ASSIGNMENT Polynomial Word Problems Name:

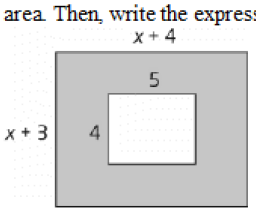
1. Donna wants to paint the walls of her bedroom. The dimensions of each wall are $(x + 1)$ by $(4x + 1)$.
   a) Write a quadratic expression that represents the area of one wall.
   b) Write a quadratic expression that represents the area of the entire room with four walls.
   c) If $x = 2$ m, what is the area of the entire room?
   d) If paint costs \$0.50/m², how much will it cost Donna to paint her room?

2. The surface area of a rectangular door locker is represented by $x^2 + 6x - 16$.
   a) What are the dimensions of the locker door?
   b) Find the actual dimensions if $x = 80$ cm.

3. The total area of the backyard is $4x^2$. Keith mowed a portion of the yard that is 2 m by 2 m. He left the rest of the yard for his father to mow.
   a) Write an expression to represent the area left for his father to mow.
   b) Factor the expressions from part a) to find expressions for the dimensions of a rectangle with an equal area to the remaining area of the backyard.
   c) Find the actual dimensions of the rectangle if $x = 2$ m.

4. The perimeter of a college campus is 70 m. The area is represented by $x^2 + 15x - 16$. Find the actual dimensions of the campus.

5. a) Find all the values of b so that $x^2 + bx + 15$ can be factored over the integers.
   b) Find all the values of b so that $4x^2 + bx + 13$ can be factored over the integers.
   c) Write an algebraic expression for the shaded area. Then, write the expression in factored form.



6. The volume of a prism is given by the expression $18\pi x^3 + 48\pi x^2 + 32\pi x$. If the prism does not have a triangular base, what shape could the prism be and what are its dimensions?

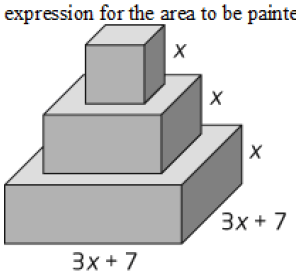
7. When the McKay family has a reunion, it is a tradition that each family member shakes hands with each of the other family members. Last year there were a total of 36 handshakes. The total number of handshakes H possible for x people is given by $H = \frac{x^2 - x}{2}$. How many people were at the McKay family reunion last year?

8. A magazine editor has asked a graphic artist to increase the length and width of a photograph in an advertisement by the same amount in order to double the area of the photograph. If the photograph was 12 cm long by 8 cm wide, what are the dimensions of the enlarged photograph?

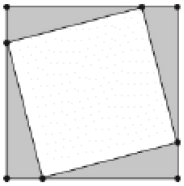
9. The area of a square garden in m² is represented by the expression $169 - 182x + 49x^2$. What is the smallest possible perimeter of the garden if x is a positive integer?

10. a) The area of an unknown shape is represented by the expression $16x^2 - 104x + 169$. If x represents an integer, what shape could this figure be?
    b) What is the smallest possible area of the figure?
    c) What is the smallest possible perimeter of the figure?

11. A pedestal for trophies is made up of three layers, each in the shape of a square-based prism. All three layers are the same height, x cm, but each base length is 2 cm less than the one immediately below it. The exposed surface area is to be covered with gold paint. Write and simplify an algebraic expression for the area to be painted.



12. The figure below consists of a small square inside a larger one. The difference in their areas (the shaded part) is 17 cm². The sum of the perimeters of the two squares is 68 m. How long are the sides of the larger square?



