(\frac{1}{2}\right)^{d/4}$$

→ think for 1 day only 0.25 of half life have passed.
for 4 days full half-life has passed.

sub d = 20 + solve

$$M = 300\left(\frac{1}{2}\right)^{20/4}$$

$$M = 300\left(\frac{1}{2}\right)^5 \quad \frac{1^5}{2^5}$$

$$= \frac{300}{1}\left(\frac{1}{32}\right)$$

$$= \frac{300}{32}$$

$$= 9.375$$

∴

Note

$$y = \text{initial value}\left(\frac{1}{2}\right)^{\frac{x}{\text{half-life}}}$$

will see this in gr. 11

4

5. The number, N, of radium atoms remaining in a sample that started at 400 atoms can be represented by the equation $N = 400 \times 2^{\frac{-t}{1600}}$, where t is the time, in years.

a) What is the half-life of radium?

$N = 400(2)^{-\frac{t}{1600}}$

$N = 400\left(\left(\frac{1}{2}\right)^{-1}\right)^{-\frac{t}{1600}}$

$N = 400\left(\frac{1}{2}\right)^{\frac{t}{1600}}$ → compared with formula pg. 4

$\therefore$ half-life is 1600 yrs.

sub $N = 200$ and solve for t.

$\frac{200}{400} = \frac{\cancel{400}}{\cancel{400}}\left(\frac{1}{2}\right)^{\frac{t}{1600}}$

$\left(\frac{1}{2}\right)^{1} = \left(\frac{1}{2}\right)^{\frac{t}{1600}}$

compare powers $\frac{1}{1} = \frac{t}{1600}$

$1600 = t$

b) How many atoms are left after 3200 years?

c) What does $t = 0$ represent?

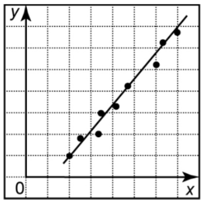
d) How many atoms were there 800 years ago? Hint: 800 years ago means $t = -800$.

e) What do negative values of t represent?

DAY 3 – Non Linear Relationships

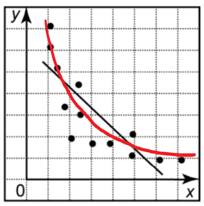
1. State whether each line of best fit is a good model for the data. Justify your answer.

a)



Line of Best fit is a good model since pts are close to the line drawn

b)

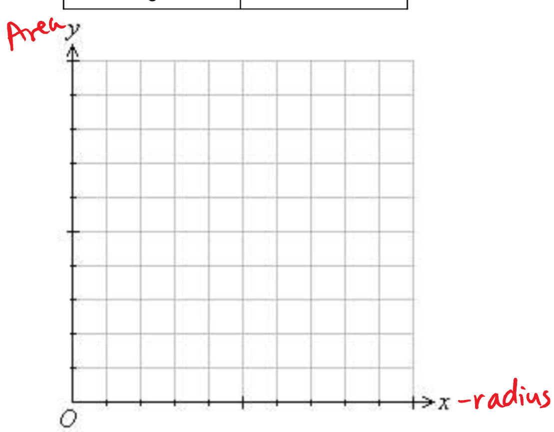


Curve of Best fit would be better to show the relationship of x to y values.

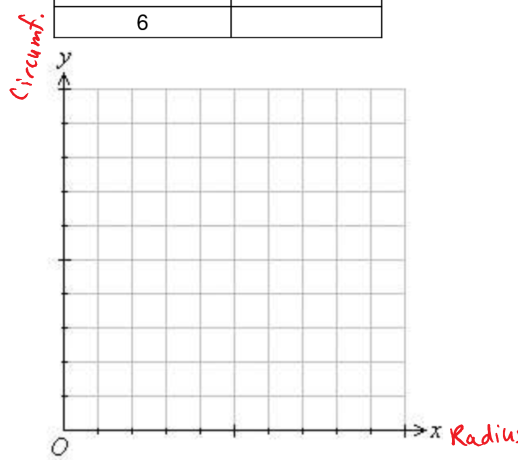
2. **a)**Complete the table of values for the relations between the area of a circle and its radius and between the circumference of a circle and its radius. Then, make two scatter plots of the data.

Radius(cm)	$A = \pi r^2$ (cm^2)
1	$3(1)^2 = 3$
2	$3(2)^2 = 12$
3	
4	
5	
6	

$\pi \approx 3$



Radius(cm)	$C=2\pi r$ (cm)
1	
2	
3	
4	
5	
6	



b) Describe the two relations.

c) Draw lines or curves of best fit for the data.

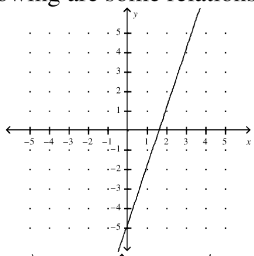
d) Use your models to predict the area and circumference for a radius of 2.5 cm.

e) Use your models to predict the area and circumference for a radius of 8 cm.

3. The following are some relations and their equations and table of values.

difference = next y − prev. y

gr. 9 Linear Relation



$y = 2x - 5$

Linear since no powers on variables

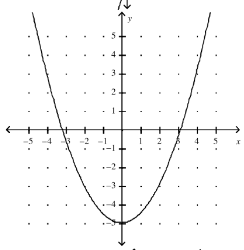
x	y
0	-5
1	-3
2	-1
3	1

Δy: +2, +2, +2

∴ Linear since 1st differences are constant.

Δy pos ∴ graph will rise ↗

gr. 10 Quadratic Relation



$y = 0.5x^2 - 5$

Quadratic since there's x^2 as highest power

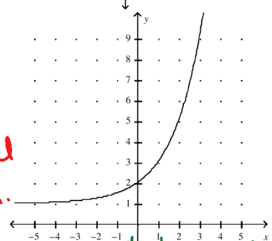
x	y
0	-5
2	-3
4	3
6	13

Δy: +2, +6, +10
ΔΔy: +4, +4

∴ Quadratic since 2nd differences are constant.

