

Unit2JOURNAL

August 24, 2016 2:11 PM

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Unit2JOURNAL

UNIT 2 – Analytic Geometry JOURNAL



Big idea/Learning Goals

You will learn how mathematics can be used to represent distances, midpoints, and specific lines (such as bisectors, medians and tangents). You will use these concepts to find shortest distances and altitudes. Keep in mind that although there are few applications to real life in this unit, you must learn these basic concepts so that you can continue building upon them in your later studies. Think of this unit as a learning curve to be able to communicate well in the mathematical language. Also if you ever take the Calculus course later on, you'd learn how vectors help you solve shortest distance problems in a faster way.

Date	Topics	Finished the journal?	How many questions did you finish from HW?	Questions to ask the teacher:
2days	Midpoint & Slope DAY 1 HW text pg66 #1cd,3cd,4,6 DAY 2 HW text pg66 #12,15,19,20		/7 /13	<i>Tentative TEST date: Wed. Oct 26 Thurs. Oct 27</i>
	Distance/Length DAY 3 HW text pg77 #1bc,3bc,6,8,10,14,15		/15	
	Circles DAY 4 HW text pg97 #7,8,9,13,15,17		/17	
3days	Apply Slope, Midpoint & Length DAY 5 HW text pg89 #3,6,16,27 DAY 6 HW text pg89 #12,24 DAY 7 HW text pg89 # #20,21		/8 /3 /7	



Reflect – previous TEST mark _____, Overall mark now _____.

Calculate your potential final mark, show your calculations here:

$$\text{potential final mark} = (\text{overall mark now})(0.10) + (\text{future unit marks})(0.60) + (\text{future final exam marks})(0.30)$$

$$= (\quad)(0.10) + (\quad)(0.60) + (\quad)(0.30)$$

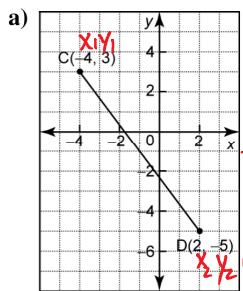
$$= \quad$$

Looking back, what can you improve upon? _____

S

DAY 1 & 2 – Midpoint & Slope of a Line Segment

1. Determine the coordinates of the midpoint of each



$$M =$$

b) $\left(-\frac{x_1}{2}, -\frac{y_1}{2} \right)$ and $\left(\frac{x_2}{2}, \frac{y_2}{2} \right)$

$$M = \left(\frac{-\frac{x_1}{2} + \frac{x_2}{2}}{2}, \frac{-\frac{y_1}{2} + \frac{y_2}{2}}{2} \right)$$

$$M = \left(\frac{\left(\frac{x_2}{2}\right)}{2}, \frac{\left(\frac{y_2}{2}\right)}{2} \right)$$

$$M = \left(\frac{\frac{x_2}{2}}{2}, 0 \right)$$

$$M = \left(\frac{x_2}{4}, 0 \right)$$

$$M = \left(\frac{1}{2}, 0 \right)$$

NOTES: pt. A (x_1, y_1) pt. B (x_2, y_2)

MIDPOINT Formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

How to find the OTHER endpoint?

2. The endpoints of the diameter of a circle are A(-5, -3) and B(3, 7). Find the coordinates of the centre of this circle.

3. One endpoint of a diameter of a circle centred at (3, -4) is (-5, 2). Find the coordinates of the other endpoint of this diameter.



$$(3, -4) = \left(\frac{-5+x}{2}, \frac{2+y}{2} \right)$$

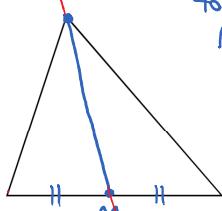
$$(2) 3 = \frac{-5+x}{2} \rightarrow (2) -4 = \frac{2+y}{2} \rightarrow \text{SAMPLE}$$

$$6 = -5 + x \quad -8 = 2 + y$$

$$11 = x \quad -10 = y \quad \therefore \text{other endpoint is } (11, -10)$$

NOTES: $y = mx + b$
MEDIAN line definition:

- join one vertex (corner of \triangle)
to the opposite side's
Midpoint.



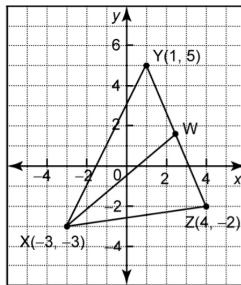
Action Plan:

1. Find midpoint of the side opposite the given vertex. $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$
2. Find slope between given vertex and midpoint found in step 1. $m = \frac{y_2 - y_1}{x_2 - x_1}$
3. Record $y = mx + b$, sub in "m" and any pt. on the line. Find b .
4. Rewrite the equation.

