

## Problem-Solving: Identifying Methods

When solving problems, very often you need to examine carefully the given information and look for clues. In this chapter you have acquired various methods of finding the factors of algebraic expressions. Look for clues to identify the method needed to factor the expressions. For example, the expression

$$m^2 + 6m + 9 - 4n^2$$

appears in a form to which you cannot apply any methods that you know. The square term  $-4n^2$  may suggest a difference of squares. Thus, you write

$$(m^2 + 6m + 9) - 4n^2 = (m + 3)^2 - (2n)^2$$

$$= (m + 3 - 2n)(m + 3 + 2n)$$

Now you can apply the difference of squares method to factor the expression.

In the following exercise, look for clues to help you identify the method you need to use. Look out for expressions that can not be factored.

### Exercise

- |               |                            |               |                          |    |                    |
|---------------|----------------------------|---------------|--------------------------|----|--------------------|
| 1             | $4x^3 + x^2$               | 2             | $3x - 9xy$               | 3  | $6x^4 - 12x$       |
| 4             | $9x^4 - 16y^2$             | 5             | $x^2 + 9x + 20$          | 6  | $5x^2 - 5y^2$      |
| 7             | $y^2 - 13y + 42$           | 8             | $6x^2 - 13x - 5$         | 9  | $x^2 + xy - 12y^2$ |
| 10            | $9x^2 + 27x + 8$           | 11            | $-4 + 25x^2$             | 12 | $y^3 - y - 6$      |
| 13            | $3y^2 - 9y^3$              | 14            | $5 + 6x + x^2$           | 15 | $18x^2 - 25x - 3$  |
| 16            | $9x^2 + 1$                 | 17            | $6x^2 - 28x - 10$        | 18 | $x^4 - 64$         |
| <del>19</del> | $m^4 + 3m^2 + 4$           | 20            | $-(9x^2 - y^4)$          |    |                    |
| 21            | $x^4 - 3x^2 - 4$           | 22            | $10x^2 + 7x + 1$         |    |                    |
| 23            | $4x^6 - y^6$               | 24            | $x^2 + 4x - 21$          |    |                    |
| 25            | $2x^4 - 3x^2 - 2$          | <del>26</del> | $y^4 + 2y^2 + 9$         |    |                    |
| 27            | $2x^2 - 4xy + 8x$          | 28            | $x^2 - 121$              |    |                    |
| 29            | $2x^2 - 2x - 28$           | 30            | $(2x + y)^2 - z^2$       |    |                    |
| 31            | $m^4 - 6m^2 - 27$          | <del>32</del> | $4y^4 - 16y^2 + 9$       |    |                    |
| 33            | $x^4 + 2x^2 - 15$          | <del>34</del> | $8y^3 + 1$               |    |                    |
| 35            | $9x^{10} - 4$              | 36            | $2y^2 + 24y + 40$        |    |                    |
| 37            | $5x^2 - 20$                | 38            | $3x^2 - 27x + 54$        |    |                    |
| 39            | $x^4 - 225y^2$             | 40            | $4x^2 - 28x - 32$        |    |                    |
| 41            | $x^6 - y^6$                | 42            | $3x^5 + 15x^3 + 12x$     |    |                    |
| 43            | $-75 + 12x^4$              | 44            | $3x^2yz^3 + 18xy^2$      |    |                    |
| 45            | $y^4 - 17y^2 + 16$         | 46            | $4(x - y)^2 - (x + y)^2$ |    |                    |
| 47            | $(x - y)^2 - 9(2x + y)^2$  | 48            | $m^2 + 6m + 9 - 4n^2$    |    |                    |
| 49            | $16y^2 - a^2 - 6ab - 9b^2$ | 50            | $x^3 - 2x^2 - 9x + 18$   |    |                    |

$$(c - x)(c + x)(c - x)^2 \cdot 05 \quad (qE + u + f^2)(qE - d - f^2) \cdot 6p$$

$$(u^2 + E + w)(u^2 - E + w) \cdot 8p$$

$$(4p - xE)(kE + xE) \cdot 7p \quad (f - xE)(kE - x) \cdot 9p$$

$$(1 - \Delta)(1 + \Delta)(4 - \Delta)(4 + \Delta) \cdot 5p \quad (6q + f^2x)(kE \cdot 7p$$