ΔΔy pos ∴ graph will open up
ΔΔy neg

gr. 11 Exponential Relation.



$y = 2^x + 1$

Neither since variable is not in the base.

x	y
0	1
1	2
2	4
3	8

Δy: +1, +2, +4
ΔΔy: 1, 2

∴ Neither since 1st or 2nd differences are not constant.

↳ 1st ratios will be constant ↳ next y ÷ prev. y.

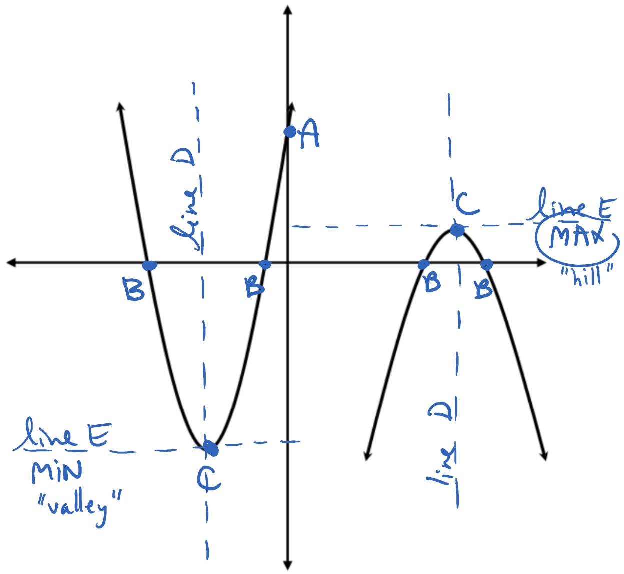
NOTES:	Linear (degree 1)	Quadratic (degree 2)	Neither
Graph		not	
Equations	$y = mx + b$ $Ax + By + C = 0$ - no powers on the variables	$y = x^2$ $y = 2x^2 - 3x$ - highest power on x must be 2	$y = 2^x$ $y = x^3$ $y = \sin x$ $y = \lvert x \rvert$ $y = x^{-2}$
Table	• 1st differences in y values must be constant • Must check before you start that x's go up evenly.	• 2nd differences in y must be the same • check x's before you start. (can't do the question unless x's go up evenly)	• neither 1st nor 2nd differences repeat.

bring colouring pencils to next class

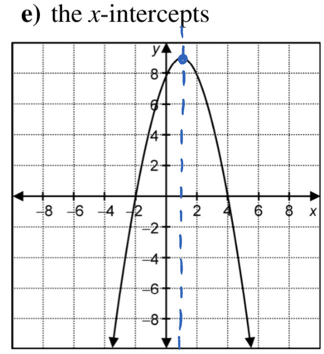
Name: ______________________

DAY 4 – Quadratic Relations

1. Vocabulary:



2. For the graph, identify
   a) the coordinates of the vertex
   b) the equation of the axis of symmetry
   c) the y-intercept
   d) the maximum or minimum value
   e) the x-intercepts



a) vertex = $(1, 9)$

b) a.of.s $x = 1$

c) y-int = 8 OR $(0, 8)$

d) MAX value = 9
opt. val $y = 9$ (MAX)

e) $(-2, 0)$ $(4, 0)$ = x-int

x-int = -2 AND 4

NOTES:

A. y-intercept – where graph crosses y-axis
- Record y-int = # OR (0, #)
- sub $x = 0$ and solve for y.

B. x-intercept "zeros" – where graph crosses x-axis
- Record (#, 0) OR x-int = # AND # ↖ not comma!
- sub $y = 0$ and solve for x ★ Hard to do

C. vertex – turning point of the U-shape curve
- Record vertex = (#, #)

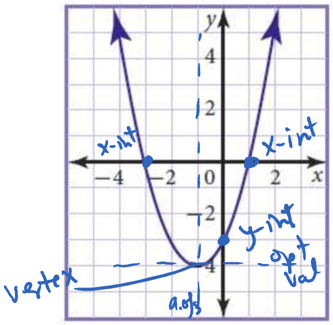
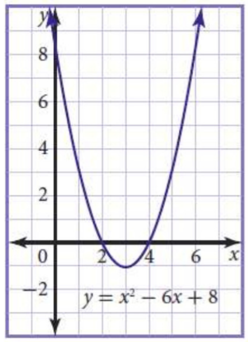
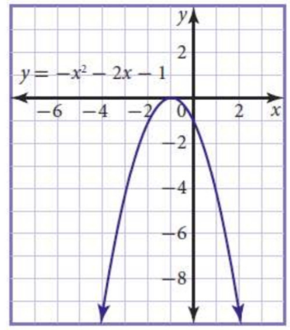
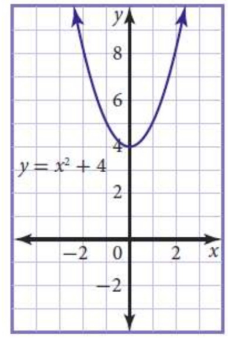
D. axis of symmetry – the vertical line through the vertex
- the "mirror" where one side of the curve reflects to get the other side.
- Record as a.of s. = x = #

E. optimal value "best" – the horizontal line through the vertex
- how high/low does it go?
- Record opt. val. = y = #, MAX/MIN

8

"Parabola" – the graph of the U-shape curve

3. Identify all the key features of the following graphs

a.	b.	c.	d.
			