NOTES: point A (x_1, y_1) pt. B (x_2, y_2)
SLOPE Formula:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

5. Find the slope of the median shown.



$$m =$$

6. a) Draw $\triangle JKL$ with vertices $J(-6, 4)$, $K(-4, -5)$, and $L(6, 1)$.
b) Draw the median from vertex J . Then, find an equation in slope y-intercept form for this median.

$$\textcircled{c} \quad M_{KL} = \left(\frac{-4+6}{2}, \frac{-5+1}{2} \right)$$

$$\textcircled{1} \quad M_{KL} = \left(\frac{2}{2}, \frac{-4}{2} \right)$$

$$\textcircled{2} \quad m_{JM} = \frac{-2 - 4}{1 - (-6)}$$

$$m_{JM} = \frac{-6}{7}$$

$$\textcircled{3} \quad y = mx + b$$

$$\text{sub in } m = \frac{-6}{7} \text{ pt. } M(1, -2)$$

$$-2 = \left(\frac{-6}{7}(1) \right) + B$$

$$-2 = -\frac{6}{7} + B$$

$$-\frac{2}{1} + \frac{6}{7} = B$$

$$-\frac{8}{7} = B$$

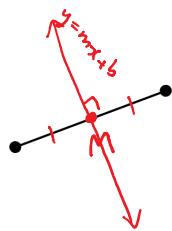
$$\textcircled{4} \quad \therefore \text{the median line from } J \text{ is } y = \frac{-6}{7}x - \frac{8}{7}$$

↳ "perpendicular"

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NOTES:
PERPENDICULAR BISECTOR line definition:



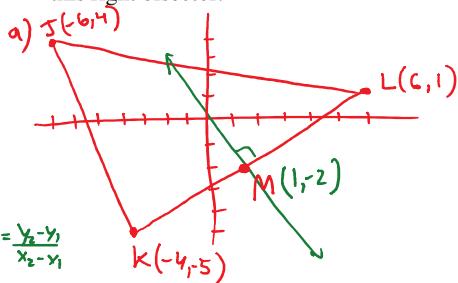
- a line that cuts the given line segment in half at 90° .

Action Plan:

- ① Find Midpoint of given line segment
 $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$
- ② Find slope of same line segment
- ③ Find \perp slope (negative reciprocal)
- ④ Record $y = mx + b$ and sub in the \perp slope AND pt. ON the line, the mid point and find b
- ⑤ Rewrite equation.

8. Find the perpendicular bisector equation of line segment AB if A(-4, 5) and B(2, -3)

7. a) Draw $\triangle JKL$ with vertices $J(-6, 4)$, $K(-4, -5)$, and $L(6, 1)$.
b) Draw the right bisector of KL . Then, find an equation in slope y-intercept form for this right bisector.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{b) } M_{KL} = \left(\frac{-4+6}{2}, \frac{-5+1}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{-4}{2} \right)$$

$$\text{c) } m_{KL} = \frac{1 - (-5)}{6 - (-4)}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$\text{d) } m_{\perp} = -\frac{5}{3}$$

$$\text{e) } y = mx + b \quad \text{sub } m = -\frac{5}{3} \text{ and } M(1, -2)$$

$$-2 = -\frac{5}{3}(1) + b$$

$$-2 = -\frac{5}{3} + b$$

$$+\frac{5}{3} \quad \nearrow$$

$$-\frac{2}{3} + \frac{5}{3} = b$$

$$-\frac{1}{3} = b$$

f) $\therefore \perp$ bisector equation is

$$y = -\frac{5}{3}x + -\frac{1}{3}$$

NOTES:Parallel lines have the same slope

$$m_1 = \frac{2}{3}$$

$$m_2 = \frac{2}{3}$$

Vertical lines:

$$x = \#$$

$$x = -4$$

have undefined slope

Perpendicular lines have negative reciprocal slopes

$$m_1 = -\frac{5}{7}$$

$$m_2 = \frac{1}{5}$$

switch sign

flip fraction

Horizontal lines:

have \neq slope

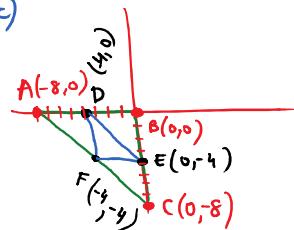
$$y = 0x + \#$$

$$y = \#$$

$$y = 3$$

10. a) Draw $\triangle ABC$ with vertices A(-8, 0), B(0, 0), and C(0, -8).
- b) Construct the midpoints of AB, BC, and AC and label them D, E, and F, respectively. (in same order)
- c) Join the midpoints to form $\triangle DEF$.
- d) Show that the line segment DE is parallel to the line segment AC.

a) b) c)



11. Find the median equation line from vertex A(-4, -6) to base BC where B(-3, 1) and C(6, 0)

d) For parallel need
 m_{AC} to be same as m_{DE}

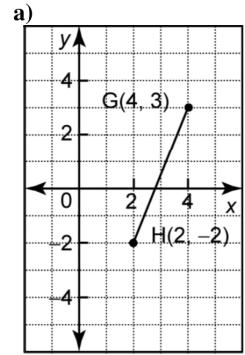
$$m_{AC} = \frac{0 - (-8)}{-8 - 0} = \frac{8}{-8} = -1$$

$$m_{DE} = \frac{-4 - 0}{0 - (-4)} = \frac{-4}{4} = -1$$

Since (\because) the slopes are the same
therefore (\therefore) lines AC and DE are parallel.

