# UNIT 3 – Quadratic Relations JOURNAL

## Big idea/Learning Goals

Not everything in real life can be modeled by a linear relations which look like: _____. Non-linear relations can look like _____ (quadratics – study this year) or _____ (exponentials – study next year). Since the last two types involve exponents we shall start this unit with laws of exponents. Exponents were invented as a shortcut of writing something that is repeated, or to avoid clumsy denominators that take up a lot of space. You will also learn how different versions of equations tell you different things, how to graph quadratics from different forms and how to interpret these graphs.

| Date | Topics | Finished the journal? Made corrections? | Did you do the HW? Checked if it was correct? | Tentative TEST date: _____ Questions to ask the teacher: |
|---|---|---|---|---|
| 2days | **Exponent Laws**<br>DAY 1 HW text pg199 #1,2,3,8<br><br>DAY 2 HW text pg 199 #4,6,11 | | | |
| | **Non Linear relationships**<br>DAY 3 HW text pg166 #1,2,3,7 | | | |
| | **Quadratic Relations**<br>DAY 4 HW text pg172 #1,2,3,9 | | | |
| 2days | **Transformation of Quadratics**<br>DAY 5 HW text pg178 #4,6,7,8,11<br><br>DAY 6 HW Handout find online on mrsk.ca website under this unit and this topic | | | |
| | **Vertex Form**<br>DAY 7 HW text<br>pg185 #1efgh, 2efgh,5,6,9,10,14 | | | |
| 3days | **Factored & Standard Forms**<br>DAY 8 HW text pg192 #4,5,8,10,13<br><br>DAY 9 & 10 HW Handout find online on mrsk.ca website under this unit and this topic | | | |

## Reflect – previous TEST mark _____, Overall mark now_____.

Calculate your potential final mark to see how averages work. Show your calculations here:

potential final mark = (overall mark now)(weight so far) + (future marks)(weight to come)

= ( )( ) + ( )( )

=

Were you able to attain your set goal before? Looking back, what else can you improve upon? Be specific in your planning.

_____

_____

_____

_____

_____

_____

_____

_____

# DAY 1 & 2 – Exponent Laws

1. Summarize the exponent laws you learned in grade 9 and provide examples.

| *LAW* | *GENERALIZATION* | *EXPLANATION* | *EXAMPLE* |
|---|---|---|---|
| Multiplication | | | $3^4 \cdot 3^5$ |
| Division | | | $\frac{7^5}{7}$ |
| Power of a Power | | | $\left(3^2\right)^5$ |
| Power of a Product | | | $\left(5x^6y^2\right)^3$ |
| Power of a Quotient | | | $\left(\frac{2x^4}{3^2y^3}\right)^5$ |
| Zero Exponent | | | $8^0$ |
| Negative Exponent | | | $\frac{3x^{-2}}{(2y)^{-1}}$ |
| Power of Sum/Diff | | | $\left(2^3+x\right)^2$ |

Practice

2. $\frac{8^{-2}}{3}$

3. $\frac{3}{5^{-2}}$

4. $\frac{(4a)^{-1}}{5b^{-3}}$

5. $3x^3 \cdot 4x^4$

6. $\frac{8x^6}{12x^4}$

7. $\left(3xy^3\right)^4$

Solving if variable is in the base:

8. $x^3 = 27$

9. $2x^2 - 4 = 12$

10. $x^{-4} = \frac{1}{16}$

Solving if variable is in the exponent:

11. $3^x = 243$

12. $6^x + 5 = 221$

13. $\left(\frac{1}{3}\right)^x = \frac{1}{81}$

14. Determine the value of $x$ that makes each statement true.

**a)** $x^{-4} = \frac{1}{81}$

**b)** $\left(\frac{1}{2}\right)^x = \frac{1}{64}$

**c)** $\left(\frac{3}{4}\right)^x = \frac{64}{27}$

**d)** $5^x = \frac{1}{25}$

**NOTES:**

1. Rewrite each power with a positive exponent.

**a)** $2^{-3}$ **b)** $4^{-1}$ **c)** $3^{-2}$

**d)** $(-4)^{-2}$ **e)** $-3^{-2}$ **f)** $(-14)^{-3}$

2. Evaluate.

**a)** $4^{-2}$ **b)** $3^{0}$ **c)** $10^{-4}$ **d)** $(-3)^{-2}$

**e)** $-8^{-2}$ **f)** $-7^{0}$ **g)** $\left(\frac{1}{3}\right)^{-3}$ **h)** $\left(-\frac{3}{7}\right)^{-2}$

3. Evaluate.

**a)** $3^4 + 3^{-1}$ **b)** $2^0 - 2^{-2}$

**c)** $(3 + 2)^0$ **d)** $9 + 9^{-2} + 9^0$

4. The half-life of radon-222 is 4 days. Determine the remaining mass of 300 mg of radon-222 after 20 days

5. The number, $N$, of radium atoms remaining in a sample that started at 400 atoms can be represented by the equation $N = 400 \times 2^{\frac{-t}{1600}}$, where $t$ is the time, in years.