Max or Min ?	Max or Min ?	Max or Min ?	Max or Min ?
Optimal Value $y=-4$	Optimal Value	Optimal Value	Optimal Value
Axis of symm $x=-1$	Axis of symm	Axis of symm	Axis of symm
Vertex $(-1,-4)$	Vertex	Vertex	Vertex
Zeros/x-int -3 AND 1	Zeros/x-int	Zeros/x-int	Zeros/x-int none
Y-intercept $(0,-3)$	Y-intercept	Y-intercept	Y-intercept

4. Use finite differences to determine whether each relation is linear, quadratic, or neither.

a)

x	y
0	3
1	6
2	9
3	12
4	15

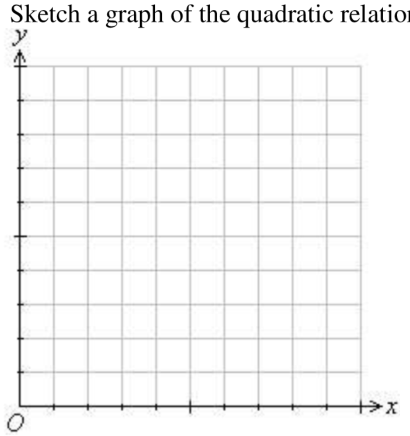
d)

x	y
–5	–125
–3	–27
–1	–1
1	1
3	27

5. The table shows the height of a ball as it moves, where x represents the distance along the ground and h represents the height above the ground, in metres.

Distance (m)	Height (m)
0	12
1	14
2	14
3	12
4	8
5	2

a) Sketch a graph of the quadratic relation.



b) Verify that $h = -x^2 + 3x + 12$ can be used to model the flight path of the ball.

test pt. (2, 14) if it is on the curve

h	$\neq -x^2 + 3x + 12$
14	$-2^2 + 3(2) + 12$
	$-4 + 6 + 12$
	$2 + 12$
	14 ✓

∴ the equation matches point given.

f) What is the exact maximum height of the ball?

Vertex will be at $x = 1.5$ right in the middle of repetition.

$h = -(1.5)^2 + 3(1.5) + 12$

$h = -2.25 + 4.5 + 12$

$h = 14.25$

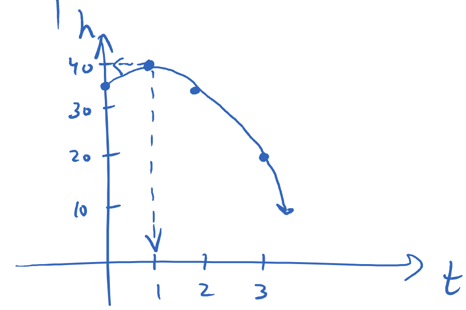
∴ exact max height is 14.25 m

6. A ball is thrown upward with an initial velocity of 10 m/s. Its approximate height, h, in metres, above the ground after t seconds is given by the relation $h = -5t^2 + 10t + 35$.

a) Sketch a graph of the quadratic relation.

x (t)	y (h)
0	$-5(0)^2 + 10(0) + 35 = 35$
1	$-5(1)^2 + 10(1) + 35 = 40$ ✱ vertex
2	$-5(2)^2 + 10(2) + 35 = 35$
3	$-5(3)^2 + 10(3) + 35 = 20$

choose any



b) Find the maximum height of the ball.

Max height is 40 m (opt. val)

c) How long does it take the ball to reach this maximum height?

It takes 1 sec (a. of s.)

d) Find when the ball is at the ground level

x	y
4	$-5(4)^2 + 10(4) + 35 = -5$
~~5~~	

The ball lands on the ground between 3 sec and 4 sec.

shifts
reflection
stretch /compress

DAY 5 & 6 – Transformations of Quadratics

NOTES: Basic $y = x$

In grade 9 LINES:

$y = mx + b$

$Ax + By + C = 0$

$y - y_1 = m(x - x_1)$

Basic $y = x^2$ "Parent"

Now in grade 10 for QUADRATICS:

$y = a(x-h)^2 + k$ vertex form "transformed" form

$y = a(x-r)(x-t)$ factored form

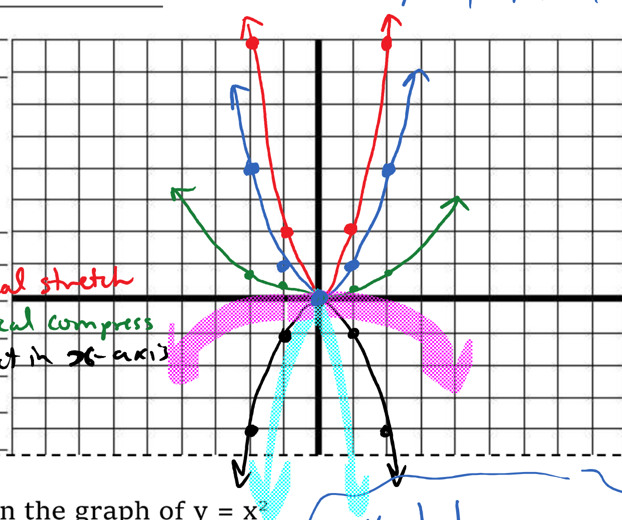
$y = ax^2 + bx + c$ standard form (expanded)

In this investigation you will graph different parabolas and determine the link between the equation in "vertex form" $y=a(x-h)^2+k$ and the transformations from the basic parabola $y = x^2$.

x	−2	−1	0	1	2
$y = x^2$	4	1	0	1	4

Parabola Investigation

Basic Equation	$y = x^2$	
Vertex Form $y = a(x-h)^2+k$ Change values for a keep h=0 and k=0 for now		
values	**Equations**	**Colour**
a=2	$y = 2x^2$	vertical stretch
a=0.2	$y = 0.2x^2$	vertical compress
a=-1	$y = -1x^2$	reflect in x-axis
a=-2	$y = -2x^2$	
a=-0.2	$y = -0.2x^2$	



NOTES:

What effect does changing "a" have on the graph of $y = x^2$

- if a is pos → opens up
- if a is neg → opens down
- if $|a| > 1$ → vertically stretched (narrow)
  ignore neg, if # is bigger than 1 ....
- if $|a| < 1$ → vertically compressed (wide)

* $|x|$ means ignore neg on x.