DAY 3 – Distance or Length of a Line Segment

1. Calculate the length of the line segment defined by each pair of endpoints.



$$D = \sqrt{ }$$

NOTES:

DISTANCE Formula: or Length $A(x_1, y_1) \quad B(x_2, y_2)$

$$D = L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

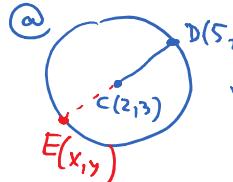
b) $\left(\frac{-3}{4}, \frac{-2}{5}\right)$ and $\left(\frac{1}{4}, \frac{3}{5}\right)$

$$D = \sqrt{\left(\frac{-3}{4} - \frac{1}{4}\right)^2 + \left(\frac{-2}{5} - \frac{3}{5}\right)^2}$$

$$= \sqrt{\left(\frac{-4}{4}\right)^2 + \left(\frac{-5}{5}\right)^2}$$

$$= \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2} \approx 1.4$$

2. The endpoint of a radius of a circle with centre C(2, 3) is D(5, 5). Determine
 a) the length of the diameter of the circle
 b) the coordinates of the endpoint E of the diameter DE of the circle



$$\text{radius} = L_{CD} = \sqrt{(2-5)^2 + (3-5)^2} \\ = \sqrt{(-3)^2 + (-2)^2} \\ = \sqrt{9+4} \\ = \sqrt{13}$$

$$\therefore \text{diameter} = 2 \times \text{radius} \\ = 2\sqrt{13} \quad \cancel{\text{not } \sqrt{26}}$$

b) $M_{ED} = \left(\frac{+}{2}, \frac{+}{2} \right)$

$$(2, 3) = \left(\frac{5+x}{2}, \frac{5+y}{2} \right)$$

p+t C is middle

separate:

$$2(2) = \left(\frac{5+x}{2} \right) \times 2 \quad \text{and} \quad 2(3) = \left(\frac{5+y}{2} \right) \times 2$$

$$4 = 5+x$$

$$-1 = x$$

$$6 = 5+y$$

$$1 = y$$

$$\therefore \text{pt. E is } (-1, 1)$$

4. Describe what you would check, what formula(s) you would use to CLASSIFY these shapes

Quadrilaterals - 4 sided shape

PARALLELGRAM - use $m = \frac{y_2 - y_1}{x_2 - x_1}$ and show 2 pairs of opposite sides are the same slope.

RECTANGLE
look at slopes already found and see if 2 adjacent (beside) sides are negative reciprocal slopes.

Square
- use
 $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to show ALL sides are equal.

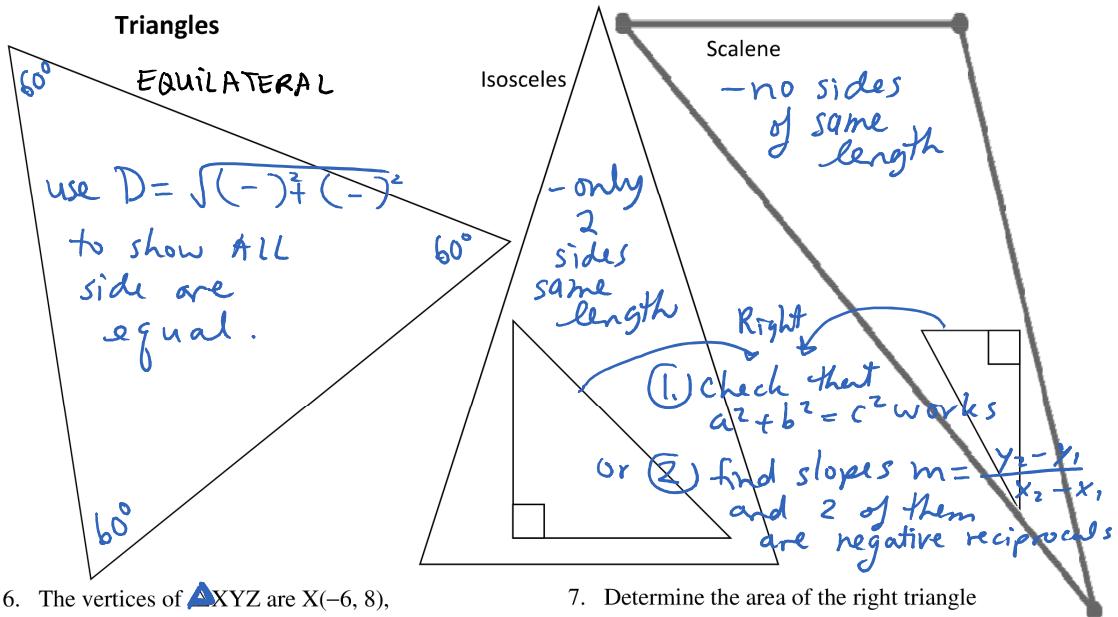
Rhombus
X use
 $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to show ALL sides are equal.
not right angle.