ANSWERS

1. ANS:
   a) $(x+1)(4x+1)=4x^2+5x+1$
   b) $4(x+1)(4x+1)=16x^2+20x+4$
   c) $108m^2$
   d) $54
2. ANS:
   a) Length = x+8 Width = x – 2
   b) L=88cm, W=78cm
3. ANS:
   a)$4x^2 – (2)(2)$
   b) $4(x^2 – 1)=4(x-1)(x+1)$
   L=4(x-1) and W=x+1
   or L=(x-1) and W=4(x+1)
   c) L=4m , W=3m
   or L=1m, W=12m
4. ANS:
   Factor (x+16)(x-1)
   L=x+16 and W=x-1
   P=2L+2W
   70=2(x+16)+2(x-1)
   70=4x + 30
   40=4x
   x=10
   Therefore L=26m and W=9m
5. ANS:
   a) The possible number pairs with a product of 15 are 3 and 5, –3 and –5, 1 and 15, and –1 and –15. For each pair, b is the sum, i.e., 8, –8, 16, and –16.
   b) The possible factors are as follows:
   $(2x+1)(2x+13)$ which gives $b = 28$
   $(2x-1)(2x-13)$ which gives $b = -28$
   $(4x+1)(x+13)$ which gives $b = 53$
   $(4x-1)(x-13)$ which gives $b = -53$
   $(4x+13)(x+1)$ which gives $b = 17$
   $(4x-13)(x-1)$ which gives $b = -17$
   c) The area of the outer rectangle = $(x+3)(x+4)$
   $= x^2 + 7x + 12$
   The unshaded area = 4×5
   $= 20$
   The shaded area = $x^2 + 7x + 12 - 20$
   $= x^2 + 7x - 8$
   $= (x+8)(x-1)$
6. ANS:
   Answers may vary.
   Since the volume of a prism is given by area of the base × the height, factor the algebraic expression:
   $V = 18\pi x^3 + 48\pi x^2 + 32\pi x$
   $= 2\pi x(9x^2 + 24x + 16)$
   $= 2\pi x(3x+4)^2$
   Scenario 1: This expression could represent the volume of a square-based prism, whose height is $2\pi x$ and each side of the square base is $(3x+4)$.
   Scenario 2: This expression could also represent the volume of a rectangular-based prism, with the area of the base being $2\pi x(3x+4)$ and the height being $(3x+4)$.
   Scenario 3: This expression can also be written in the form $V = \pi r^2 h$
   $= \pi(3x+4)^2(2x)$
   This format can represent the volume of a cylinder with $r = (3x+4)$ and $h = 2x$.
7. ANS:
   Substitute 36 handshakes for *H:*
   $\frac{x^2 - x}{2} = 36$
   $x^2 - x = 72$
   $x^2 - x - 72 = 0$
   $(x+8)(x-9) = 0$
   There are 36 handshakes when $x = -8$ and $x = 9$. There could not have been -8 people at the reunion, so there were 9 people at the reunion.
8. ANS:
   If the width and length are each extended by x, the new dimensions, in centimetres, are $(x+12)$ and $(x+8)$.
   Since the area has doubled,
   $(x+12)(x+8) = 2(12)(8)$
   $x^2 + 20x + 96 = 192$
   $x^2 + 20x - 96 = 0$
   $(x+24)(x-4) = 0$
   The area is doubled when $x = -24$ and $x = 4$. The photograph's dimensions cannot be increased by –24 cm, so the dimensions should be increased by 4 cm, and the new dimensions will be 16 cm and 12 cm.

9. ANS:

$A = 169 - 182x + 49x^2$

$= (13 - 7x)^2$

The side of the square is $(13 - 7x)$ m, and the perimeter is $4(13 - 7x)$ m. Since x is a positive integer, its smallest possible value is 1. When $x = 1$, the perimeter $= 4(13 - 7)$

$= 24$

The smallest possible perimeter is 24 m.

10. ANS:

a) $A = 16x^2 - 104x + 169$

$= (4x - 13)^2$

Because the two sides are equal in length, the shape is a square.

b) Since $(4x - 13)$ must be positive, $4x$ must be greater than 13, so x must be at least 4. When $x = 4$,

$A = (16 - 13)^2$

$= 9$

The smallest possible area is 9 square units.

c) The perimeter is given by $4(4x - 13)$. When $x = 4$, the perimeter $= 4(16 - 13)$

$= 12$

The smallest possible perimeter is 12 units.

11. ANS:

The bottom prism has a square base with sides measuring $(3x + 7)$ cm, the middle prism has a square base with sides measuring $(3x + 5)$ cm, and the top prism has a square base with sides measuring $(3x + 3)$ cm.

The exposed area on the top prism is

$A = Area_{\text{top}} + 4(Area_{\text{side}})$

$= (3x + 3)^2 + 4(3x + 3)(x)$

$= (9x^2 + 18x + 9) + 12x^2 + 12x$

$= 21x^2 + 30x + 9$

The exposed area on the middle prism is

$A = (Area_{\text{top}} - Area_{\text{base of top prism}}) + 4(Area_{\text{side}})$

$= [(3x + 5)^2 - (3x + 3)^2] + 4(3x + 5)(x)$

$= 9x^2 + 30x + 25 - 9x^2 - 18x - 9 + 12x^2 + 20x$

$= 12x^2 + 32x + 16$

The exposed area on the bottom prism is

$A = (Area_{\text{top}} - Area_{\text{base of middle prism}}) + 4(Area_{\text{side}})$

$= [(3x + 7)^2 - (3x + 5)^2] + 4(3x + 7)(x)$

$= 9x^2 + 42x + 49 - 9x^2 - 30x - 25 + 12x^2 + 28x$

$= 12x^2 + 40x + 24$

The total exposed surface area is

$= 21x^2 + 30x + 9 + 12x^2 + 32x + 16 + 12x^2 + 40x + 24$

$= 45x^2 + 102x + 49$

The total area to be painted is $45x^2 + 102x + 49$ cm^2.

12. ANS:

If the length of the side of the larger square is x cm, and the length of the side of the smaller square is y cm, $4x + 4y = 68$.

The area of the shaded part is represented by the equation $x^2 - y^2 = 17$ or $(x + y)(x - y) = 17$.

$4x + 4y = 68$

$x + y = \frac{68}{4}$

$= 17$

Substitute $x + y = 17$ into $(x + y)(x - y) = 17$:

$17(x - y) = 17$

$x - y = 1$

$x = y + 1$

Substitute $x = y + 1$ into $x + y = 17$:

$(y + 1) + y = 17$

$2y = 16$

$y = 8$

Substitute $y = 8$ into $x = y + 1$:

$x = 9$

Each side of the larger square measures 9 cm.