$$(5 + f^2)(5 - f^2)(E \cdot 7p \quad (1 + f^2)(4 + f^2)x \cdot 7p$$

$$(f^4 - f^2)(f^4 + f^2) \cdot 7p \quad (1 + x)(8 - x) \cdot 7p \cdot 0p$$

$$(451 - f^2)(451 + f^2) \cdot 6E \quad (E - x)(9 - x) \cdot 8E$$

$$(2 - x)(2 + x) \cdot 5E \quad (2 + \Delta)(01 + \Delta) \cdot 2 \cdot 9E$$

$$(2 + f^2)(2 - f^2) \cdot 5E \quad (1 + kE - f^2 \Delta)(1 + kE) \cdot 7E$$

$$(E - f^2)(E + f^2) \cdot 7E \quad (kE + E - f^2)(kE - E - f^2) \cdot 7E$$

$$(E + w)(E - w)(E + w) \cdot 7E \quad (z + f + xE)(z - f + xE) \cdot 0E$$

$$(41 - x - f^2)(2 \cdot 6E \quad (11 - x)(11 + x) \cdot 8E$$

$$(4 + kE - x) \cdot 7E \quad (kE + E + f^2 \Delta)(kE - E + f^2 \Delta) \cdot 9E$$

$$(2 - f^2)(1 + f^2) \cdot 5E \quad (E - x)(L + x) \cdot 9E$$

$$(f^4 + f^2)(f^4 - f^2) \cdot 7E \quad (1 + xE)(1 + xE) \cdot 7E$$

$$(2 + x)(2 - x)(1 + f^2) \cdot 7E \quad (x^2 - f^2)(x^2 + f^2) \cdot 0E$$

$$(u + z + f^2)(u - z + f^2) \cdot 61 \quad (8 - f^2)(8 + f^2) \cdot 81$$

$$(5 - x)(1 + xE) \cdot 2 \cdot 71 \quad (a + 1) \cdot (kE \cdot 91$$

$$(E - xE)(1 + xE) \cdot 51 \quad (1 + x)(E + x) \cdot 41 \quad (kE - 1) \cdot (kE \cdot 71$$

$$(E + kE + f^2 \Delta)(2 - \Delta) \cdot 71 \quad (2 - x)(2 + x) \cdot 5 \cdot 11$$

$$(8 + xE)(1 + xE) \cdot 01 \quad (kE - x)(kE + x) \cdot 6$$

$$(1 + xE)(5 - xE) \cdot 8 \quad (L - \Delta)(9 - \Delta) \cdot 7 \quad (f - x)(f + x) \cdot 5 \cdot 9$$

$$(5 + x)(4 + x) \cdot 5 \quad (4p + f^2)(kE - f^2)(E) \cdot 4$$

$$(2 - f^2)(xy)(kE - 1) \cdot xE \cdot 2 \quad (1 + xp) \cdot 2 \cdot 1$$

# Inventary: Factoring Skills

Sometimes it is necessary to use more than one factoring skill to factor certain algebraic expressions.

**Example** Factor  $a^4 - 5a^2 - 36$ .

**Solution**  $a^4 - 5a^2 - 36 = (a^2 - 9)(a^2 + 4)$  Think: Check to see if any factors can be factored again.  
 $= (a - 3)(a + 3)(a^2 + 4)$

## Exercise

**B** You must be able to recognize which skills must be used to factor a polynomial. When you factor the following polynomials, always check for a common factor first. There are at least two expressions that cannot be factored. Can you find others?