**a)** What is the half-life of radium?

**b)** How many atoms are left after 3200 years?

**c)** What does $t = 0$ represent?

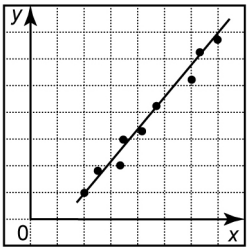
**d)** How many atoms were there 800 years ago?
Hint: 800 years ago means $t = -800$.

**e)** What do negative values of $t$ represent?

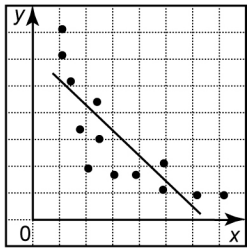
# DAY 3 – Non Linear Relationships

1. State whether each line of best fit is a good model for the data. Justify your answer.

**a)**

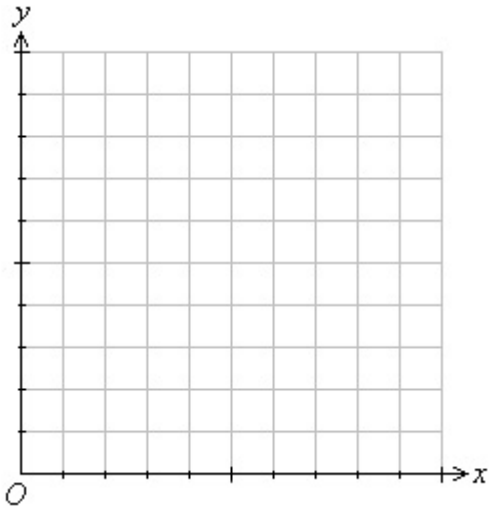



**b)**

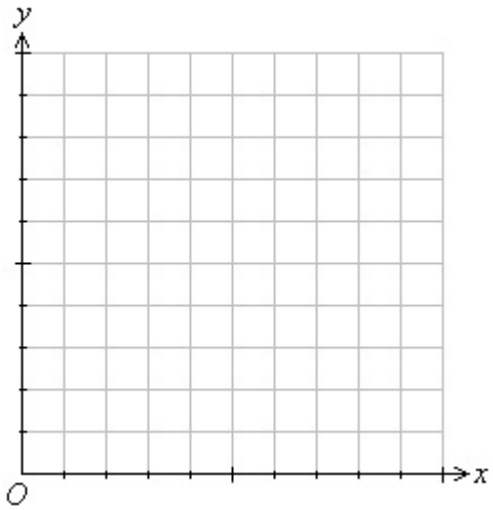



2. **a)**Complete the table of values for the relations between the area of a circle and its radius and between the circumference of a circle and its radius. Then, make two scatter plots of the data.

| Radius(cm) | $A = \pi r^2$ (cm$^2$) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |




| Radius(cm) | $C=2\pi r$ (cm) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |



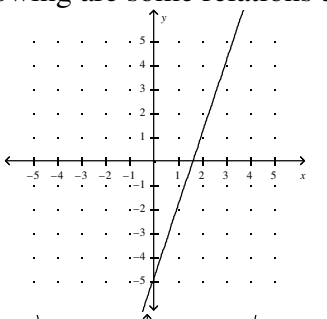

**b)** Describe the two relations.

**c)** Draw lines or curves of best fit for the data.

**d)** Use your models to predict the area and circumference for a radius of 2.5 cm.

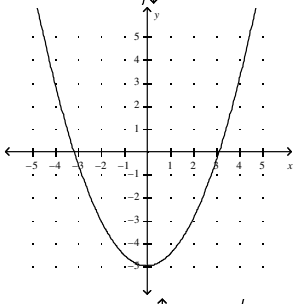
**e)** Use your models to predict the area and circumference for a radius of 8 cm.