1. State the transformations performed on $y = x^2$ in each of the following quadratics

a. $y = 1/3x^2$ b. $y = -5x^2$ c. $y = -0.001x^2$

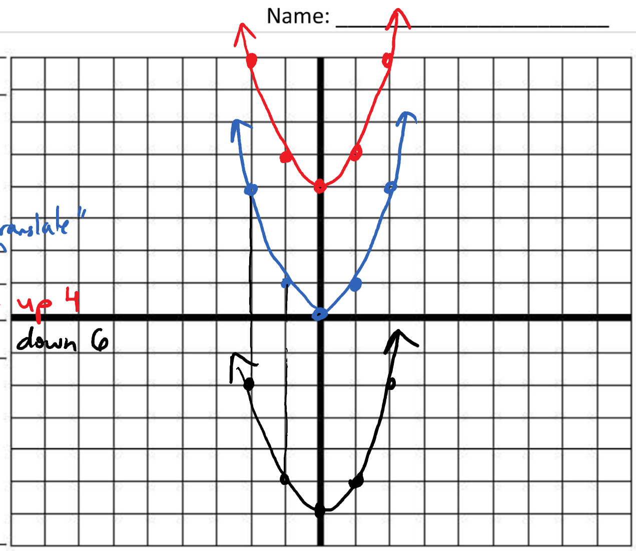
$a = +\frac{1}{3}$
→ no reflection
→ vertically compressed.

$a = -5$
→ reflect in x-axis
→ vertical stretch

x	-2	-1	0	1	2
$y=x^2$	4	1	0	1	4

Basic Equation	$y = x^2$	
Vertex Form $y = a(x-h)^2+k$ (a = 1, h = 0, k = 4) Change values for **k** keep a=1 and h=0 for now		
values	**Equations**	**Colour**
k=4	$y = x^2 + 4$	shift up 4
k=-6	$y = x^2 - 6$	shift down 6
What effect does changing k have on the graph of $y = x^2$? • if k is pos → shift up • if k is neg → shift down		

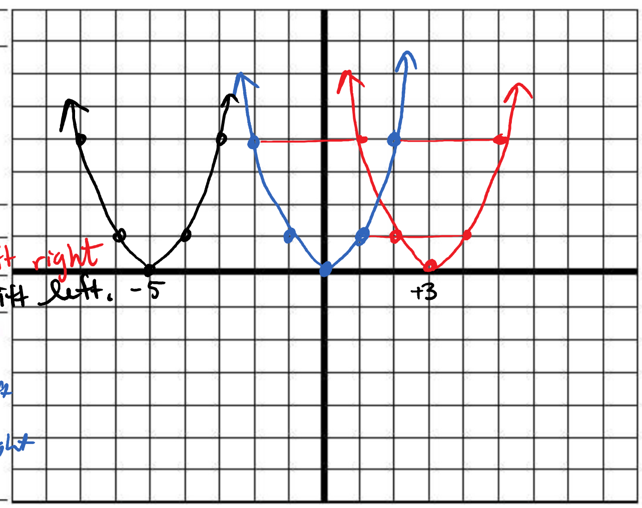
"translate"



2. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = 2x^2 - 9$ b. $y = -0.5x^2 - 16$ c. $y = -3x^2 + 9$

$a = -3$ → reflect in x-axis → vertical stretch
$K = 9$ → shift up

Basic Equation	$y = x^2$	
Vertex Form $y = a(x-h)^2+k$ (a = 1, h = 3, k = 0) Change values for **h** keep a=1 and k=0 for now		
values	**Equations**	**Colour**
h=3	$y = (x-3)^2$	shift right
h=-5	$y = (x--5)^2 = (x+5)^2$	shift left.
What effect does changing h have on the graph of $y = x^2$ • # in equation is pos → h neg → shift left • # in equation (in brackets) is neg → h pos → shift right		



3. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = (x+2)^2$ b. $y = (x-4)^2 - 7$ c. $y = -1(x+4)^2 + 3$ d. $y = 2(x-1)^2$

$a = -1$ → reflect in x-axis → nothing, same width as basic
$h = -4$ * switch! shift left.
$k = 3$ shift up

$y = x^2$ Basic

 $\boxed{y = a(x-h)^2 + k}$ → vertex/transformed. Name: ______________________

1.

$y = -2(x+4)^2 + 8$ left up

i) state the coordinates of the vertex

Vertex $= (-4, 8)$

ii) state all the transformations

$a = -2$ → reflect in x-axis → vertical stretch

$h = -4$ → shift left

$k = 8$ → shift up.

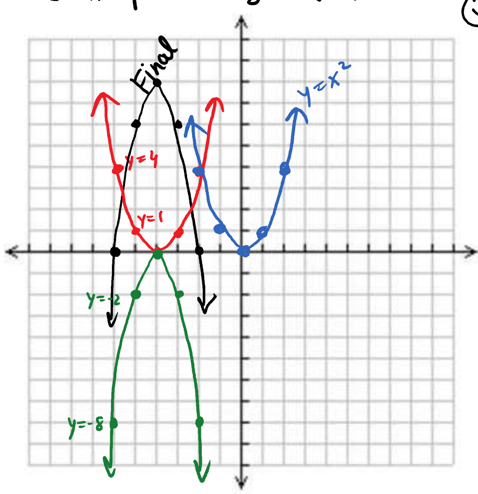
iii) sketch the graph on grid (show step by step transformations)