Trapezoid
use $m = \frac{y_2 - y_1}{x_2 - x_1}$ and show ^{ONLY} 2 opposite sides are same slope

Kite
use
 $D = \sqrt{(\)^2 + (\)^2}$ to show 2 adjacent sides are same length

Rhombus
use lengths found + see if ALL are the same.

5. Describe what you would check, what formula(s) you would use to CLASSIFY these shapes



6. The vertices of $\triangle XYZ$ are $X(-6, 8)$, $Y(-2, -4)$, and $Z(4, 6)$.

- a) Determine the exact length of each side of this triangle.

- b) Classify the triangle.

$$\begin{aligned} D_{xy} &= \sqrt{(-6 - -2)^2 + (8 - -4)^2} \\ &= \sqrt{(-4)^2 + (12)^2} \\ &= \sqrt{16 + 144} \\ &= \sqrt{160} \end{aligned}$$

$$\begin{aligned} D_{yz} &= \sqrt{(-2 - 4)^2 + (-4 - 6)^2} \\ &= \sqrt{(-6)^2 + (-10)^2} \\ &= \sqrt{36 + 100} \\ &= \sqrt{136} \end{aligned}$$

$$\begin{aligned} D_{xz} &= \sqrt{(-6 - 4)^2 + (8 - 6)^2} \\ &= \sqrt{(-10)^2 + 2^2} \\ &= \sqrt{100 + 4} \\ &= \sqrt{104} \end{aligned}$$

\therefore this Δ is right?
scalene \nwarrow non-right?

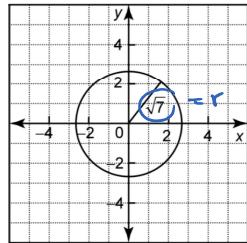
DAY 4 – Circles**NOTES:**

EQUATION of a CIRCLE: centered at origin
 $x^2 + y^2 = r^2$
 where $r = \text{radius}$
 $(x,y) = \text{general pt. on circle.}$

Point inside/on/outside:



1. Determine an equation for the circle.



3. State the radius of the circle defined by the equation and give the coordinates of one point on the circle
- $x^2 + y^2 = 1.44$

2. Determine whether each point is on, inside, or outside the circle defined by
- $x^2 + y^2 = 26$
- .

- a) (1, 3) b) (-4, 6) c) (1, 5)

$$\begin{array}{r} x^2 + y^2 = 26 \\ (1)^2 + (3)^2 \end{array}$$

$$\begin{array}{r} 1+9 \\ 10 \\ 10 = 26 \end{array}$$

$\therefore (1,3)$ is inside the circle

$$\begin{array}{r} x^2 + y^2 = 26 \\ (-4)^2 + (6)^2 \end{array}$$

$$\begin{array}{r} 16+36 \\ 52 \\ 52 \leq 26 \end{array}$$

$\therefore (-4,6)$ is outside the circle.

$$\begin{array}{r} x^2 + y^2 = 26 \\ 1^2 + 5^2 \end{array}$$

$$\begin{array}{r} 1+25 \\ 26 \\ 26 = 26 \end{array}$$

$\therefore (1,5)$ is on the circle's edge.

4. The point A(4, b) lies on the circle defined by
- $x^2 + y^2 = 25$
- .
- $\Rightarrow r=5$

- a) Find the possible value(s) of b.