3. The following are some relations and their equations and table of values.



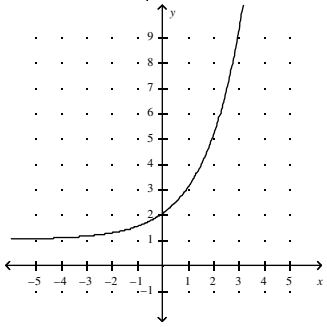
$y = 2x - 5$

| x | y |
|---|---|
| 0 | -5 |
| 1 | -3 |
| 2 | -1 |
| 3 | 1 |



$y = 0.5x^2 - 5$

| x | y |
|---|---|
| 0 | -5 |
| 2 | -3 |
| 4 | 3 |
| 6 | 13 |



$y = 2^x + 1$

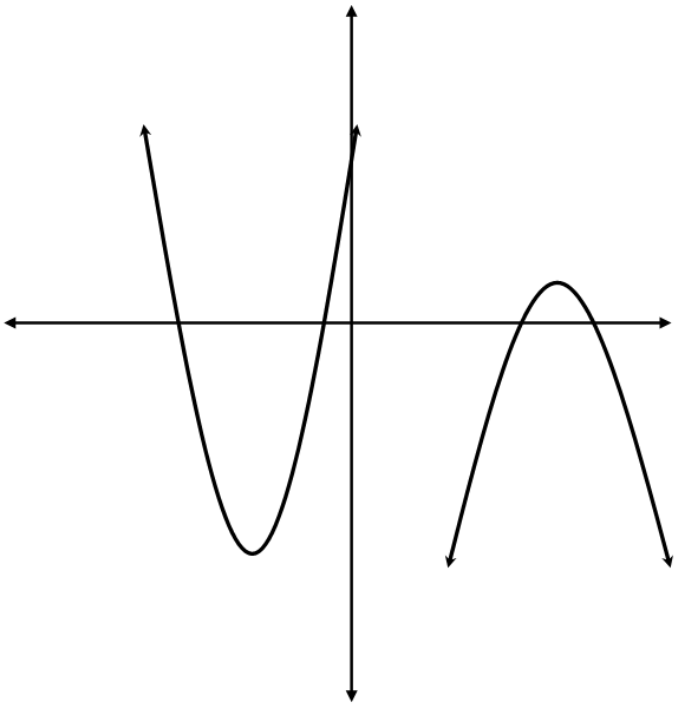
| x | y |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |

**NOTES:**

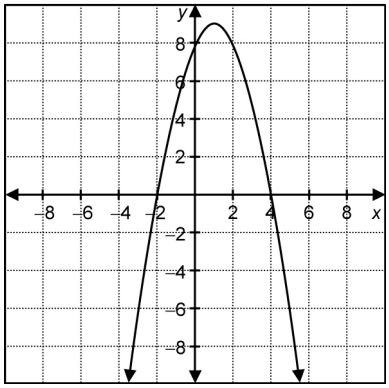
 bring colouring pencils to next class Name: _____

## DAY 4 – Quadratic Relations

1. Vocabulary:



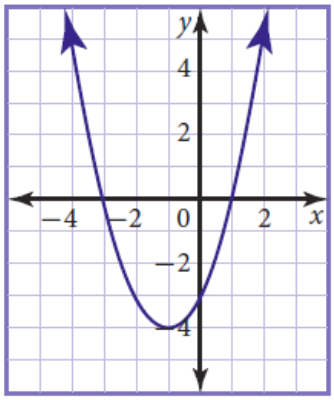
2. For the graph, identify
   **a)** the coordinates of the vertex
   **b)** the equation of the axis of symmetry
   **c)** the *y*-intercept
   **d)** the maximum or minimum value
   **e)** the *x*-intercepts




**NOTES:**

3. Identify all the key features of the following graphs

a.




Max or Min ?

Optimal Value

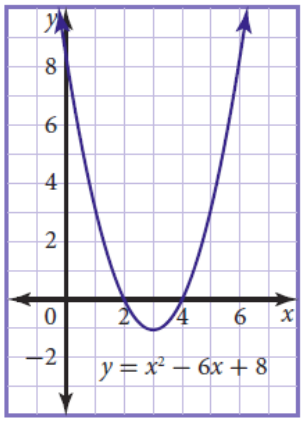
Axis of symm

Vertex

Zeros/x-int

Y-intercept

b.




Max or Min ?

Optimal Value

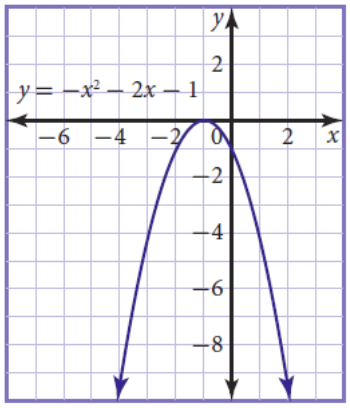
Axis of symm

Vertex

Zeros/x-int

Y-intercept

c.




Max or Min ?

Optimal Value

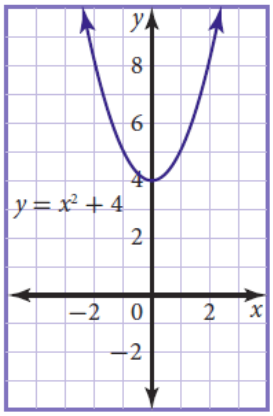
Axis of symm

Vertex

Zeros/x-int

Y-intercept

d.




Max or Min ?

Optimal Value

Axis of symm

Vertex

Zeros/x-int

Y-intercept

4. Use finite differences to determine whether each relation is linear, quadratic, or neither.

**a)**

| *x* | *y* |
|---|---|
| 0 | 3 |
| 1 | 6 |
| 2 | 9 |
| 3 | 12 |
| 4 | 15 |

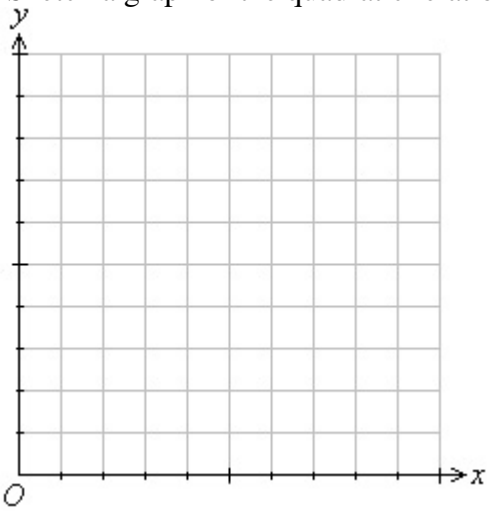
**d)**

| *x* | *y* |
|---|---|
| –5 | –125 |
| –3 | –27 |
| –1 | –1 |
| 1 | 1 |
| 3 | 27 |