Basic $y = x^2$

shift left 4 : $y = (x+4)^2$

$y = -2(x+4)^2$ reflect and stretch at the same time take all y-values, mult. by "a" $= -2$

shift up 8 : $y = -2(x+4)^2 + 8$ Final ☺



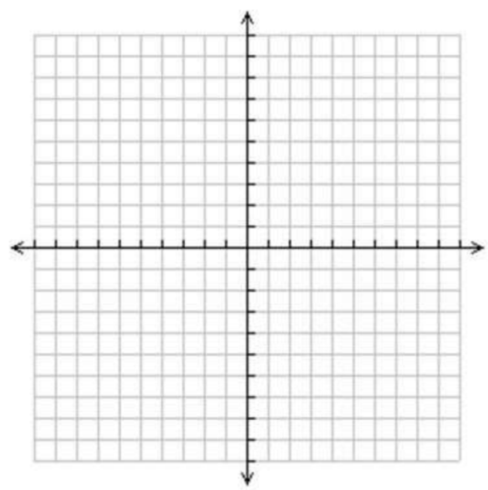
2.

$y = 0.5(x-2)^2 - 6$

i) state the coordinates of the vertex

ii) state all the transformations

iii) sketch the graph on grid (show step by step transformations)



NOTES:

(1.) Plot basic $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

(2.) Follow BEDMAS to apply transformations in the following order:

→ shift left/right

→ reflection/stretch/compress (change y-values by mult. by "a")

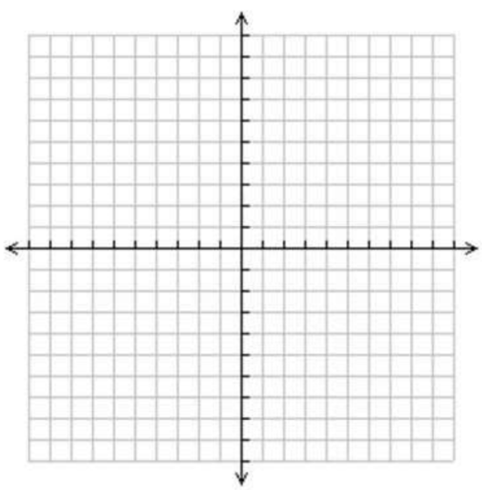
→ shift up/down → can count squares from previous shape

3.
$y = -(x-6)^2 + 2$

i) state the coordinates of the vertex

ii) state all the transformations

iii) sketch the graph on grid (can use step pattern)



4.
$y = 1.5(x+5)^2 - 4 = \overset{a}{1.5}(x - \overset{h}{(-5)})^2 \overset{k}{(-4)}$

left, down

i) state the coordinates of the vertex

vertex = $(-5,-4)$

ii) state all the transformations

$a = 1.5 \rightarrow$ vertical stretch

$h = -5 \rightarrow$ shift left

$k = -4 \rightarrow$ shift down

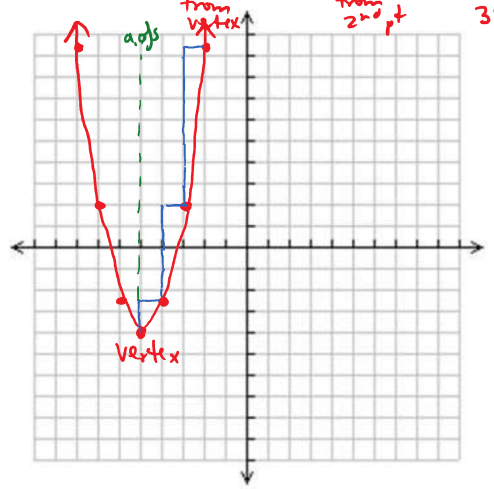
iii) sketch the graph on grid (can use step pattern) SHORT CUT. only works with quadratics.

Vertex $(-5,-4) \rightarrow$ move from origin (x,y)

step : $(1, 3, 5)(1.5)$

$= \frac{1.5}{1}, \frac{4.5}{1}, \frac{7.5}{1} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

from vertex, from 2nd pt, from 3rd pt.



NOTES:

1. Identify the vertex + plot it.
2. Find step pattern: $(\frac{1}{1}, \frac{3}{1}, \frac{5}{1}) \cdot a$

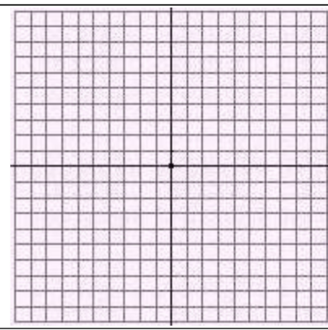
Think of step pattern as $\frac{\text{rise}}{\text{run}}$ that keeps changing from the preceeding point you drew (do not start back at vertex)

* if Run is not 1 ex. $(1,3,5)\cdot(\frac{1}{2}) = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$
make it ONE by using decimals $= \frac{0.5}{1}, \frac{1.5}{1}, \frac{2.5}{1}$

DAY 7 – Vertex Form $y=a(x-h)^2+k$

1. Complete the table for each parabola.

$-2(x--4)^2+3$

Property	$y = (x - 3)^2 - 2$	$y = -2(x + 4)^2 + 3$
Vertex		$(-4, 3)$
stretch or compression & direction of opening		vertical stretch opens down
values that x may take	any x's	any x's
values that y may take	-2 $y \geq -2$	y ~~must be below~~, or ~~equal to~~ 3 $y \leq 3$
step pattern		$(1,3,5)\cdot(-2) = \frac{-2}{1}, \frac{-6}{1}, \frac{-10}{1} \ldots$
sketch		