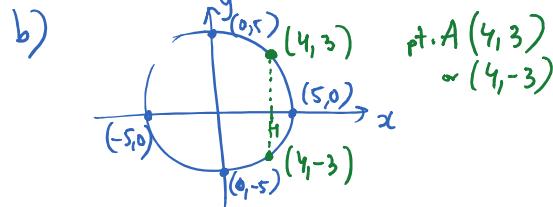
- b) Use a graph to show that the point(s) corresponding to the possible value(s) of b are on the circle.

$$(4)^2 + (b)^2 = 25$$

$$b^2 = 25 - 16$$

$$\sqrt{b^2} = \pm\sqrt{9}$$

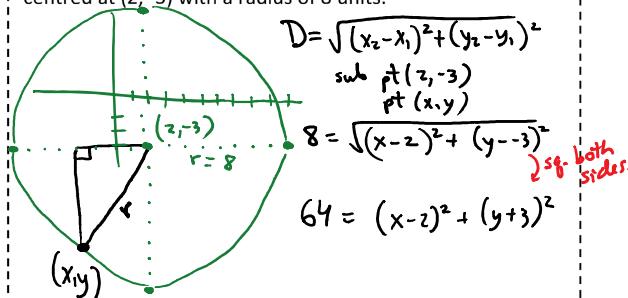
$$b = 3 \text{ or } b = -3$$



\therefore two y-values for single value $x=4$

NOTES:

Suppose the circle is NOT centred at the origin. Develop the equation of a circle using the length formula for a circle centred at (2, -3) with a radius of 8 units.



$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{sub pt}(2, -3) \quad \text{pt}(x, y)$$

$$8 = \sqrt{(x-2)^2 + (y+3)^2}$$

sq both sides.

$$64 = (x-2)^2 + (y+3)^2$$

\therefore in general if centre (h, k)
 radius = r

$$(x-h)^2 + (y-k)^2 = r^2$$

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5. Determine an equation for the circle that has a diameter with endpoints B(-4, 7) and C(4, -7).

Method 1

$$\text{find } \frac{\text{diameter}}{2} = \text{radius}$$

$$D_{BC} = \sqrt{(4 - (-4))^2 + (-7 - 7)^2} \\ = \sqrt{8^2 + (-14)^2} \\ = \sqrt{64 + 196} \\ = \sqrt{260}$$

$$x^2 + y^2 = r^2 \quad \text{want } r$$

Method 2

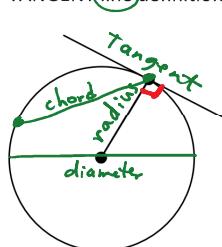
sub any pt. into

$$x^2 + y^2 = r^2 \quad \text{sub B.} \\ (-4)^2 + 7^2 = r^2 \\ 16 + 49 = r^2 \\ 65 = r^2$$

$$\therefore \text{equation is } x^2 + y^2 = 65$$

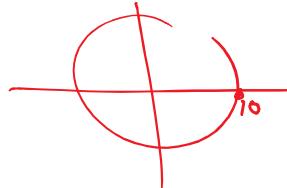
NOTES:

TANGENT line definition:



- a line to the radius at the edge of the circle.
- touches circle in one spot.

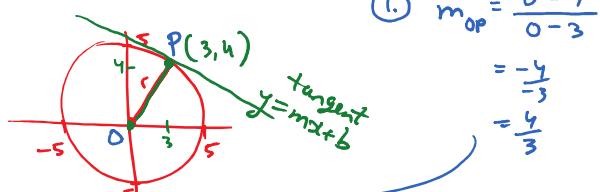
6. a) Graph the circle defined by $x^2 + y^2 = 100$.
 b) Verify algebraically that the point D(6, -8) lies on this circle.
 c) Determine an equation for the tangent line DO.



Action Plan:

- ① Find slope of radius $m = \frac{y_2 - y_1}{x_2 - x_1}$
- ② Find the slope of tangent (neg. recip)
- ③ Use $y = mx + b$ and sub in the slope and ANY pt. on tangent line, to find " b "
- ④ Rewrite equation $y = \underline{\hspace{2cm}} x + \underline{\hspace{2cm}}$

7. Find tangent equation line at point (3, 4) of circle $x^2 + y^2 = 25$



$$\textcircled{1} \quad m_{OP} = \frac{0 - 4}{0 - 3} \\ = -\frac{4}{3} \\ = \frac{4}{3}$$

$$\textcircled{2} \quad m_T = -\frac{3}{4}$$

$$\textcircled{3} \quad y = mx + b \quad \text{sub } m = -\frac{3}{4}, P(3, 4)$$

$$4 = \left(-\frac{3}{4}\right)(3) + b$$

$$\frac{4}{1} + \frac{9}{4} = b$$

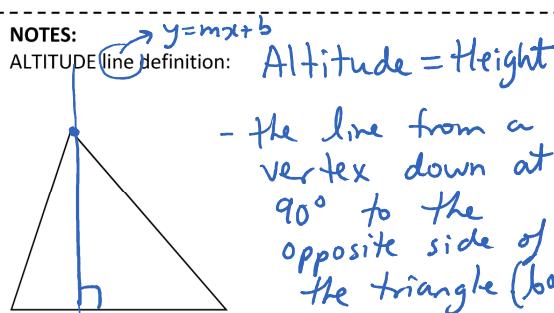
$$\frac{25}{4} = b$$

$$\therefore \text{equation of tangent line} \\ y = -\frac{3}{4}x + \frac{25}{4}$$