5. The table shows the height of a ball as it moves, where $x$ represents the distance along the ground and $h$ represents the height above the ground, in metres.

| Distance (m) | Height (m) |
|---|---|
| 0 | 12 |
| 1 | 14 |
| 2 | 14 |
| 3 | 12 |
| 4 | 8 |
| 5 | 2 |

**a)** Sketch a graph of the quadratic relation.




**b)** Verify that $h = -x^2 + 3x + 12$ can be used to model the flight path of the ball.

**f)** What is the exact maximum height of the ball?

6. A ball is thrown upward with an initial velocity of 10 m/s. Its approximate height, $h$, in metres, above the ground after $t$ seconds is given by the relation $h = -5t^2 + 10t + 35$.

**a)** Sketch a graph of the quadratic relation.

**b)** Find the maximum height of the ball.

**c)** How long does it take the ball to reach this maximum height?

**d)** Find when the ball is at the ground level

## DAY 5 & 6 – Transformations of Quadratics

**NOTES:**

**In grade 9 LINES:** **Now in grade 10 for QUADRATICS:**

In this investigation you will graph different parabolas and determine the link between the equation in "vertex form" $y=a(x-h)^2+k$ and the transformations from the basic parabola $y = x^2$

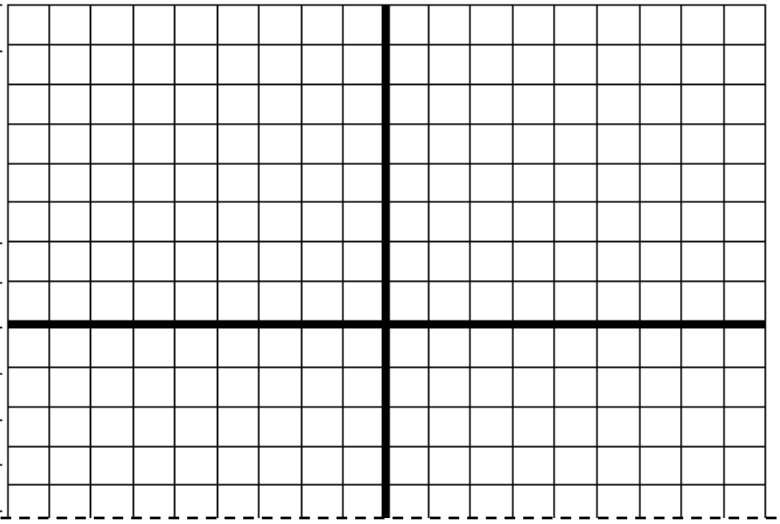
### Parabola Investigation

| Basic Equation | $y = x^2$ |
|---|---|

Vertex Form $y = a(x – h)^2+k$

Change values for a
keep h=0 and k=0 for now

| values | Equations | Colour |
|---|---|---|
| a=2 | | |
| a=0.2 | | |
| a=-1 | | |
| a=-2 | | |
| a=-0.2 | | |



**NOTES:**

**What effect does changing *"a"* have on the graph of $y = x^2$**

1. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = 1/3x^2$ b. $y = – 5x^2$ c. $y = – 0.001x^2$

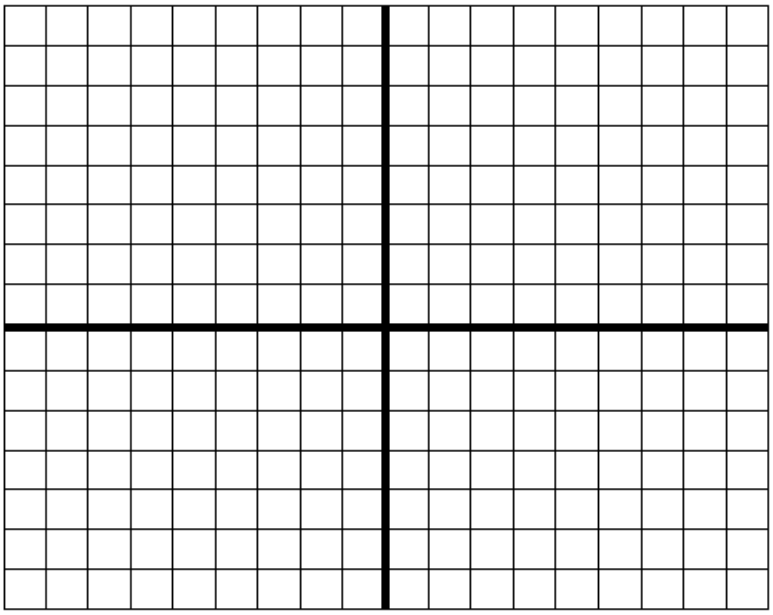
| Basic Equation | $y = x^2$ |
|---|---|

Vertex Form $y = a(x - h)^2 + k$

Change values for **k**
keep a=1 and h=0 for now

| values | Equations | Colour |
|---|---|---|
| k=4 | | |
| k=-6 | | |

What effect does changing $k$ have on the graph of $y = x^2$ ?



2. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = 2x^2 - 9$ b. $y = -0.5x^2 - 16$ c. $y = -3x^2 + 9$

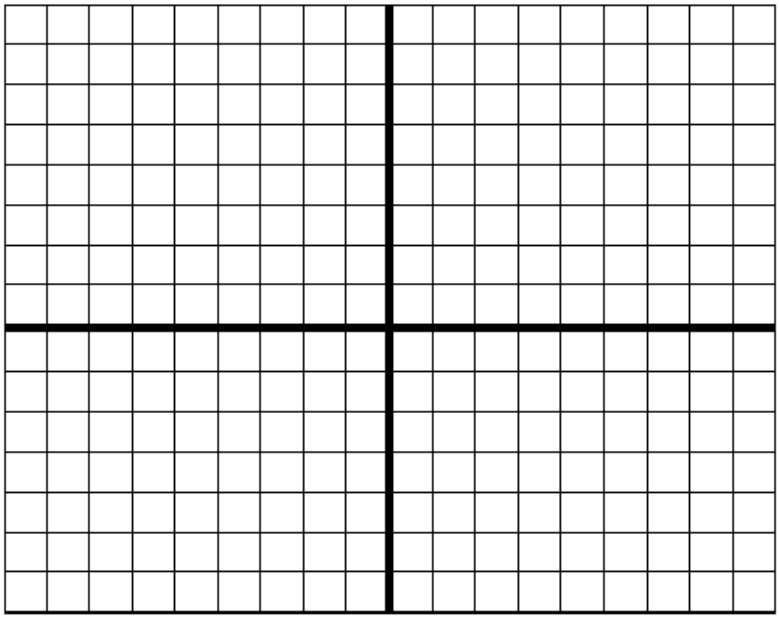
| Basic Equation | $y = x^2$ |
|---|---|

Vertex Form $y = a(x - h)^2 + k$

Change values for **h**
keep a=1 and k=0 for now

| values | Equations | Colour |
|---|---|---|
| h=3 | | |
| h=-5 | | |

What effect does changing $h$ have on the graph of $y = x^2$



3. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = (x+2)^2$ b. $y = (x - 4)^2 - 7$ c. $y = -(x+4)^2 + 3$ d. $y = 2(x - 1)^2$