2. Write an equation for the parabola that satisfies each set of conditions.

a) vertex (–2, –4), opening downward with a vertical stretch

b) The graph of $y = x^2$ is translated 9 units downward, translated 10 units to the left, reflected in the x-axis and compressed vertically

shift down 9 $\therefore k=-9$

shift left 10 $\therefore h=-10$, but in equation will be ~~negative~~ pos

reflected $\therefore$ a is neg

Compressed $\therefore$ pick $|a|$ value between 0 and 1

$\therefore y = -0.5(x+10)^2 - 9$

$y = -0.01(x+10)^2 - 9$; many answers

3. A parabola $y = ax^2 + k$ passes through the points (1, 5) and (3, 29). Find the values of a and k.

sub (1,5) into $y = ax^2 + k$

$5 = a(1)^2 + k$

$\boxed{5 = a + k}$ equation (1)

sub (3,29) into $y = ax^2 + k$

$29 = a(3)^2 + k$

$29 = a(9) + k$

$\boxed{29 = 9a + k}$ equation (2)

Solve system of equations elimination or substitution

$5 = a + k$

$29 = 9a + k$

subtract

$-24 = -8a$

$3 = a$

sub in (1)

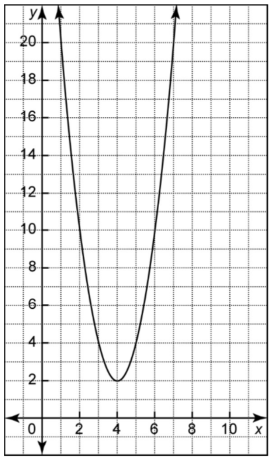
$5 = 3 + k$

$2 = k$

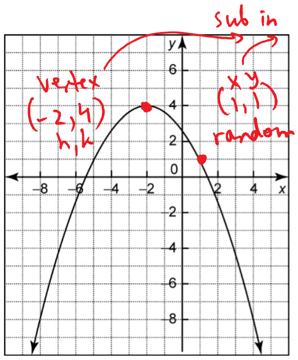
use BEDMAS – if variable already isolate
use SAMDEB – to isolate the variable

4. Write an equation for each parabola.

a)



b)



$\boxed{y = a(x-h)^2 + k}$ memorize this

vertex $(-2, 4)$ h, k — sub in

$(1, 1)$ x, y random pt

$1 = a(1 - -2)^2 + 4$

$1 = a(3)^2 + 4$

$1 = a(9) + 4$

$1 = 9a + 4$

$1 - 4 = 9a$

$-3 = 9a$

$-\frac{1}{3} = a$

$\therefore y = -\frac{1}{3}(x+2)^2 + 4$ or $y = -\frac{1}{3}(x - -2)^2 + 4$

5. Find an equation for the parabola with vertex $(-3, 1)$ that passes through the point $(-2, -1)$.

6. A rocket travels according to the equation $h = -4.9(t - 6)^2 + 182$, where h is the height, in metres, above the ground and t is the time, in seconds.

a) Sketch a graph of the rocket's motion.

step pattern – not usefull if Big #'s are there

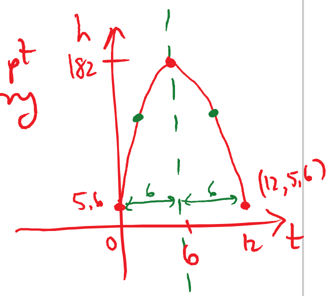
instead – plot vertex
– find random pt
– use symmetry

vertex $= (6, 182)$

sub $t = 0$ for random pt.

$h = -4.9(0-6)^2 + 182$

$h = 5.6$



b) Find the maximum height of the rocket. → from vertex

182 m

c) How long does it take the rocket to reach its maximum height? → from vertex

6 sec

d) How high was the rocket above the ground when it was fired? → initial value at $t = 0$

5.6 m

NOTES: Finding An Equation

- always call vertex (h, k)
  random pt. (x, y)
- Sub into $\boxed{y = a(x-h)^2 + k}$ to find "a"
- when recording final answer keep x and y as variable the a, h, k should be #'s

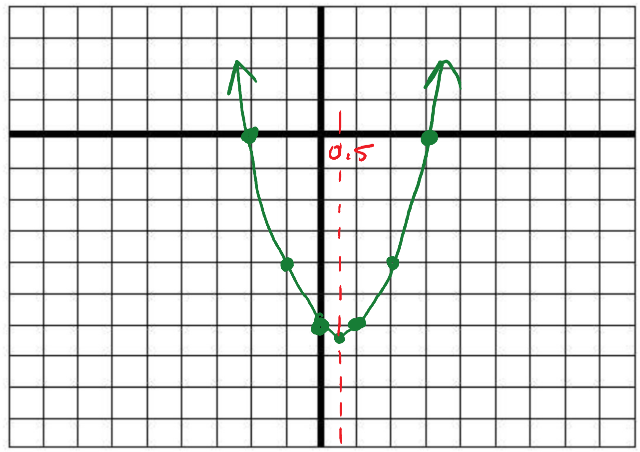
16

DAY 8 & 9 & 10– Factored Form y = a(x-r)(x-t) and Standard Form y = ax² + bx + c

1. Table of Values

x	y
-2	0
-1	-4
0	-6
0.5	-6.25
1	-6
2	-4
3	0

a. Plot the points and draw a curve of best fit



MAX

min

b. What is the y-intercept? y-int = −6 (0,−6)

c. What is the direction of opening? up

d. Is the vertex at max/min? MIN

e. What are the zeros/x-int? X-int = −2 AND 3 (−2,0) (3,0)

f. What is the axis of symmetry? X = 0.5

g. What is the Optimal Value? y = −6.25 MIN

h. What is the vertex? (0.5, −6.25)