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DAY 5 & 6 & 7 – Apply Slope, Midpoint & Length Formulas**NOTES:**

ALTITUDE line Definition:

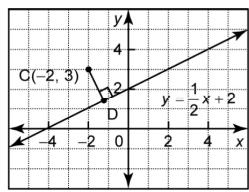

 $y = mx + b$
Altitude = Height

- the line from a vertex down at 90° to the opposite side of the triangle (base)

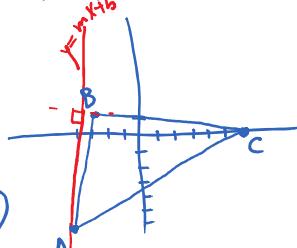
Action Plan:

- ① Find slope of the base (side across given vertex)
- ② Find \perp slope of the altitude line
- ③ use $y = mx + b$, sub m and given vertex to find b
- ④ Rewrite the equation.

2. Find an equation for the line containing line segment CD.



1. Find the altitude equation line from vertex A(-4, -6) to base BC where B(-3, 1) and C(6, 0)



$$\text{① } m_{BC} = \frac{1-0}{-3-6} \\ = \frac{1}{-9}$$

$$\text{② } m_{\perp} = \frac{9}{1}$$

$$\text{③ } y = mx + b \quad \text{sub } m = 9 \text{ pt. } A(-4, -6) \\ -6 = 9(-4) + b \\ -6 = -36 + b \\ +36 \cancel{+36} \\ 30 = b$$

∴ equation of the altitude is
 $y = 9x + 30$

3. Find whether the point A(2, -5) lies on the right bisector of the line segment of P(4, 2) and Q(-4, -6)

h

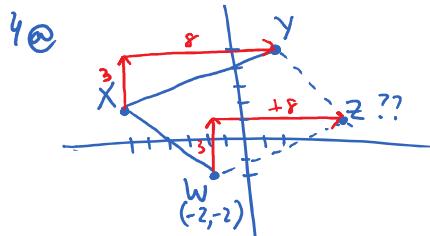
Plan (1) Find equation of the bisector of PQ.
 (2) sub in pt. A to see if LS = RS

Plan 4C

Find Midpoints of diagonals
 - if it's the same pt. that will mean bisected (cut in half)

4. The points W(-2, -2), X(-6, 2), and Y(2, 5) are three vertices of parallelogram WXYZ.

- a) Find the coordinates of vertex Z.
 b) Find the length of the diagonals XZ and WY.
 c) Show that the diagonals XZ and WY bisect each other. → Plan.



$$m_{XY} = \frac{2-5}{-6-2}$$

$$= \frac{-3}{-8}$$

$$= \frac{3}{8}$$

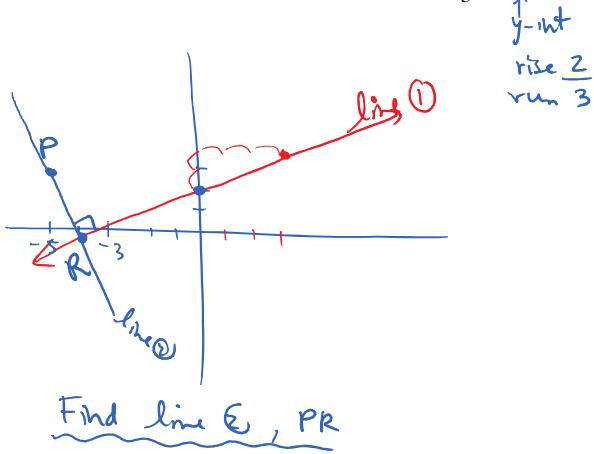
this means from pt. X rise 3, run 8 to get to pt. Y

rise 3 from -2 = 1
 run 8 from -2 = 6
 \therefore pt. Z (6, 1)

can't reduce slope for this method.

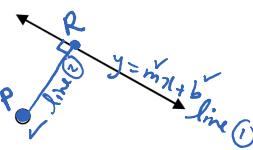
5. Determine the shortest distance from

the point $P(-5, 3)$ to the line $y = \frac{2}{3}x + 2$



NOTES:

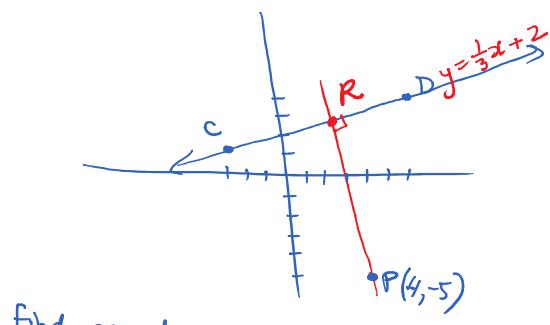
SHORTEST DISTANCE:



Action Plan:

- ① Find the equation of the 2nd perp line by
 - find m_{perp} from the given line
 - find b from $y = m_1x + b$ and substituting in a point on 2nd line.
- ② Find P.O.I. from two equations by substitution/elimination method.
- ③ Find distance between P.O.I and given point $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

6. Determine the shortest distance from the point $P(4, -5)$ to the line joining $C(-3, 1)$ and $D(6, 4)$



Find equation of line PR

$$m_b = -3$$

$$y = mx + b \text{ sub } P(4, -5), m = -3 \\ -5 = (-3)(4) + b$$

$$-5 = -12 + b$$

$$\boxed{7 = b}$$

∴ line PR is $y = -3x + 7$

Find POI of $y = \frac{1}{3}x + 2$ ①
 $y = -3x + 7$ ②

sub ① into ②

~~$\frac{1}{3}x + 2 = -3x + 7$~~

$$1x + b = -9x + 21$$

$$10x = 15$$

$$x = 1.5 \quad \text{pt. } R(1.5, ?)$$

Find equation of line CD

$$m_{CD} = \frac{4-1}{6-(-3)} \\ = \frac{3}{9} \\ = \frac{1}{3}$$

$$y = mx + b \text{ sub } C(-3, 1), m = \frac{1}{3}$$

$$1 = \left(\frac{1}{3}\right)(-3) + b$$

$$1 = -1 + b \\ 2 = b$$

∴ line CD is $y = \frac{1}{3}x + 2$

→ sub $x = 1.5$ into ②

$$y = -3(1.5) + 7$$

$$y = -4.5 + 7$$

$$y = 2.5$$

∴ POI or pt. R $(1.5, 2.5)$
 $\text{P } (4, -5)$

Find distance between P and R

$$D_{PR} = \sqrt{(4-1.5)^2 + (-5-2.5)^2}$$

$$= \sqrt{(2.5)^2 + (-7.5)^2}$$

$$= \sqrt{6.25 + 56.25}$$

$$= \sqrt{62.5}$$

≈ 7.9 is the shortest distance.

$$D_{HP} = \sqrt{ }$$

7. A triangle has vertices G(-5, -4), H(-1, 8), and I(3, -6)

- a) Find the equation of the altitude from H.
- b) Find the length of the altitude
- c) Find the area of this triangle.

a) Find altitude $y = mx + b$ from H.

$$\begin{aligned} m_{GI} &= \frac{-6 - (-4)}{3 - (-5)} \\ &= \frac{-2}{8} \\ &= -\frac{1}{4} \end{aligned}$$

$$m_H = 4$$

$$\begin{aligned} y &= mx + b \quad \text{sub } m = 4, H(-1, 8) \\ 8 &= (4)(-1) + b \\ 8 &= -4 + b \\ 12 &= b \end{aligned}$$

\therefore equation of altitude is $y = 4x + 12$

b) Find $y = mx + b$ for line ①

sub $m = -\frac{1}{4}$ I(3, -6)

$$\begin{aligned} -6 &= \left(-\frac{1}{4}\right)(3) + b \\ -6 &= -\frac{3}{4} + b \\ -6 + \frac{3}{4} &= b \\ -\frac{21}{4} &= b \end{aligned}$$

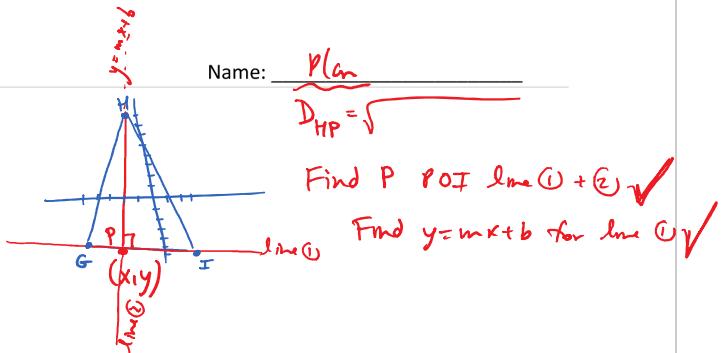
\therefore line ① equation $y = -\frac{1}{4}x - \frac{21}{4}$

c) $A_{\Delta} = \frac{BH}{2}$ or $\frac{1}{2}BH$

$$A = \frac{1}{2} \left(\frac{\sqrt{68}}{1} \right) \left(\frac{\sqrt{45968}}{17} \right)$$

$$A = \frac{\sqrt{3125824}}{34} = \frac{1768}{34}$$

$\therefore A = 52 \text{ units}^2$



Find POI of line ① + ② ✓

Find $y = mx + b$ for line ② ✓

Find POI of two lines

$$\begin{aligned} ① \quad y &= -\frac{1}{4}x - \frac{21}{4} \\ ② \quad y &= 4x + 12 \end{aligned}$$

sub ① into ②

$$-\frac{1}{4}x - \frac{21}{4} = 4x + 12$$

$$-x - 21 = 16x + 48$$

$$-17x = 69$$

$$x = -\frac{69}{17}$$

$$\text{sub } ② \quad y = 4\left(-\frac{69}{17}\right) + 12$$

$$y = -\frac{276}{17} + \frac{12 \cdot 17}{17}$$

$$y = -\frac{32}{17}$$

$$\therefore \text{POI is } \left(-\frac{69}{17}, -\frac{32}{17}\right) = \text{pt. P}$$