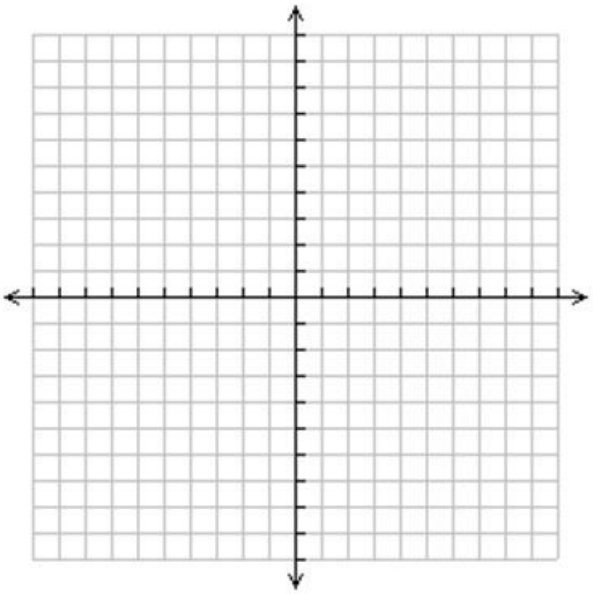
**1.**

$y = -2(x+4)^2 + 8$

**i)** state the coordinates of the vertex

**ii)** state all the transformations

**iii)** sketch the graph on grid (show step by step transformations)



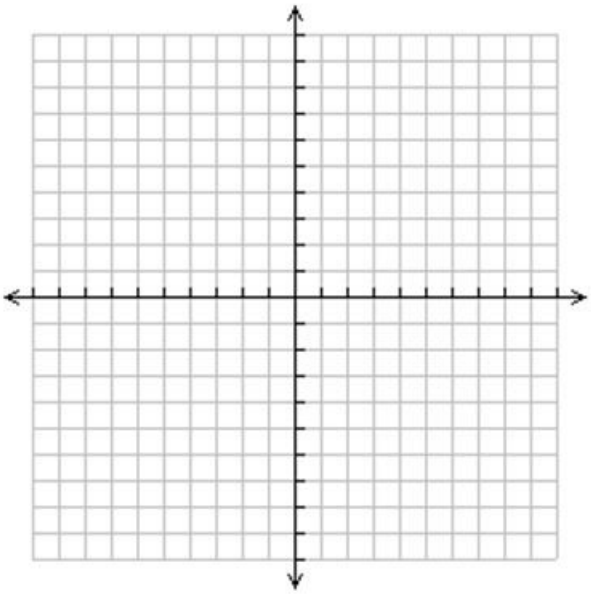
**2.**

$y = 0.5(x-2)^2 - 6$

**i)** state the coordinates of the vertex

**ii)** state all the transformations

**iii)** sketch the graph on grid (show step by step transformations)



**NOTES:**

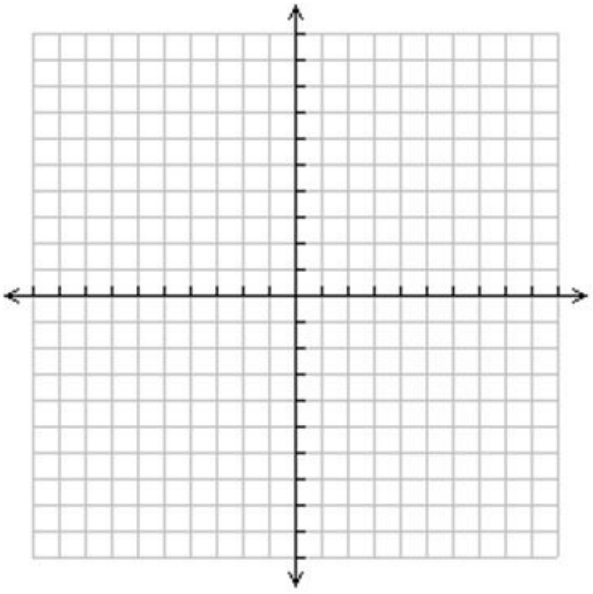
**3.**

$y = -(x-6)^2 + 2$

**i)** state the coordinates of the vertex

**ii)** state all the transformations

**iii)** sketch the graph on grid (can use step pattern)



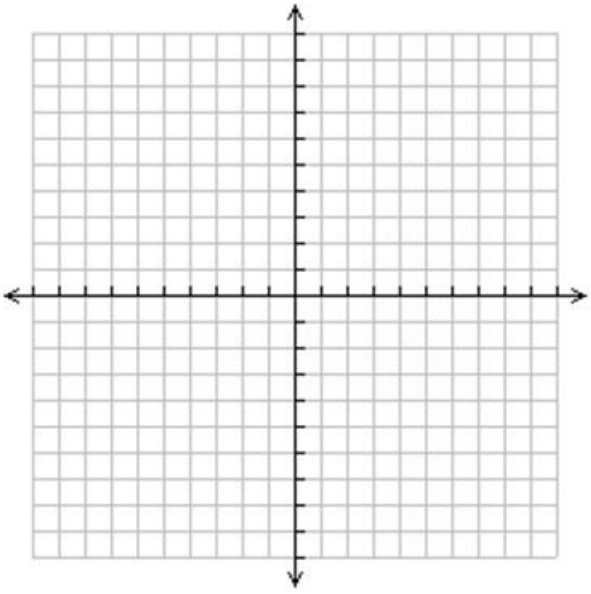
**4.**

$y = 1.5(x+5)^2 - 4$

**i)** state the coordinates of the vertex

**ii)** state all the transformations

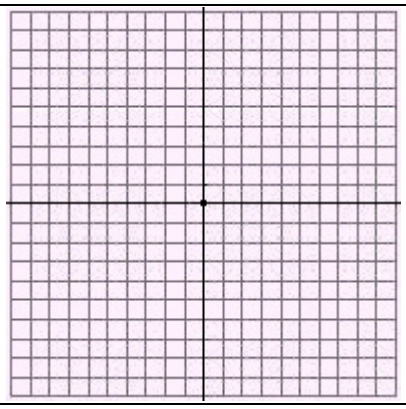
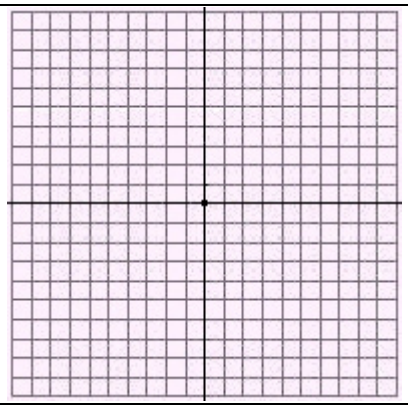
**iii)** sketch the graph on grid (can use step pattern)



**NOTES:**

## DAY 7 – Vertex Form $y=a(x-h)^2+k$

1. Complete the table for each parabola.

| **Property** | $y = (x-3)^2 - 2$ | $y = -2(x+4)^2 + 3$ |
|---|---|---|
| Vertex | | |
| stretch or compression & direction of opening | | |
| values that $x$ may take | | |
| values that $y$ may take | | |
| step pattern | | |
| sketch |  |  |

2. Write an equation for the parabola that satisfies each set of conditions.

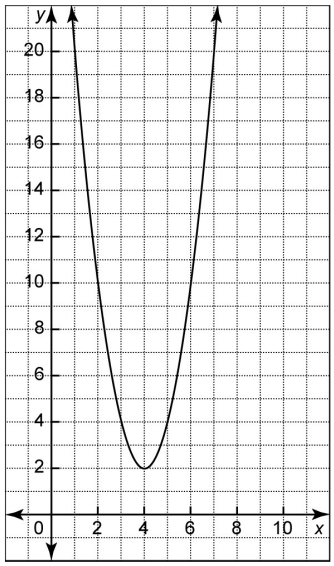
**a)** vertex $(-2, -4)$, opening downward with a vertical stretch

**b)** The graph of $y = x^2$ is translated 9 units downward, translated 10 units to the left, reflected in the $x$-axis and compressed vertically

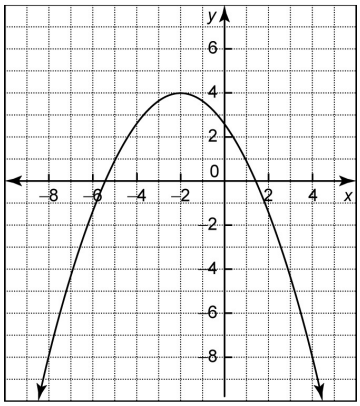
3. A parabola $y = ax^2 + k$ passes through the points $(1, 5)$ and $(3, 29)$. Find the values of $a$ and $k$.