2. Check that BOTH of the following equations model this parabola

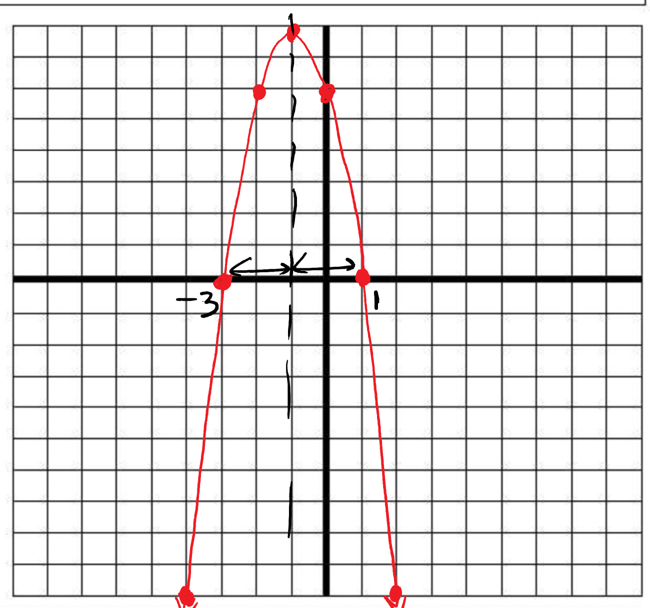
Standard form $y = x^2 - x - 6$

Factored form $y = (x - 3)(x + 2)$

3. Table of Values

x	y
-4	-10
-3	0
-2	6
-1	8
0	6
1	0
2	-10

a. Plot the points and draw a curve of best fit



b. What is the y-intercept? y-int = 6

c. What is the direction of opening? down

d. Is the vertex at max/min? MAX

e. What are the zeros/x-int? X-int = −3 AND 1

f. What is the axis of symmetry? X = −1

g. What is the Optimal Value? y = 8 MAX

h. What is the vertex? (−1, 8)

4. Check that BOTH of the following equations model this parabola

Standard form $y = -2x^2 - 4x + 6$

Factored form $y = -2(x - 1)(x + 3)$

5. What are the answers to ☆ ?

NOTES:

- What part of equation will tell you direction of opening?

  The "a" in ANY form will tell if parabola opens up → a is pos OR down → a is neg.

- What part of the equation will tell you the y-intercept?

  The "c" in standard form: $y = ax^2 + bx + c$ is the y-int OR sub $x = 0$ and solve if you have other forms.

- What part of the equation will tell you the zeros/x-int?

  The "r" and "t" in factored form: $y = a(x-r)(x-t)$ are the x-int OR sub $y = 0$ and solve if you have other forms (not easy)

- How to get axis of symmetry from just zeros?

  $\boxed{\text{a.o.s} = \frac{\text{add zeros}}{2}}$ *record as an equation $\boxed{x = \#}$

- How to get Max/Min optimal value?

  opt.val = sub in for x the a.o.s. # and solve for y

  *record as an equation $\boxed{y = \#,\ \text{MAX/MIN}}$

6. Sketch each parabola. Label the vertex and the *x*-intercepts.

a) $y = -\frac{1}{2}(x-3)(x-7)$

x-int = 3 AND 7

$\text{a.o.s} = \frac{(3+7)}{2}$

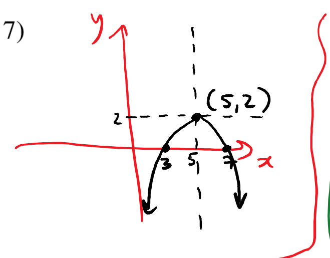
$= \frac{10}{2}$

$x = 5$

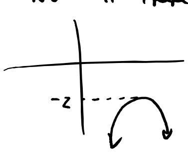
$\text{opt.val} = -\frac{1}{2}(5-3)(5-7)$

$= +\frac{1}{\cancel{2}}(\cancel{2})(+2)$

$y = +2$, MAX



Note if there are no zeros



MAX $y = -2$
can be negative.

b) $y = 2(x-8)(2x+3)$ — $2(x-8)\left(\frac{2x}{2}+\frac{3}{2}\right)$, (2)

This is factored form but not in proper form (x must have no #)

$y = 4(x-8)(x - -1.5)$

x-int: 8 AND −1.5

OR can work with original and sub $y = 0$ and solve

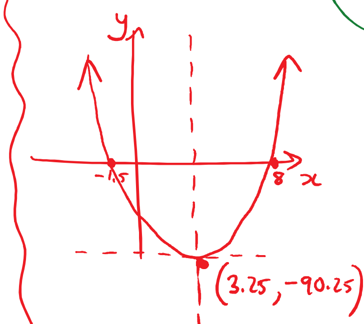
$0 = 2(x-8)(2x+3)$

then use property of Ø

$0 \neq 2$, $x - 8 = 0$, $2x + 3 = 0$

$x = 8$, $2x = -3$

$x = -\frac{3}{2}$



$\text{a.o.s} = \frac{-1.5+8}{2}$

$= \frac{6.5}{2}$

$x = 3.25$

$\text{opt.val} = 4(x-8)(x+1.5)$

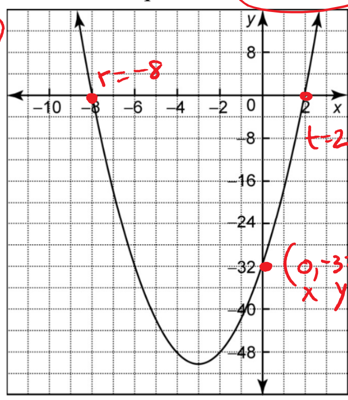
$= 4(3.25-8)(3.25+1.5)$

$= 4(-4.75)(4.75)$

$y = -90.25$, MIN

7. Determine an equation in factored form

a)



$y = a(x-r)(x-t)$

$-32 = a(0 - -8)(0-2)$

$-32 = a(8)(-2)$

$\frac{-32}{-16} = \frac{-16a}{-16}$

$2 = a$

$\therefore$ equation is $y = 2(x+8)(x-2)$

b) the parabola has one of the zeros at 2, axis of symmetry at -0.5 and goes through the point (4, -28)

/3

c) the quadratic has only one zero at 3 and is reflected in the x-axis and has a vertical compression factor.