Find length HP

$$D_{HP} = \sqrt{\left(\frac{-69}{17} - (-1)\right)^2 + \left(\frac{-32}{17} - \frac{8}{17}\right)^2}$$

$$= \sqrt{\left(\frac{-52}{17}\right)^2 + \left(\frac{-208}{17}\right)^2}$$

$$= \sqrt{\frac{2704}{289} + \frac{43264}{289}}$$

$$= \sqrt{\frac{45968}{289}} \div 12.6$$

\therefore length of altitude is $\frac{\sqrt{45968}}{17}$

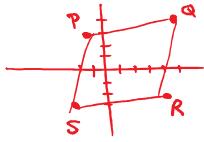
$$D_{GI} = \sqrt{(-5 - 3)^2 + (-4 - (-6))^2}$$

$$= \sqrt{(-8)^2 + 2^2}$$

$$= \sqrt{64 + 4}$$

$$= \sqrt{68}$$

8. A quadrilateral has vertices P(-1, 3), Q(5, 4), R(4, -2), and S(-2, -3). What type of quadrilateral is PQRS? Explain.



$$\begin{aligned} m_{PA} &= \frac{3-4}{-1-5} & \xleftarrow{\text{same?}} m_{SR} &= \frac{-3-2}{-2-4} \\ &= \frac{-1}{-6} & &= \frac{1}{6} \\ &= \frac{1}{6} & & \end{aligned}$$

$$\begin{aligned} m_{RS} &= \frac{3-(-3)}{-1-(-2)} & \xleftarrow{\text{same?}} m_{QR} &= \frac{4-(-2)}{5-4} \\ &= \frac{6}{1} & &= \frac{6}{1} \end{aligned}$$

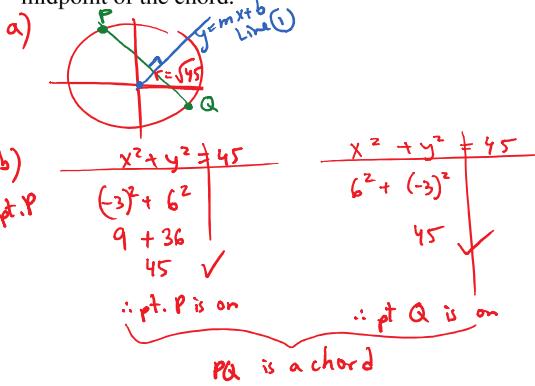
It is a parallelogram since
2 pairs of opposite sides are same slope.
It is not a rectangle since adjacent
slopes are not negative reciprocals.

BUT it is a rhombus?

$$\begin{aligned} D_{PS} &= \sqrt{(-1-2)^2 + (3-(-3))^2} & \xleftarrow{\text{same?}} D_{PA} &= \sqrt{(-1-5)^2 + (3-4)^2} \\ &= \sqrt{1^2 + 6^2} & &= \sqrt{(-6)^2 + (-1)^2} \\ &= \sqrt{1+36} & &= \sqrt{37} \\ &= \sqrt{37} & & \end{aligned}$$

∴ it is a rhombus.

9. a) Graph the circle defined by $x^2 + y^2 = 45$.
b) Verify algebraically that the line segment joining P(-3, 6) and Q(6, -3) is a chord of this circle.
c) Find an equation of the line that passes through the origin and is perpendicular to the chord PQ.
d) Verify that this line passes through the midpoint of the chord.



$$\begin{aligned} m_{PQ} &= \frac{-3-6}{-6-3} \\ &= \frac{-9}{-9} \\ &= -1 \\ m_{\perp} &= 1 \end{aligned}$$

$$\begin{aligned} b &= 0 \quad \text{since origin} \\ \therefore \text{line ① is } y &= 1x + 0 \text{ i.e. } y = x \end{aligned}$$

d) $M_{PA} = \left(\frac{-3+6}{2}, \frac{6+3}{2} \right)$

$$\begin{aligned} &= \left(\frac{3}{2}, \frac{9}{2} \right) \\ \text{sub } &\quad LS = RS \\ &\quad y = x \\ &\quad \frac{3}{2} = \frac{9}{2} \quad \checkmark \end{aligned}$$

∴ Midpoint is on line ①