4. Write an equation for each parabola.

**a)**




**b)**




5. Find an equation for the parabola with vertex (−3, 1) that passes through the point (−2, −1).

6. A rocket travels according to the equation $h = -4.9(t - 6)^2 + 182$, where $h$ is the height, in metres, above the ground and $t$ is the time, in seconds.

**a)** Sketch a graph of the rocket's motion.

**b)** Find the maximum height of the rocket.

**c)** How long does it take the rocket to reach its maximum height?

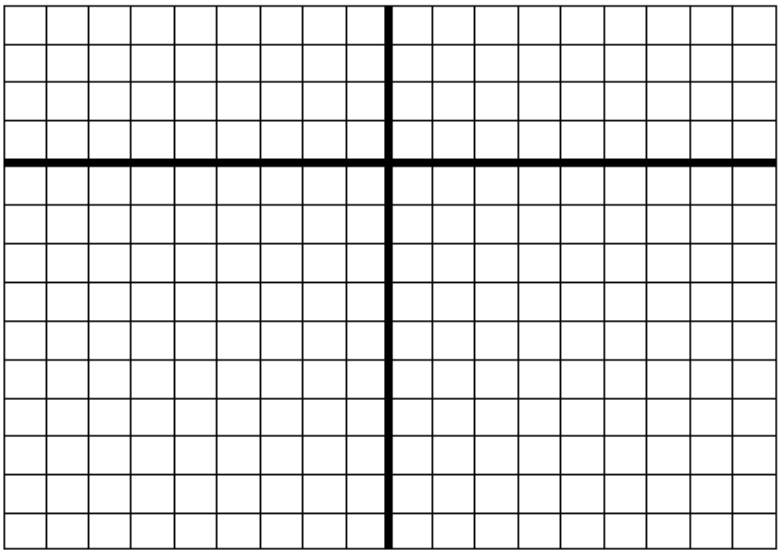
**d)** How high was the rocket above the ground when it was fired?

**NOTES:**

# DAY 8 & 9 & 10– Factored Form y = a(x-r)(x-t) and Standard Form $y = ax^2 + bx + c$

| 1. Table of Values | |
|---|---|
| x | y |
| -2 | 0 |
| -1 | -4 |
| 0 | -6 |
| 0.5 | -6.25 |
| 1 | -6 |
| 2 | -4 |
| 3 | 0 |

a. Plot the points and draw a curve of best fit



b. What is the y-intercept?

c. What is the direction of opening?

d. Is the vertex at max/min?

e. What are the zeros/x-int?

f. What is the axis of symmetry?

g. What is the Optimal Value?

h. What is the vertex?

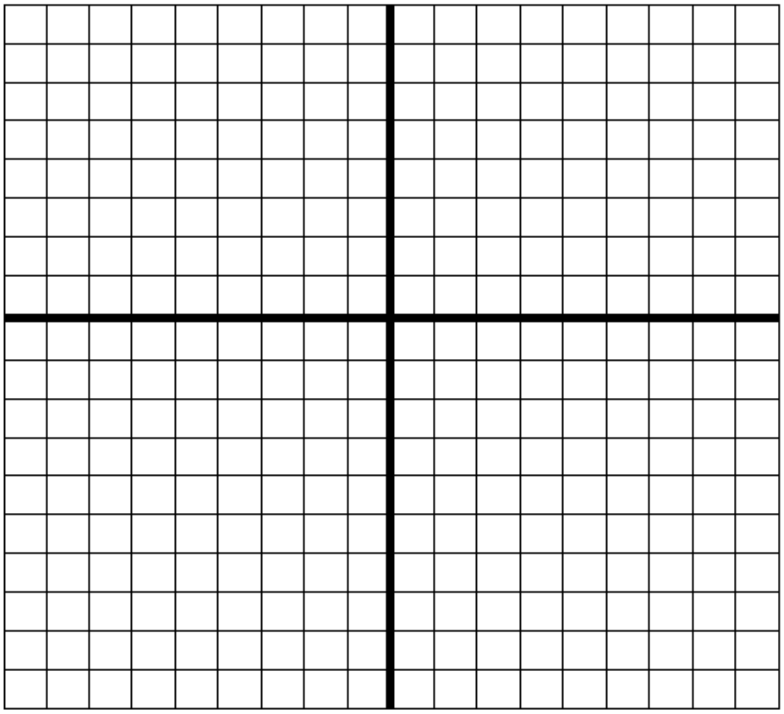
**2.** Check that BOTH of the following equations model this parabola

**Standard form** $y = x^2 - x - 6$

**Factored form** $y = (x - 3)(x + 2)$

| 3. Table of Values | |
|---|---|
| x | y |
| -4 | -10 |
| -3 | 0 |
| -2 | 6 |
| -1 | 8 |
| 0 | 6 |
| 1 | 0 |
| 2 | -10 |

a. Plot the points and draw a curve of best fit



b. What is the y-intercept?

c. What is the direction of opening?

d. Is the vertex at max/min?

e. What are the zeros/x-int?

f. What is the axis of symmetry?

g. What is the Optimal Value?

h. What is the vertex?