$y = a(x-r)(x-t)$

both r and $t = 3$

$y = a(x-3)(x-3)$

$y = a(x-3)^2 \rightarrow$ vertex form

reflection $\rightarrow$ a is neg

compress $\rightarrow |a|$ is smaller than 1

$\therefore a = -0.5$

$a = -0.1$ etc many ans.

$\therefore$ equation is

$y = -0.5(x-3)^2$

8. The path of a kicked football can be modelled by the relation $h = -0.02x(x - 45)$, where h represents the height, in metres, above the ground and x represents the horizontal distance, in metres, measured from the kicker.

a) Sketch the path of the ball.

/5

a) When the ball hits the ground, how far has it travelled?

b) What is the maximum height of the ball?

/4

c) What is the horizontal distance when this occurs?

d) If the goal post is 40 m away, will the kick clear the 3-m-high crossbar for a field goal?

For each of the following quadratic relations state the following:

a) the direction of opening, b) the zeros, c) the equation of the axis of symmetry,
d) the maximum or minimum value of y, e) the coordinates of the vertex, f) sketch using vertex and zeros

1. $y=-0.5(4-2x)(x+5)$

6

2. $y=3(2+3x)(2-x)$

$y=-9\left(x+\frac{2}{3}\right)(x-2)$

a) opens down

b) zeros/x-int = $-\frac{2}{3}$ AND 2

c) a of s $=\dfrac{-\frac{2}{3}+\frac{2\cdot 3}{1\cdot 3}}{2}$

$=\dfrac{-\frac{2}{3}+\frac{6}{3}}{2}$

$=\left(\frac{4}{3}\right)\cdot\frac{1}{2}$

$\div\frac{2}{1}$ $\times\frac{1}{2}$

$x=\frac{2}{3}$

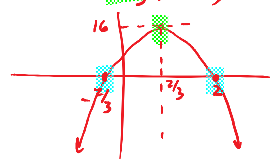
d) opt. val $=-9\left(\frac{2}{3}+\frac{2}{3}\right)\left(\frac{2}{3}-\frac{2\cdot 3}{1\cdot 3}\right)$

$=+9\left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$

$y=16$, MAX

e) vertex $\left(\frac{2}{3}, 16\right)$

f)



NOTES:

Another way to find dir. of opening

- instead of fixing the factored form you can expand (FOIL) to see "a" on x^2 term

$y=3(2+3x)(2-x)$

$y=3(4-2x+6x-3x^2)$

$a=-9$

a pos ∪

a neg ∩

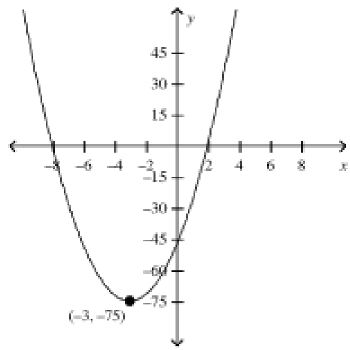
	Factored form y = (x - s)(x - t)	x-intercepts	Standard Form y = ax² + bx + c	y-intercept sub x=0
3.	$y=(2x+3)(x-1)$ $y=2(x+1.5)(x-1)$	-1.5 AND 1 or $-\frac{3}{2}$ AND 1	$y=(2x+3)(x-1)$ F O I L $y=2x^2-2x+3x-3$ $y=2x^2+1x-3$	-3 = y-int or $(0,-3)$
4.	$y=a(x-r)(x-t)$ $12=a(0-(-4))(0-3)$ $12=a(4)(-3)$ $-1=a$ $\therefore y=-1(x+4)(x-3)$	x = -4 and x = 3 r t	$y=-1(x+4)(x-3)$ $y=-1(x^2-3x+4x-12)$ F O I L $y=-x^2+3x-4x+12$ $y=-x^2-1x+12$ ← check y-int	12 (0, 12) x, y

For each of the following graphs answer these questions.

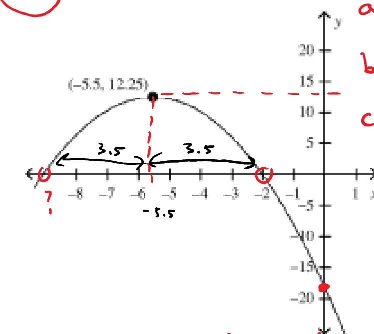
a) What is the maximum/minimum? b) When did the maximum/minimum occur? c) What are the zeros?

d) Find an equation to describe the graph. (Use factored and vertex form.)

5.



6.



a) MAX y = 12.25

b) at x = −5.5

c) x-int = −2 AND −9

d) $y = a(x-r)(x-t)$ sub r = −2, t = −9

$12.25 = a(-5.5 - -2)(-5.5 - -9)$ random pt (−5.5, 12.25) x, y

$12.25 = a(-3.5)(3.5)$

$\frac{12.25}{-12.25} = \frac{a(-12.25)}{-12.25}$

$-1 = a$ ∴ factored form

$y = -1(x+2)(x+9)$

vertex form

$y = -1(x+5.5)^2 + 12.25$

	Standard form	Factored form	x-intercepts	y-intercept
7.	$y = 2x^2 + 12x$ +0	$y = 2x\left(\frac{2x^2}{2x} + \frac{12x}{2x}\right)$ G.C.F. $y = 2x(x+6)$ or $y = 2(x-0)(x+6)$	0 AND −6	(0,0) y-int = 0
8.	$y = x^2 + 2x - 15$	$y = x^2 + 2x - 15$ x, x −1 −3 / 15 5 1 3 / −15 −5 $y = (x-3)(x+5)$	3 AND −5	−15