- |                                |                                  |                         |
|--------------------------------|----------------------------------|-------------------------|
| 1. $3a^2 + 6a$                 | 2. $2x - 8xy$                    | 3. $36a^3 - 4a^2$       |
| 4. $25a^4 - 9y^4$              | 5. $x^2 + 7x + 12$               | 6. $3a^2 - 3b^2$        |
| 7. $y^2 - 11y + 28$            | 8. $16x^2 - 8x + 1$              | 9. $a^2 - ab - 56b^2$   |
| 10. $4x^2 - 11x + 6$           | 11. $-1 + 9k^2$                  | 12. $1 + 18y + 32y^2$   |
| 13. $2y^2 - 8y^3$              | 14. $x^2 + 6x + 8$               | 15. $56x^2 + 9x - 2$    |
| 16. $-16 - 9x^2$               | 17. $16 - 28x + 10x^2$           | 18. $m^4 - 16$          |
| 19. $8 - 14y + 5y^2$           | 20. $-(1 - a^4)$                 | 21. $m^4 - 5m^2 - 36$   |
| 22. $6a^2 + 5a + 1$            | 23. $x^4 - y^4$                  | 24. $p^2 - 2pq - 63q^2$ |
| 25. $m^4 + 3m^2 - 4$           | 26. $x^2 - xy$                   | 27. $x^2 + 3xy - x$     |
| 28. $a^2 - 144$                | 29. $3a^2 - 36a + 36$            | 30. $(a + b)^2 - c^2$   |
| 31. $-a^2 - 2ab - b^2$         | 32. $x^3 + 5x^2 - 6$             | 33. $x^4 + 18x^2 + 32$  |
| 34. $m^4 - 9m^2 - 112$         | 35. $x^8 - 1$                    | 36. $2y^2 - 2y - 24$    |
| 37. $2x^2 - 8$                 | 38. $4y^2 + 8y - 60$             | 39. $m^4 - 16$          |
| 40. $2x^2 - 16x + 32$          | 41. $x^3 - xy^2$                 | 42. $x^4 - 5x^2 + 4$    |
| 43. $-48 - 3y^2$               | 44. $x^2y^3z - 2xy^2$            |                         |
| 45. $(x - y)^2 - (x + y)^2$    | 46. $9(a + b)^2 - (a - b)^2$     |                         |
| 47. $(a - b)^2 - 16(a + 2b)^2$ | 48. $25(2x + 1)^2 - (9x - 1)^2$  |                         |
| 49. $4(x - y)^2 - 16(x + y)^2$ | 50. $25(x + 2y)^2 - 9(x - 2y)^2$ |                         |

(4)  $8 + x)(c + x)2) 805 (c + x)(c + x)(c - 6)$   
 $(9 + x)(b + x61) 89 (q + x)(q + x)5x - 74$   
 $(q + x)(q + x)2) 99 axy - 5p (c - z)(x) cx 7p$   
 $(x^4 + 91)x - 7p (1 - x)(c - x)(1 + x)(c + x) 74$   
 $(x - x)(x^2 + x) 7p x^4 - x)(c - x)(c + x)(c - x)(c$   
 $+ x)(b + x) 66 (c - d)(c + d) 8c (c - x)(c + x)(c$   
 $(c + d)(b - d) 9c (1 - x)(1 + x)(1 + x^2)(1 + x^2) 5c$   
 $(b - x)(b + x)(c + x)(c + x)(c + x)(c + x)(c + x) 5c$   
 $aq 10000a 7c (q + a) - 7c (p - q + a)(c + q + a) 70c$   
 $(c1 + c21 - c^2)p^2 6c (c1 - d)(c1 + d) 8c$   
 $(1 - c)(c + x)x 7c (f - x)x 9c (1 - x)(1 + x)(c + x) 5c$   
 $(b2 + d)(b6 - d) 7c (f - x)(f + x)(f^2 + x^2) 7c$   
 $(1 + c2)(1 + c) 7c (c - x)(c + x)(c + x) 7c$   
 $(1 - x)(1 + x)(1 + x^2) 7c (f - x)(f + x)(f^2 + x^2) 7c$   
 $(c - x)(c + x)(c + x) 81 (x - z)(xc - p)x 71$   
 $pa10000y 91 (c + x)(1 - x) 51$   
 $(f + x)(c + x) 71 (f^4 - 1) c^2 (c1 (c9) + 1)(c2 + 1) 71$   
 $(1 + x)(1 - x)(1 - x)(1 - x)(1 - x)(1 - x)(1 - x) 6$   
 $(1 - x)(1 - x)(1 - x)(1 - x)(1 - x)(1 - x)(1 - x) 9$   
 $(f + x)(c + x) 5 (f^4c + c^5)x (f^4c - c^5) 7$   
 $(1 - x)(1 - x)(1 - x)(1 - x)(1 - x)(1 - x)(1 - x) 7$