**4.** Check that BOTH of the following equations model this parabola

**Standard form** $y = -2x^2 - 4x + 6$

**Factored form** $y = -2(x - 1)(x + 3)$

**NOTES:**

- What part of equation will tell you direction of opening?
- What part of the equation will tell you the y-intercept?
- What part of the equation will tell you the zeros/x-int?
- How to get axis of symmetry from just zeros?
- How to get Max/Min optimal value?

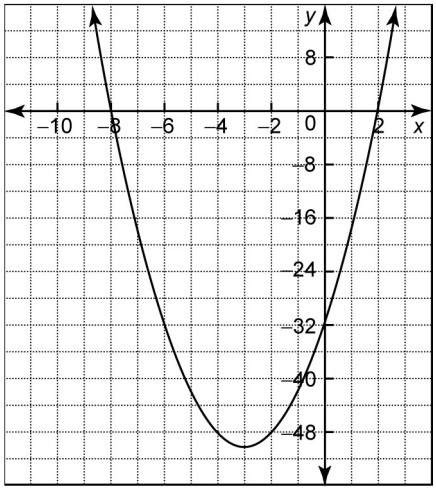
6. Sketch each parabola. Label the vertex and the $x$-intercepts.

**a)** $y = -\frac{1}{2}(x-3)(x-7)$

**b)** $y = 2(x-8)(2x+3)$

7. Determine an equation in factored form

**a)**




**b)** the parabola has one of the zeros at 2, axis of symmetry at -0.5 and goes through the point (4, -28)

**c)** the quadratic has only one zero at 3 and is reflected in the x-axis and has a vertical compression factor.

8. The path of a kicked football can be modelled by the relation $h = -0.02x(x - 45)$, where $h$ represents the height, in metres, above the ground and $x$ represents the horizontal distance, in metres, measured from the kicker.

**a)** Sketch the path of the ball.

**b)** When the ball hits the ground, how far has it travelled?

**c)** What is the maximum height of the ball?

**d)** What is the horizontal distance when this occurs?

**e)** If the goal post is 40 m away, will the kick clear the 3-m-high crossbar for a field goal?

For each of the following quadratic relations state the following:

a) the direction of opening, b) the zeros, c) the equation of the axis of symmetry,
d) the maximum or minimum value of y, e) the coordinates of the vertex, f) sketch using vertex and zeros

1. $y = -0.5(4 - 2x)(x + 5)$

2. $y = 3(2 + 3x)(2 - x)$

**NOTES:**

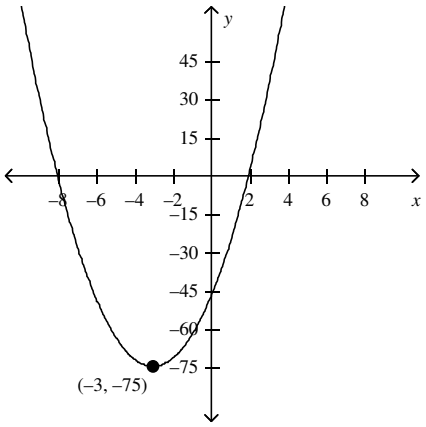
| | **Factored form** $y = (x - s)(x - t)$ | ***x*-intercepts** | **Standard Form** $y = ax^2 + bx + c$ | ***y*-intercept** |
|---|---|---|---|---|
| *3.* | *y = (2x + 3)(x - 1)* | | | |
| *4.* | | x = -4 and x = 3 | | 12 |

For each of the following graphs answer these questions.

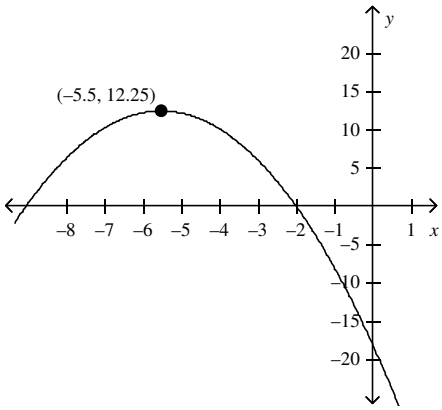
a) What is the maximum/minimum? b) When did the maximum/minimum occur? c) What are the zeros?

d) Find an equation to describe the graph. (Use factored and vertex form.)

5.




6.




| | *Standard form* | Factored form | x-intercepts | y-intercept |
|---|---|---|---|---|
| *7.* | $y = 2x^2 + 12x$ | | | |
| *8.* | $y = x^2 + 2x - 15$ | | | |